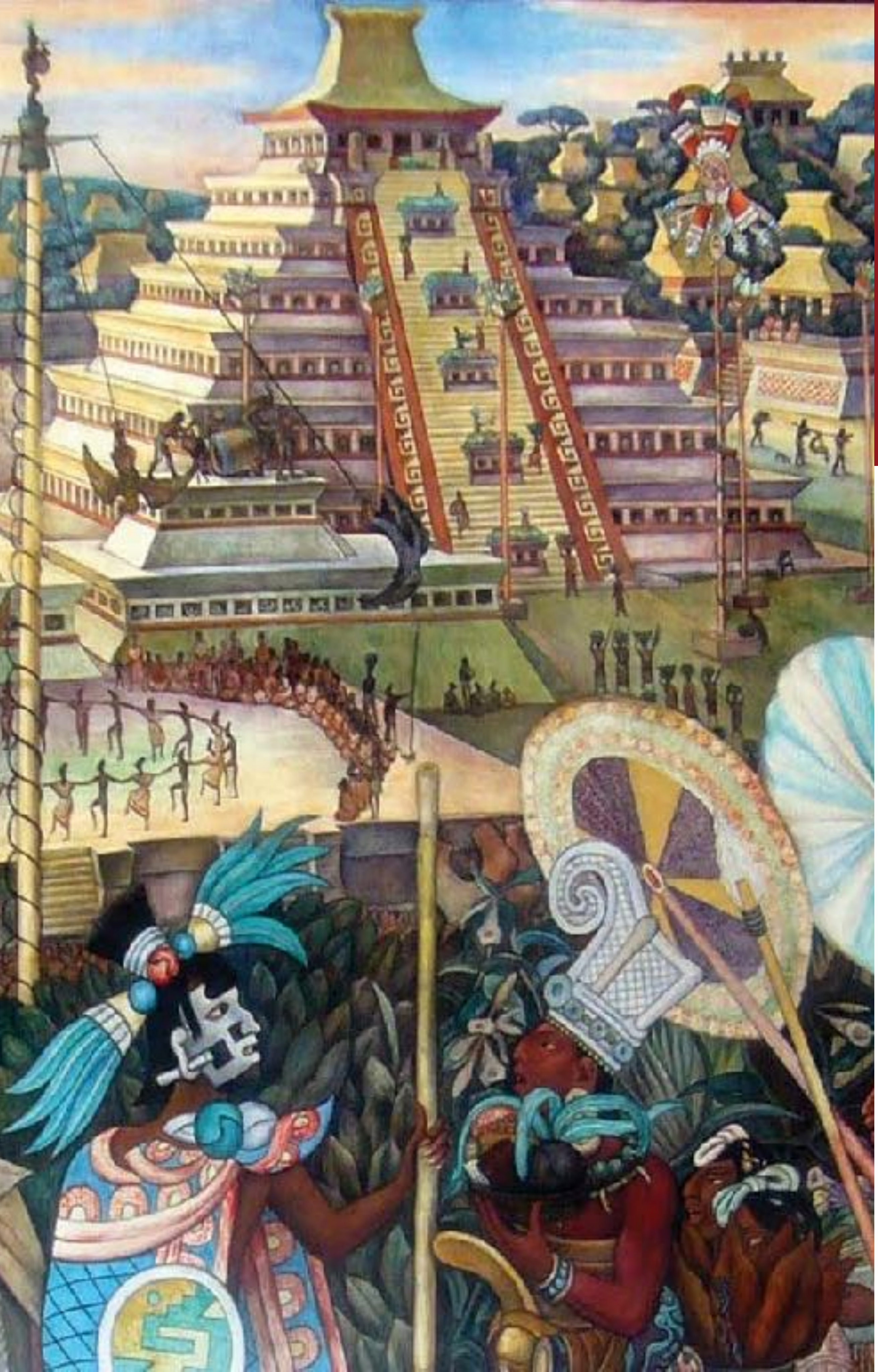

[Diego Rivera]

Gravitational Waves

Jose María Ezquiaga

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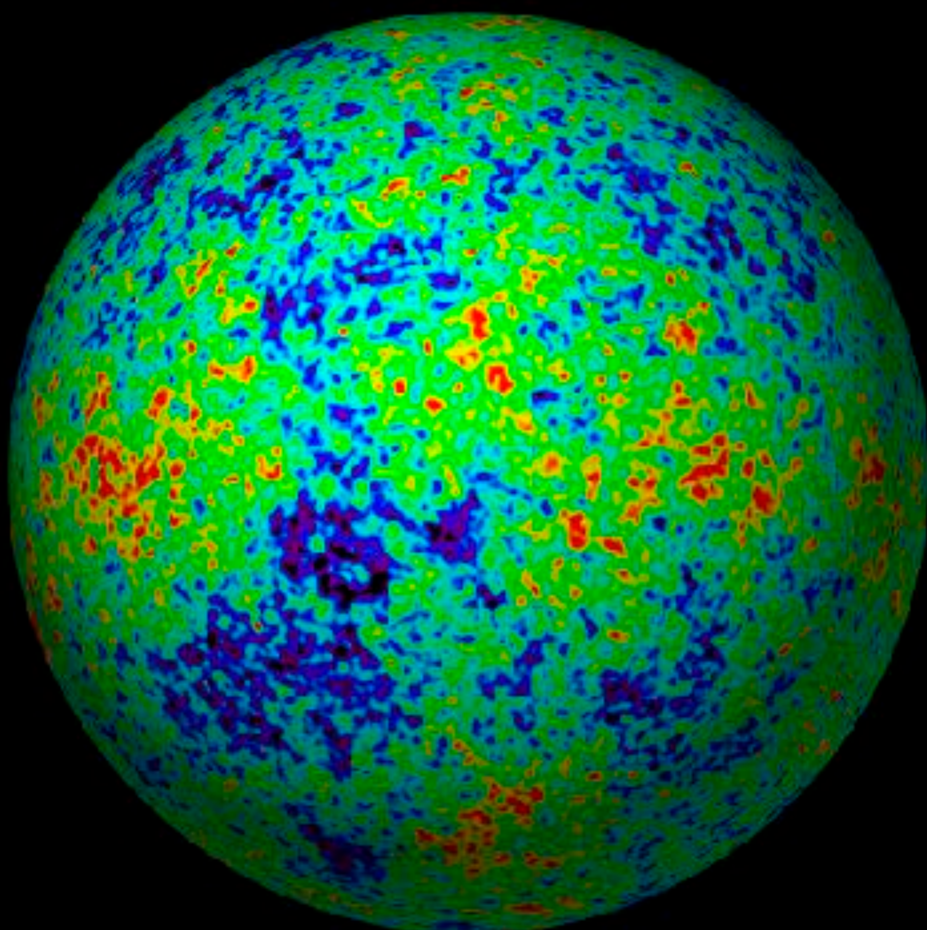
ezquiaga.github.io

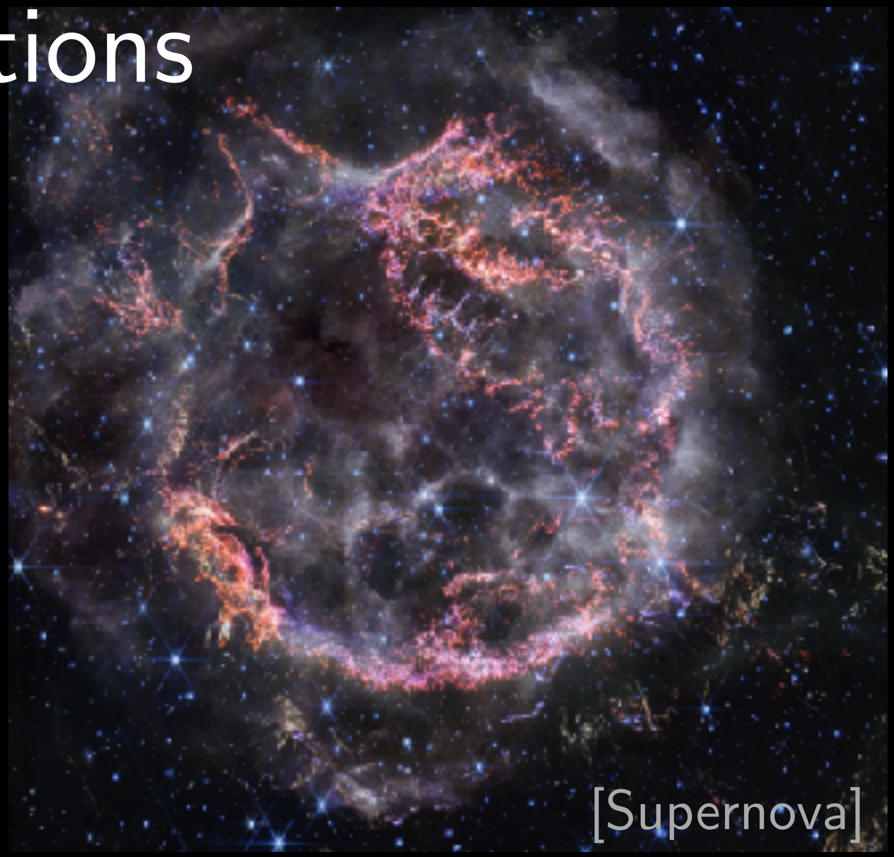
VILLUM FONDEN

KØBENHAVNS UNIVERSITET

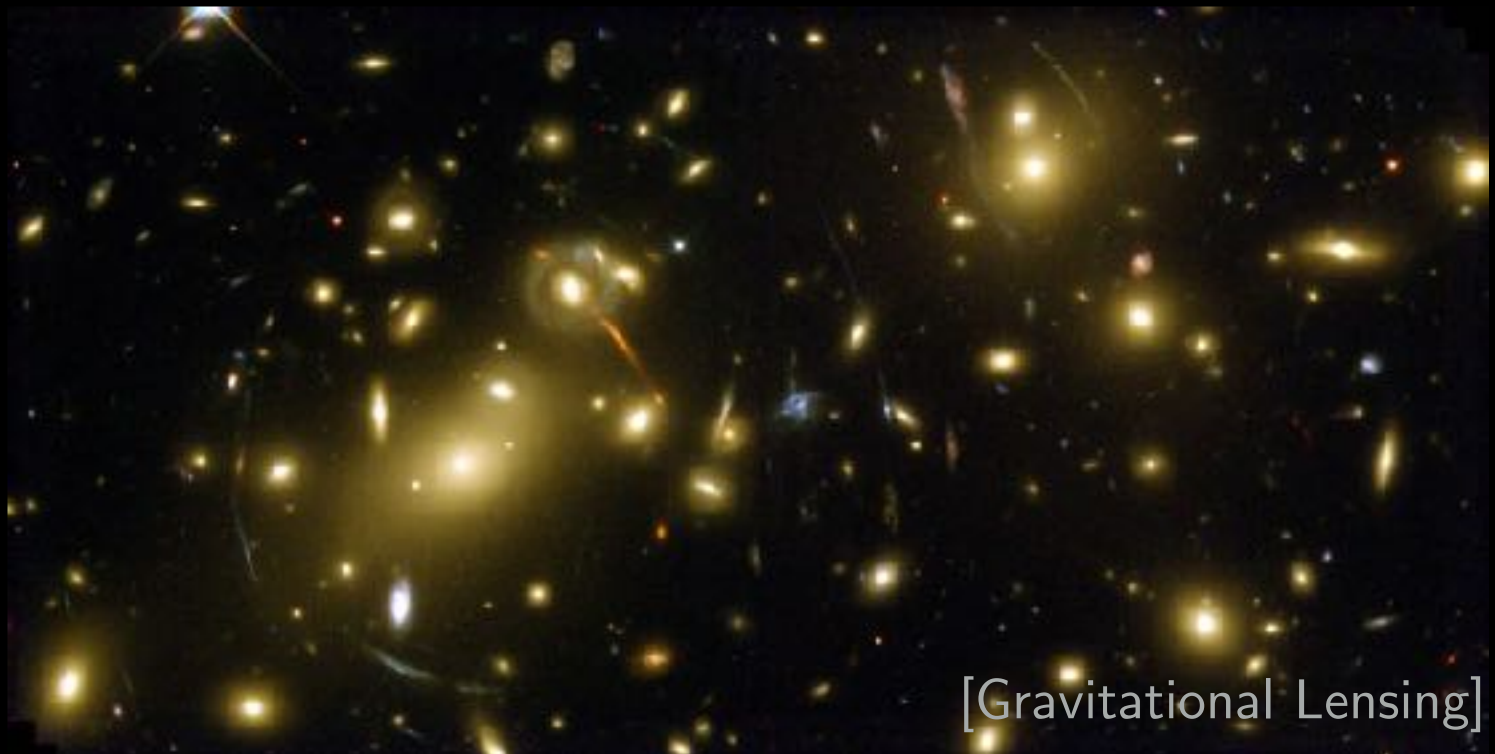
A plethora of cosmological observations



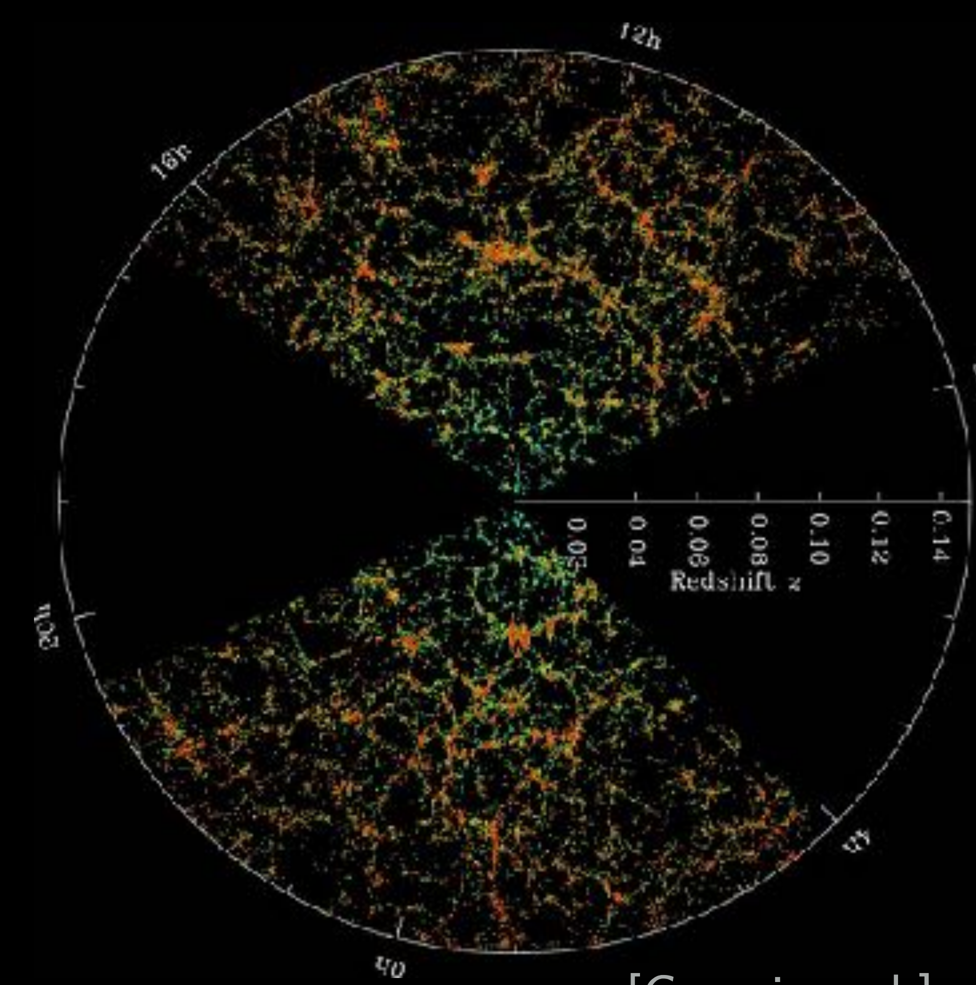
[Cosmic microwave background]



[Supernova]

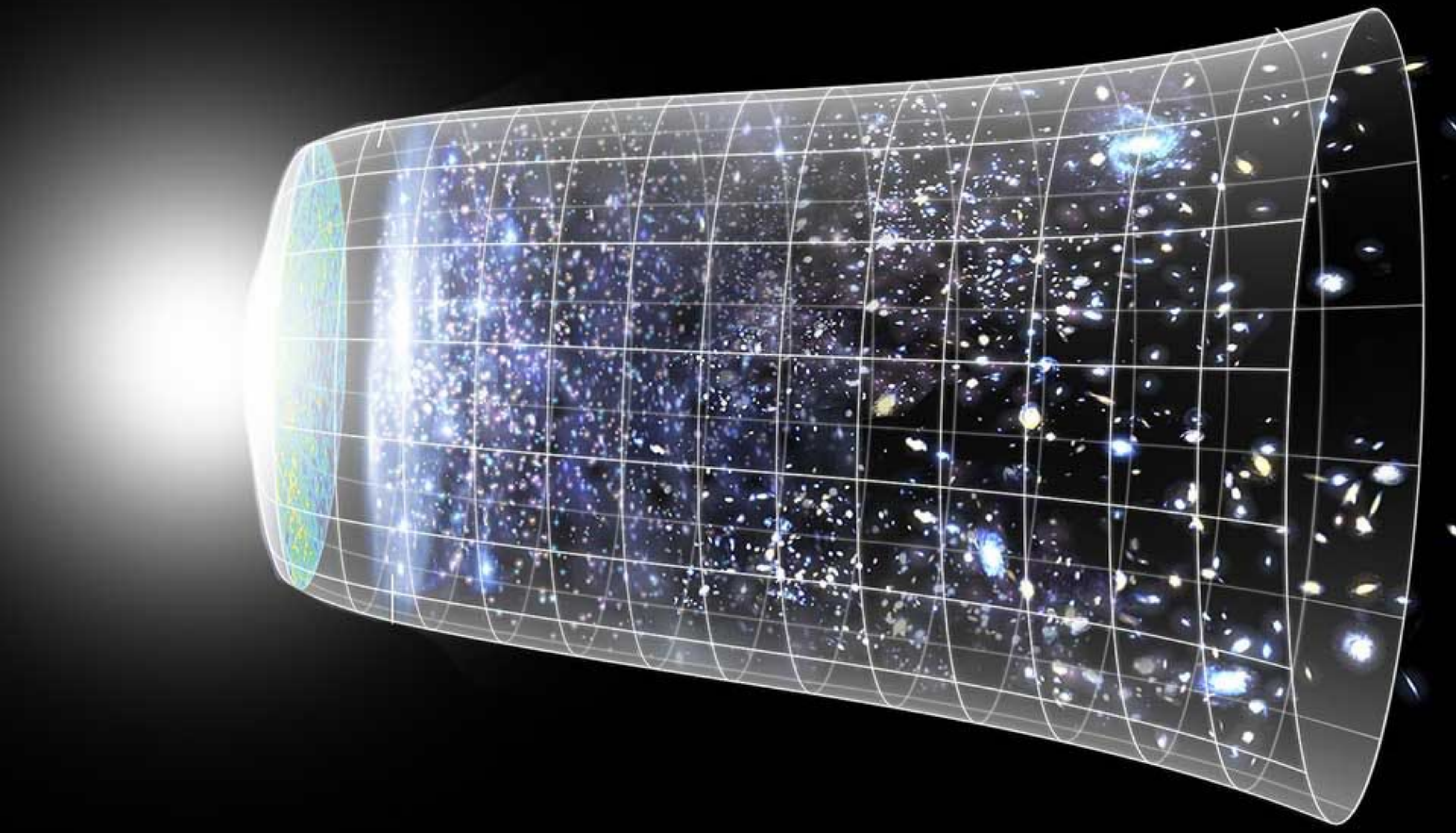


[Gravitational Lensing]



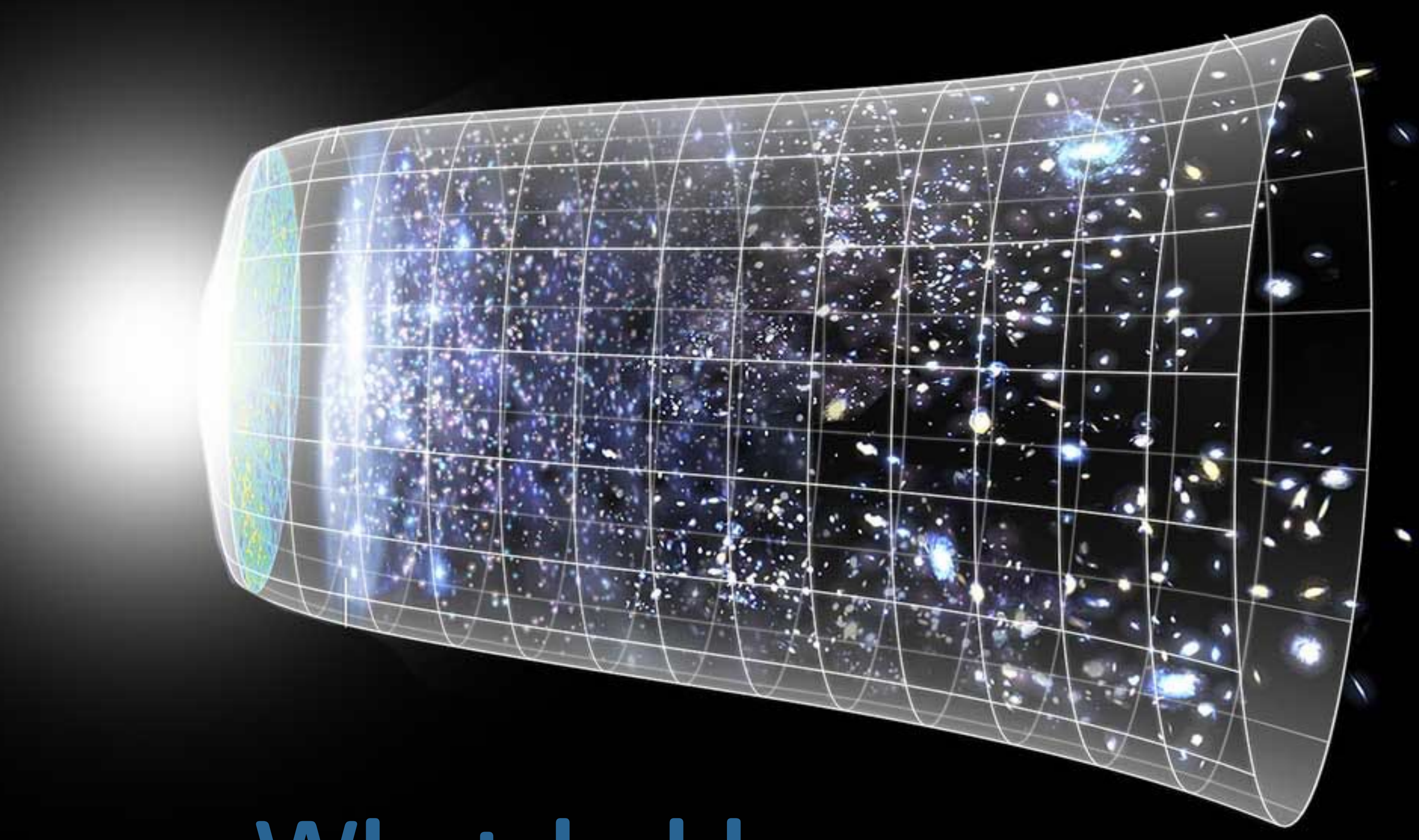
[Cosmic web]

The **standard** cosmological model...



...13.8 billion years of cosmic history

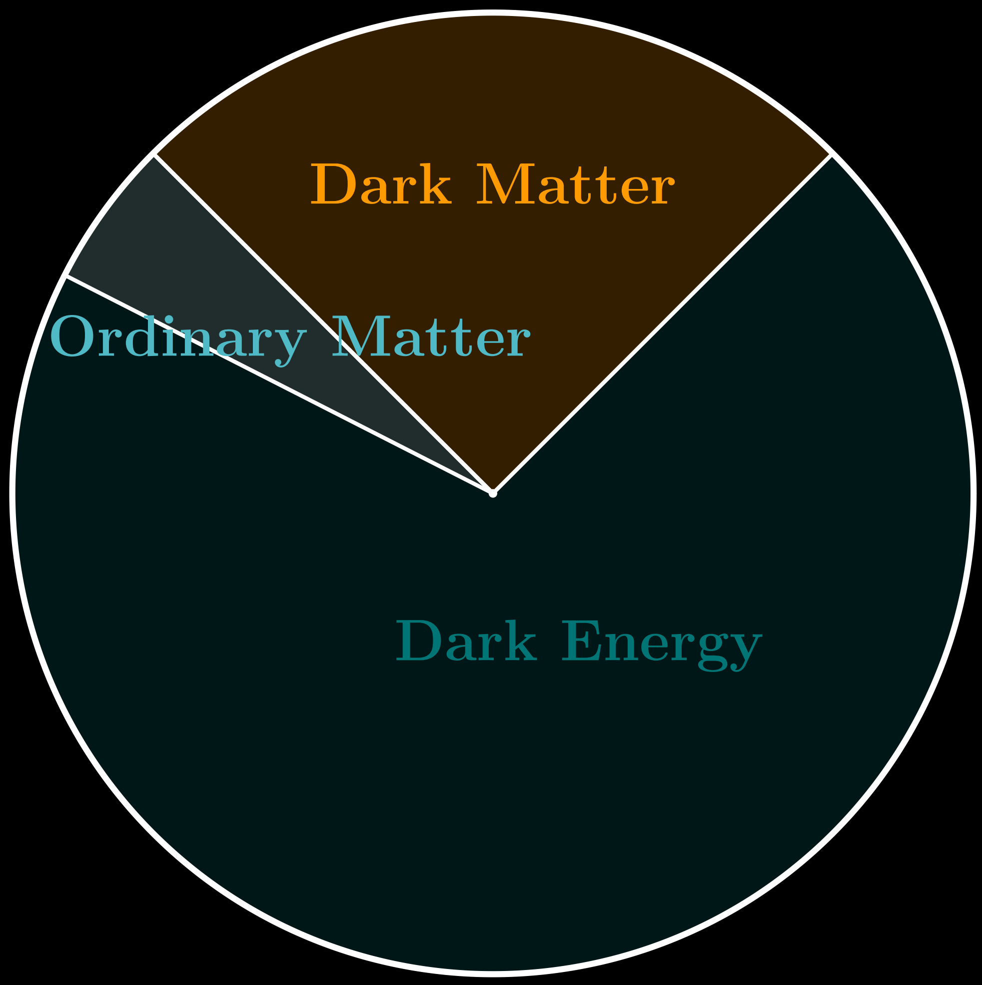
We **don't** understand the basics of our Universe



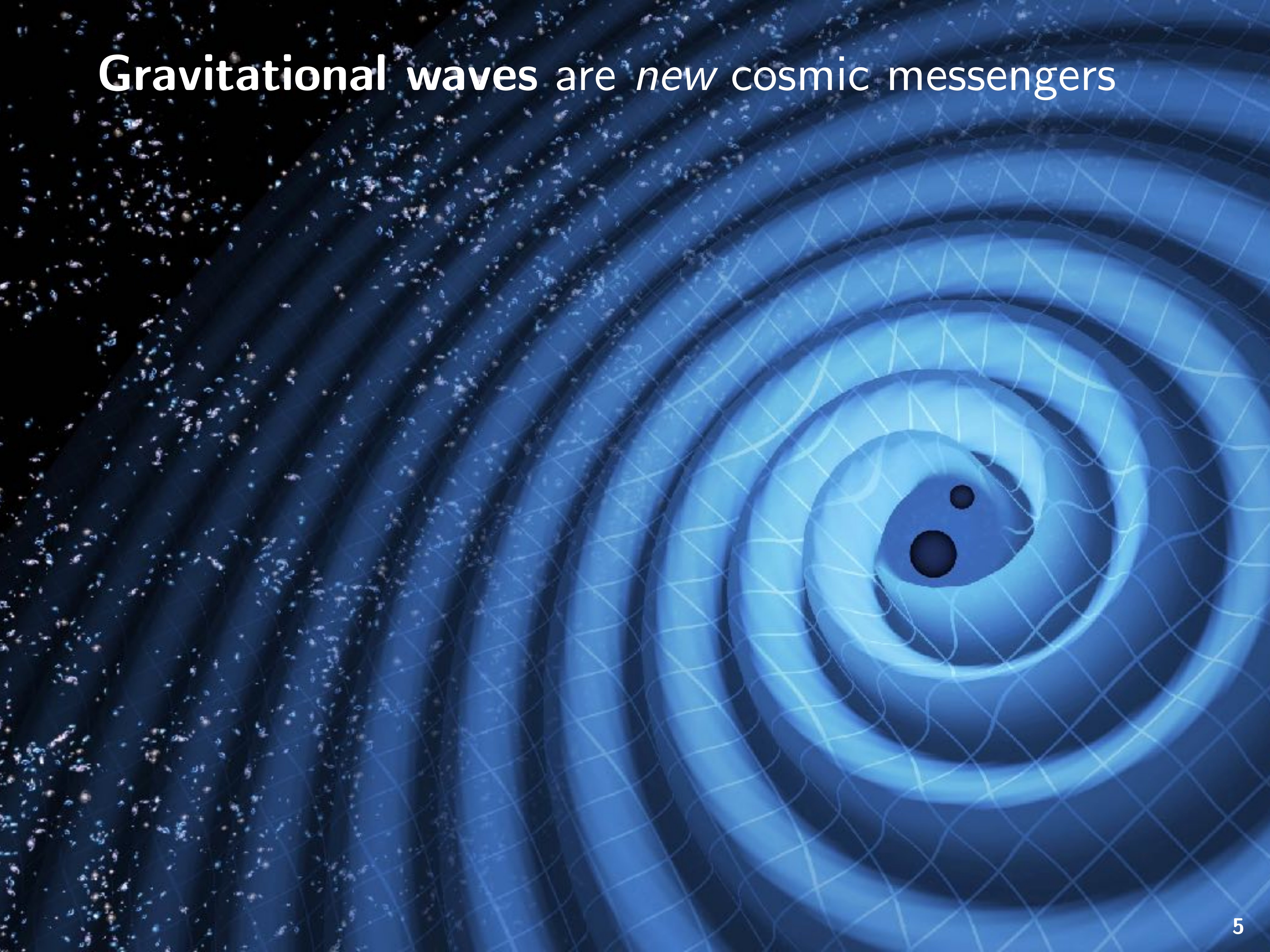
Why the Universe expands ever faster?

What holds galaxies together?

Is Einstein gravity valid at cosmological scales?

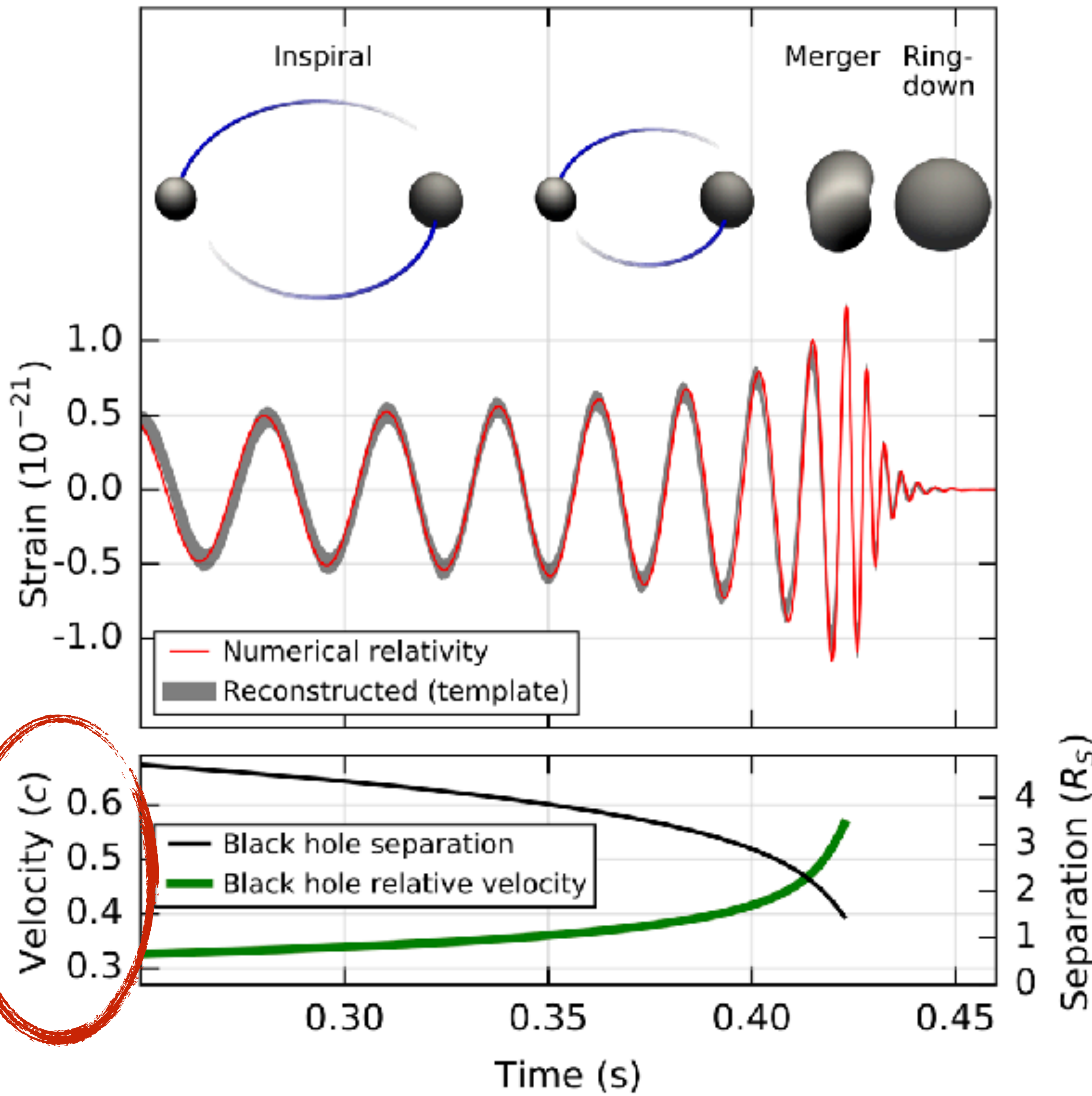


Gravitational waves are *new* cosmic messengers



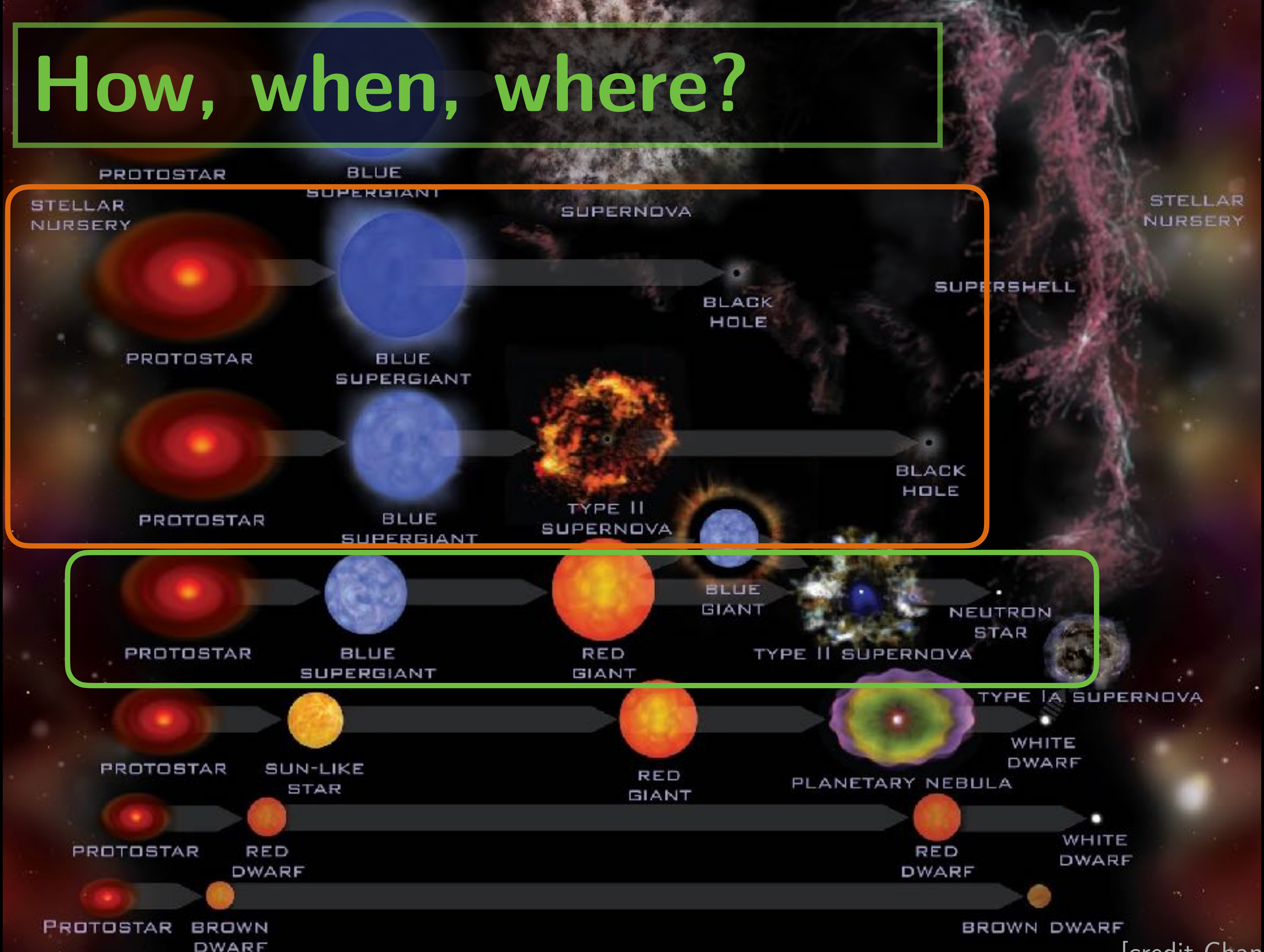
Gravitational waves from stellar-mass **binary black holes**

Strong-field gravity

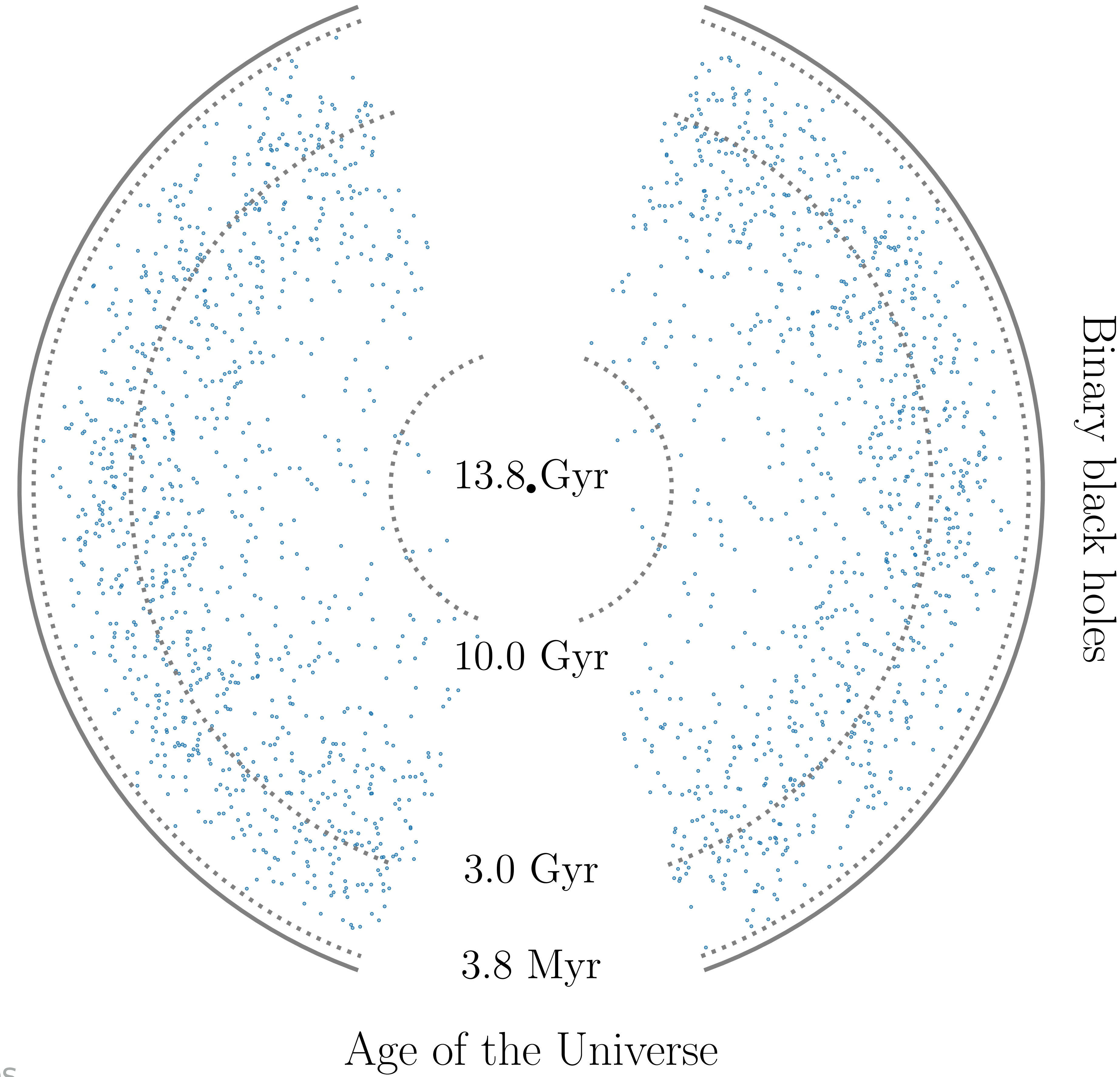


[First detection, GW150914]

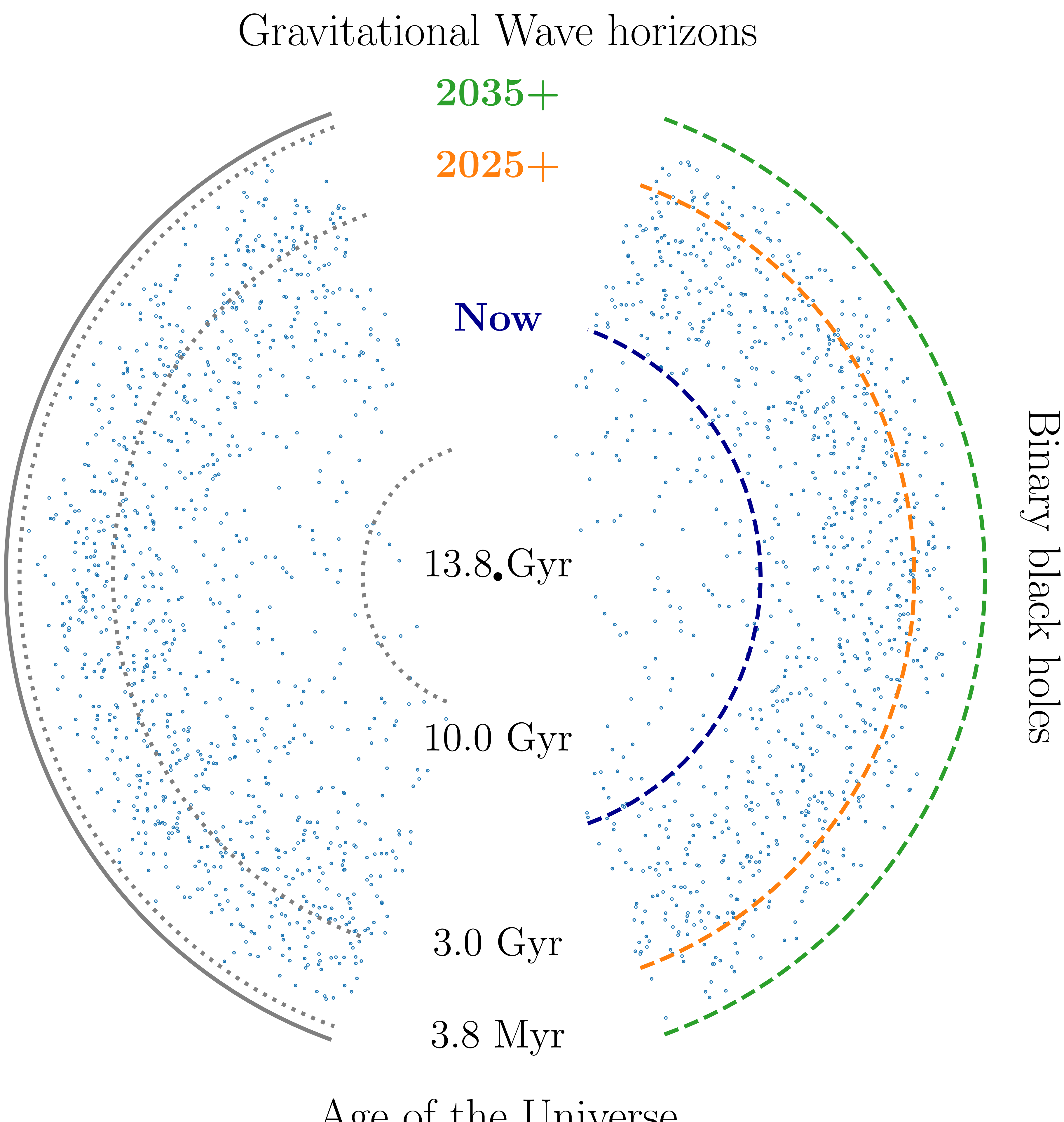
Stellar evolution



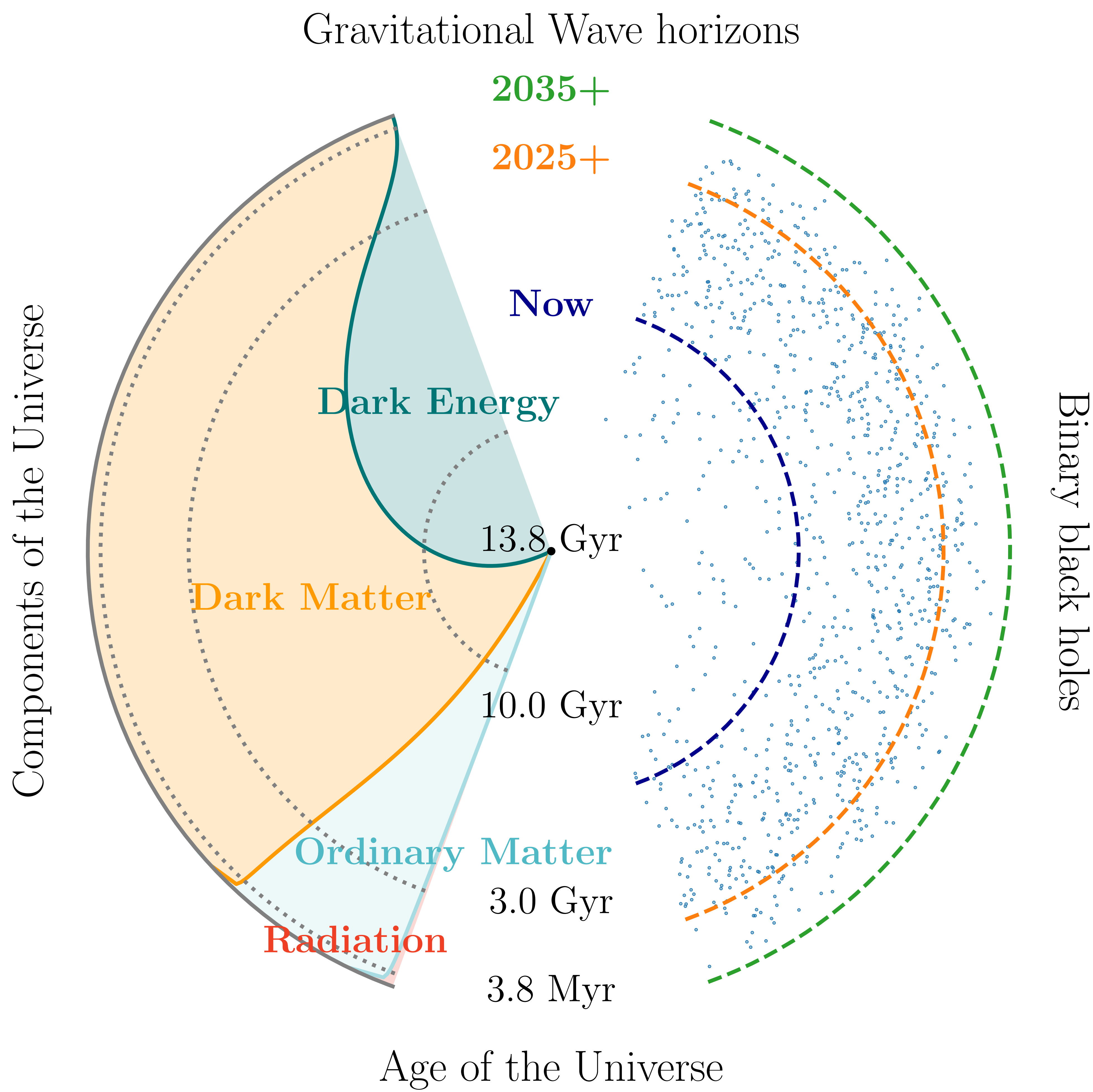
[credit Chandra]



*stellar mass binary black holes



Gravitational Wave horizons
2035+
2025+
Now
13.8 Gyr
10.0 Gyr
3.0 Gyr
3.8 Myr
Binary black holes
Age of the Universe



Gravitational Wave horizons
2035+
2025+
Now
Dark Energy
13.8 Gyr
Dark Matter
10.0 Gyr
Ordinary Matter
3.0 Gyr
Radiation
3.8 Myr
Components of the Universe
Binary black holes
Age of the Universe

The plan

0. Motivation: gravity, astrophysics, cosmology

1. A *crash-course* on gravitational waves

linearized Einstein's equations, quadrupole formula, compact binary coalescences

2. The new era of gravitational-wave astronomy

detectors, matched-filtering, data analysis, current observations, next generation detectors

3. *Standard siren* cosmology

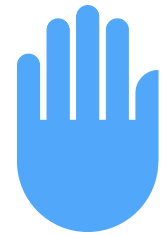
bright, dark and spectral sirens, status and future prospects

4. Gravitational wave *lensing*

lensing regimes (geometric/wave optics), current search efforts, science case

The plan - *warm up*

- Please, raise your hand if...



  - You are in the *first* year of your PhD
  - In your *second* year?
  - You have studied before a *course* with "gravitational waves" in the title
  - You have published a *paper* with the words "gravitational waves" written somewhere
  - In the *title*?
  - You have already seen the *monkeys* in the hotel :)

The plan - *practicalities*

- Please ask *questions*! (during and after the lectures)

- The goal of these lectures is to give an *overview* of gravitational wave astronomy and its application to cosmology
    - I will avoid technical derivations. Focus on compact binaries
    - There are many slides. No need to cover them all!

- Detailed derivations can be found in my lecture notes: ezquiaga.github.io/lectures/Lecture_Notes
    - Also references to seminal papers and books

- The slides contain references [in brackets] with links to papers/sources
    - QR code linking to the slides

- *Remember*, please ask *questions*!

(during and after the lectures)

1. A crash-course on gravitational waves

Gravitational waves in flat space

- Perturbations around Minkowski

$$g_{\mu\nu}(t,\vec{x}) = \eta_{\mu\nu} + h_{\mu\nu}(t,\vec{x})$$

$$|h_{\mu\nu}(t,\vec{x})| \ll 1$$

- Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

- Gravitational wave propagation

$$\boxed{\Box h_{\mu\nu} = -16\pi G \left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T \right)}$$

Gravitational wave properties

- Wave equation in vacuum $\Box h_{\mu\nu} = 0$
- Wave ansatz $h_{\mu\nu}(x) = \mathrm{Re}\left[A_{\mu\nu}(x) e^{i\theta(x)}\right]$ $k_\mu \equiv \partial_\mu \theta$

$A_{\mu\nu} \equiv A\epsilon_{\mu\nu}$

- Highly oscillatory phase: $\theta \to \theta/\varepsilon$
- Leading order: *gravitational wave follow null geodesics*

$$\eta_{\mu\nu} k^\mu k^\nu = 0$$

- Next to Leading order: *gravitons conserved + parallel transport*

$$\nabla^\mu (A^2 k_\mu) = 0 \qquad k^\alpha \nabla_\alpha \epsilon_{\mu\nu} = 0$$

Gravitational wave polarizations

- Counting degrees of freedom:

Symmetric 4D tensor $\epsilon_{\mu\nu} = \epsilon_{\nu\mu}$: **10**

Lorenz gauge $\nabla^{\mu} h_{\mu\nu} = 0$: **10 - 4 = 6**

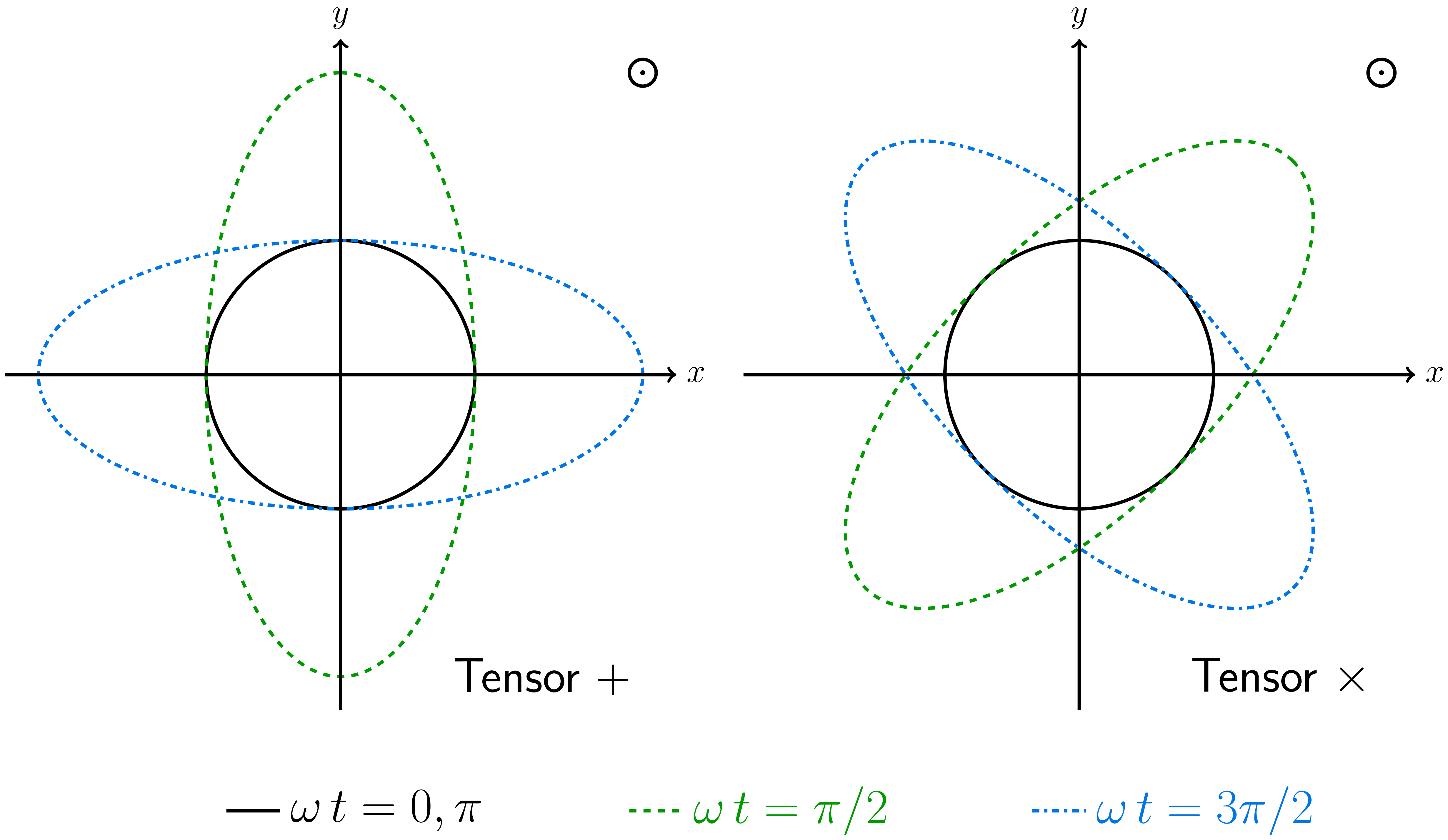
Residual gauge $\epsilon_{0\mu} = 0$: **10 - 4 - 4 = 2**

- Polarization decomposition:

$$\epsilon_{\mu\nu}(x) = \epsilon_{+}(x)\hat{\epsilon}^{+}_{\mu\nu} + \epsilon_{\times}(x)\hat{\epsilon}^{\times}_{\mu\nu}$$

$$\epsilon_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \epsilon_{+} & \epsilon_{\times} & 0 \\ 0 & \epsilon_{\times} & -\epsilon_{+} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Gravitational Wave Polarizations



Gravitational waves in curved space

- Perturbations around curved background

$$g_{\mu\nu} = g^{\mathrm{B}}_{\mu\nu} + h_{\mu\nu}$$

- Definition is not unique, short-wave approx.

$$\lambda_{\mathrm{gw}} \ll L_{\mathrm{B}} \sim |R^{\mathrm{B}}_{\alpha\beta\gamma\rho}|^{-1/2}$$

- We can fix the transverse-traceless gauge in vacuum $\nabla^{\mu} h_{\mu\nu} = h = 0$

- Wave equation

$$\Box h_{\mu\nu} + 2R^{\mathrm{B}}_{\mu\alpha\nu\beta} h^{\alpha\beta} = 0$$

New interactions! $\partial g^{\mathrm{B}}_{\mu\nu}$ $\partial\partial g^{\mathrm{B}}_{\mu\nu}$

Gravitational waves in cosmology

- Perturbations around homogeneous and isotropic backgrounds

$$g_{\mu\nu} = g_{\mu\nu}^{\rm FLRW} + h_{\mu\nu}$$

- GWs unambiguously defined + scalar-vector-tensor decomposition

- Wave equation in vacuum

$$\Box^{\rm FLRW} h_{ij} + 2R_{ijkl}^{\rm FLRW} h^{jl} = 0$$

$$\downarrow$$

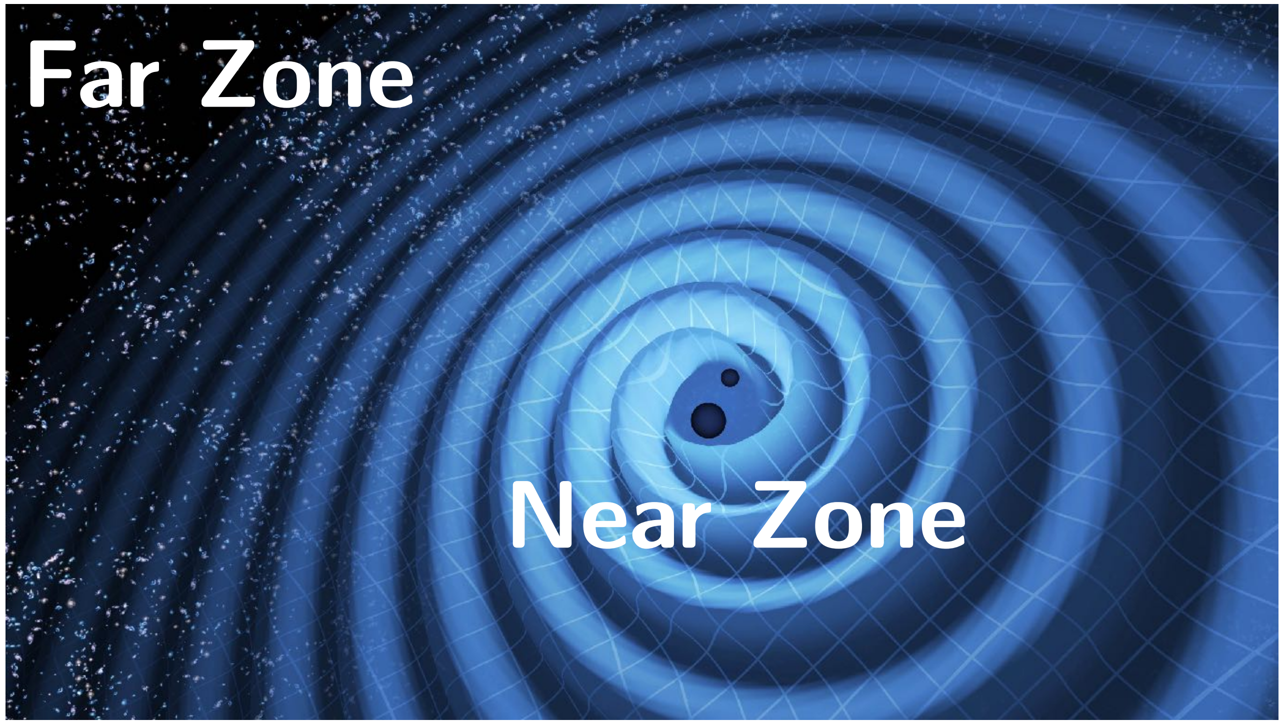
$$h''_{ij} + 2\mathcal{H}h'_{ij} + \nabla^2 h_{ij} = 0$$

$$\downarrow$$

$$h_{ij}(\eta, \mathbf{x}) \simeq \frac{1}{a(\eta)} h_{ij}^{\rm flat}(\eta, \mathbf{x})$$

Gravitational wave generation

- Different regimes



- Rewriting the field equations:

$$\Box \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu} + \mathcal{O}(h^2) \equiv -16\pi G \tau_{\mu\nu}$$

- Green's function solution:

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} h$$

$$\bar{h}_{\mu\nu}(t, \vec{x}) = 4G \int d^3x' \, \frac{\tau_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|}$$

Quadrupole formula

- Far zone solution: *expand large distances*
- Near zone solution: *expand small velocities v/c*
- Leading Newtonian limit: *match near and far zone solutions*

$$h_{ij}^{TT}(t,\vec{x}) = \frac{2G}{c^4 r}\frac{d^2 Q_{ij}^{TT}(t-r/c)}{dt^2}$$

Amplitude scales inversely with distance

Gravitational waves sourced by accelerated quadrupole moment

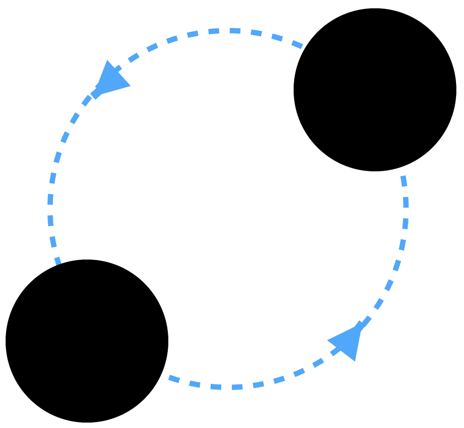
$$Q^{ij} \equiv \int d^3x \tau^{00}(x)\left(x^i x^j - \frac{1}{3}r^2\delta^{ij}\right)$$

Compact binary coalescence

- At leading order in post-Newtonian expansion

$$h_+(t) = h_c \left(\frac{1+\cos^2\iota}{2}\right) \cos\left[\Phi(t)\right]$$

$$h_\times(t) = h_c \cos\iota \, \sin\left[\Phi(t)\right]$$



$$\mathcal{M}_c = (m_1 m_2)^{3/5}/(m_1+m_2)^{1/5}$$

- Amplitude

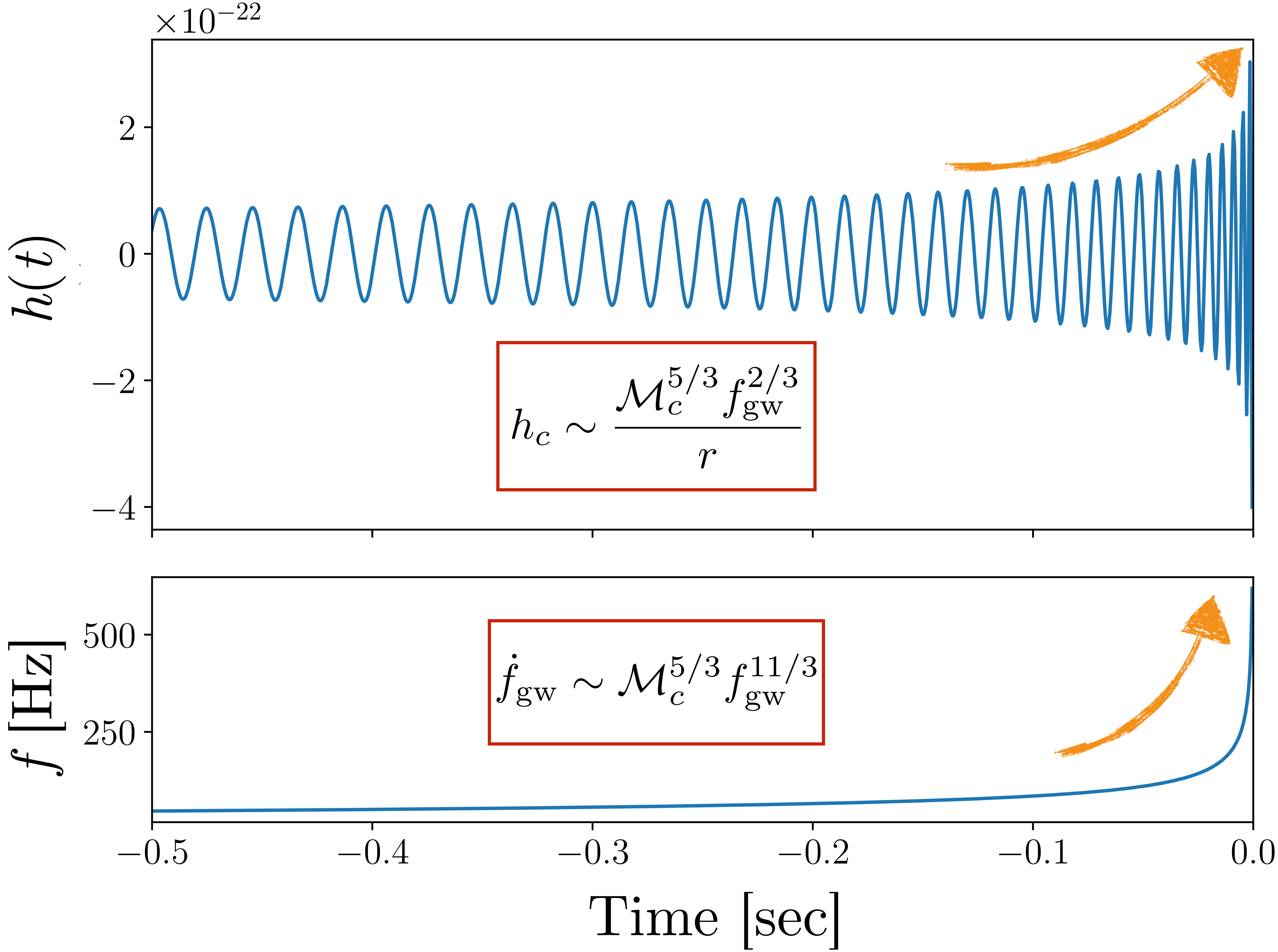
$$h_c \sim \frac{\mathcal{M}_c^{5/3} f_{\rm gw}^{2/3}}{r}$$

- Frequency

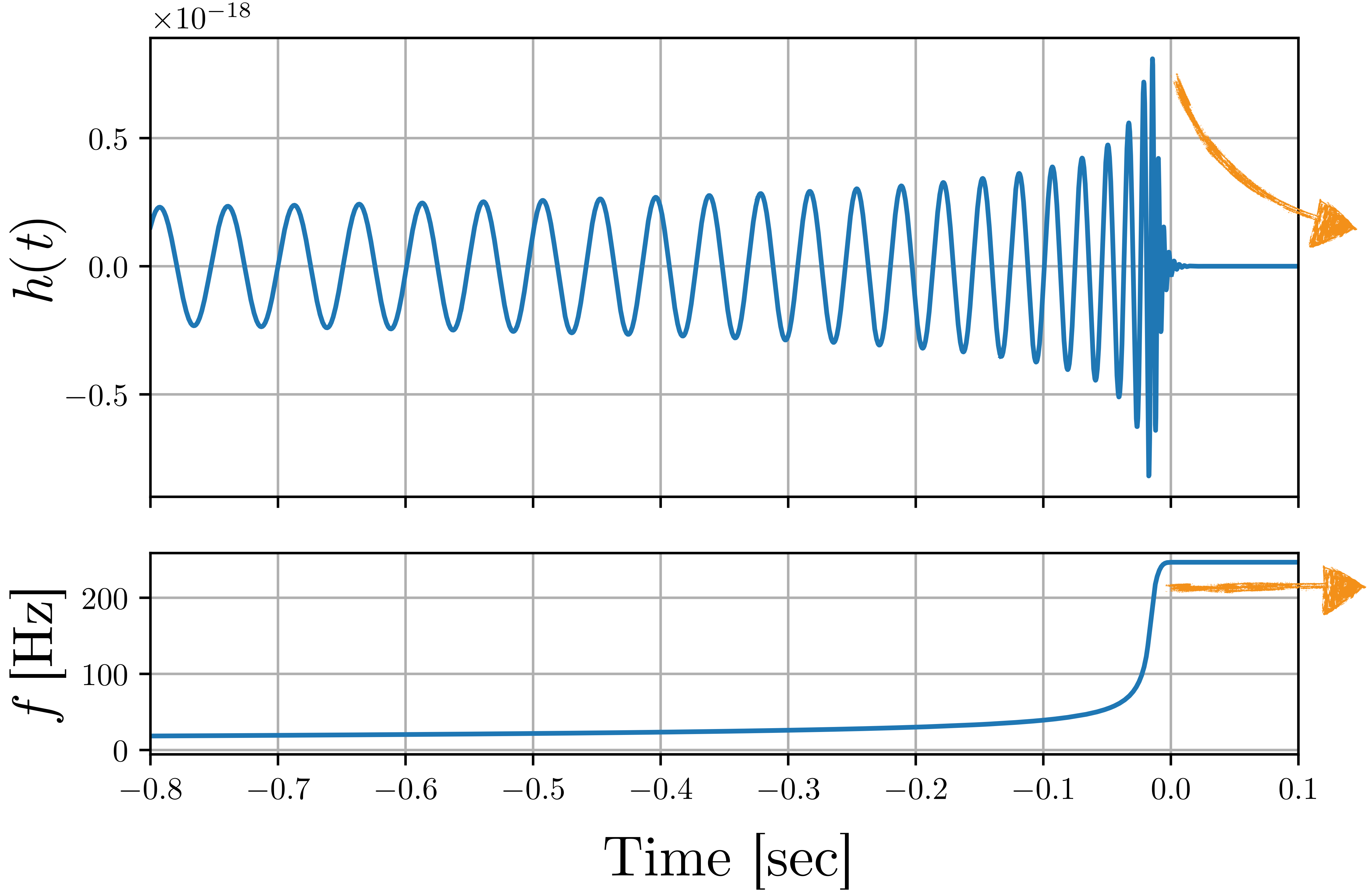
$$\dot{f}_{\rm gw} \sim \mathcal{M}_c^{5/3} f_{\rm gw}^{11/3}$$

$$f_{\rm gw} \sim d\Phi/dt$$

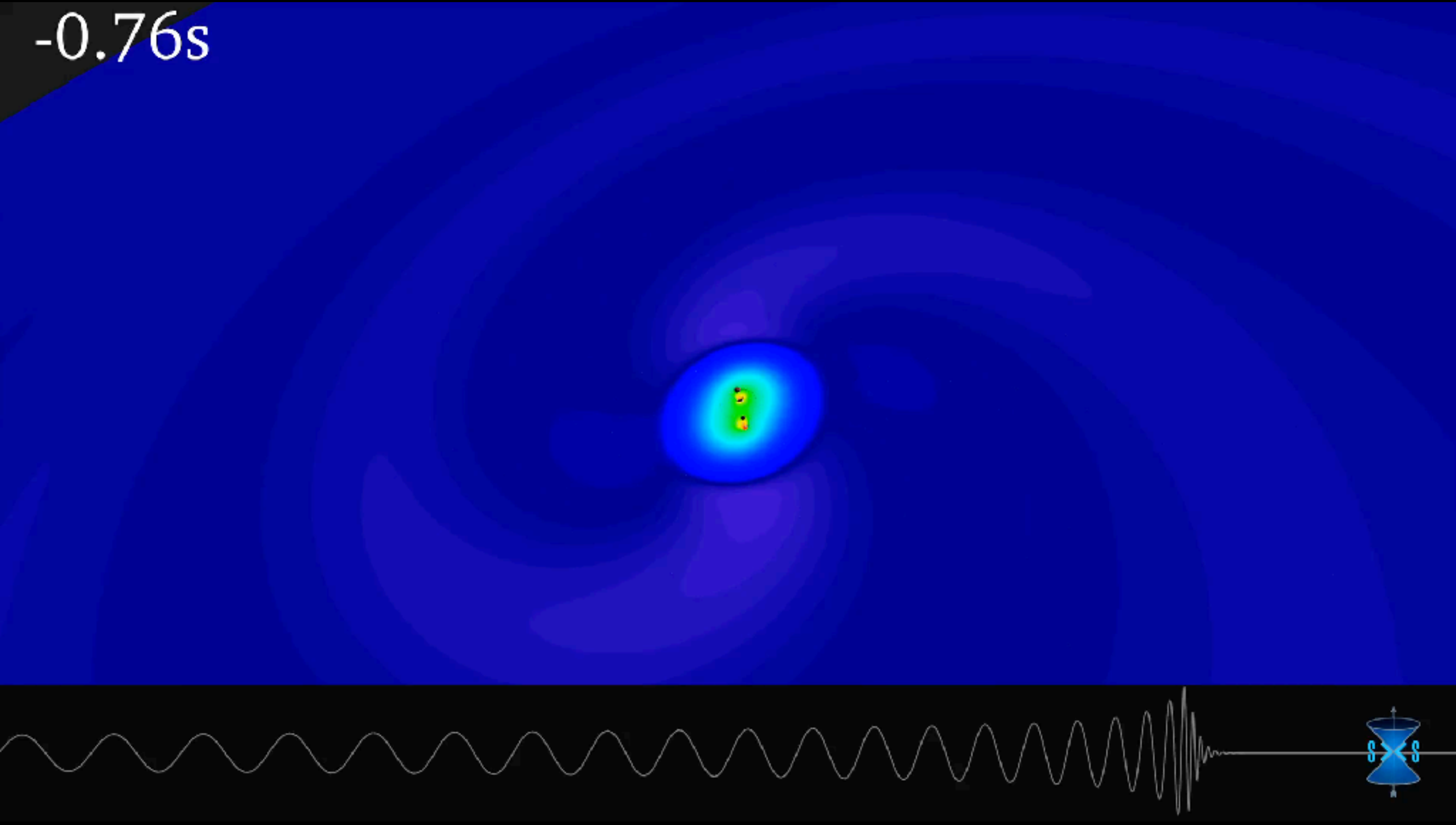
Inspiral - the "chirp"



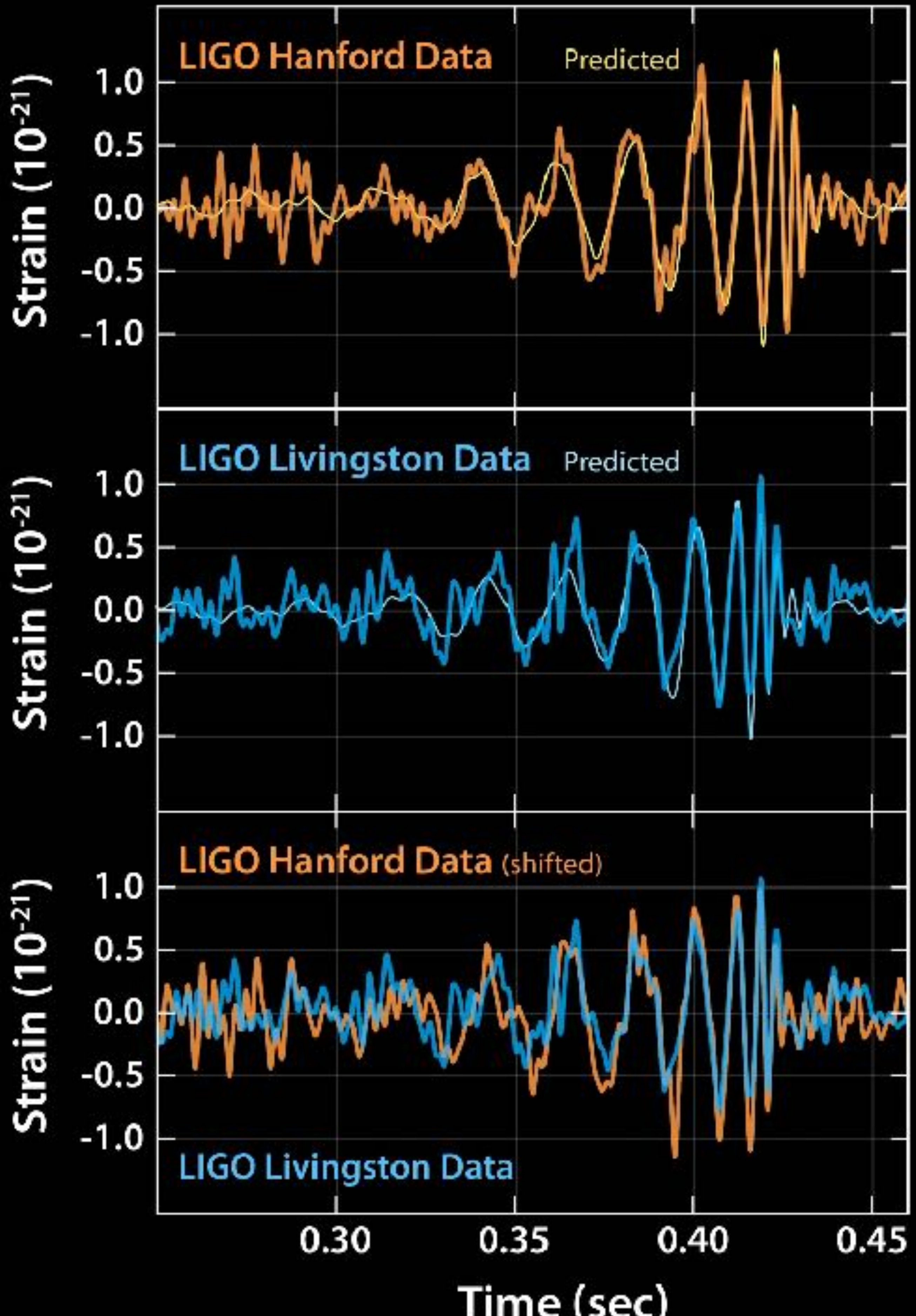
Inspiral-Merger-Ringdown



Numerical simulation of a binary black hole merger



[Credit: SxS Collaboration]


LIGO Hanford Data
Predicted
LIGO Livingston Data
Predicted
LIGO Hanford Data (shifted)
LIGO Livingston Data
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
0.30
0.35
0.40
0.45
Time (sec)

Cosmological compact binary coalescence

- Compact binaries at cosmological distances

$$h_c(t_{\rm obs}) \sim \frac{\mathcal{M}_c^{5/3} f_{\rm gw}^{2/3}}{a(t_{\rm obs})\, r}$$

$$f_{\rm gw} = (1+z) f_{\rm obs}$$

$$\mathcal{M}_z = (1+z)\mathcal{M}_c$$

$$h_c(t_{\rm obs}) \sim \frac{\mathcal{M}_z^{5/3} f_{\rm obs}^{2/3}}{d_{\rm L}^{\rm gw}}$$

$$d_{\rm L}^{\rm gw} = d_{\rm L}^{\rm em} = a_0(1+z)\int_0^{z_{\rm src}} \frac{dz}{H(z)}$$

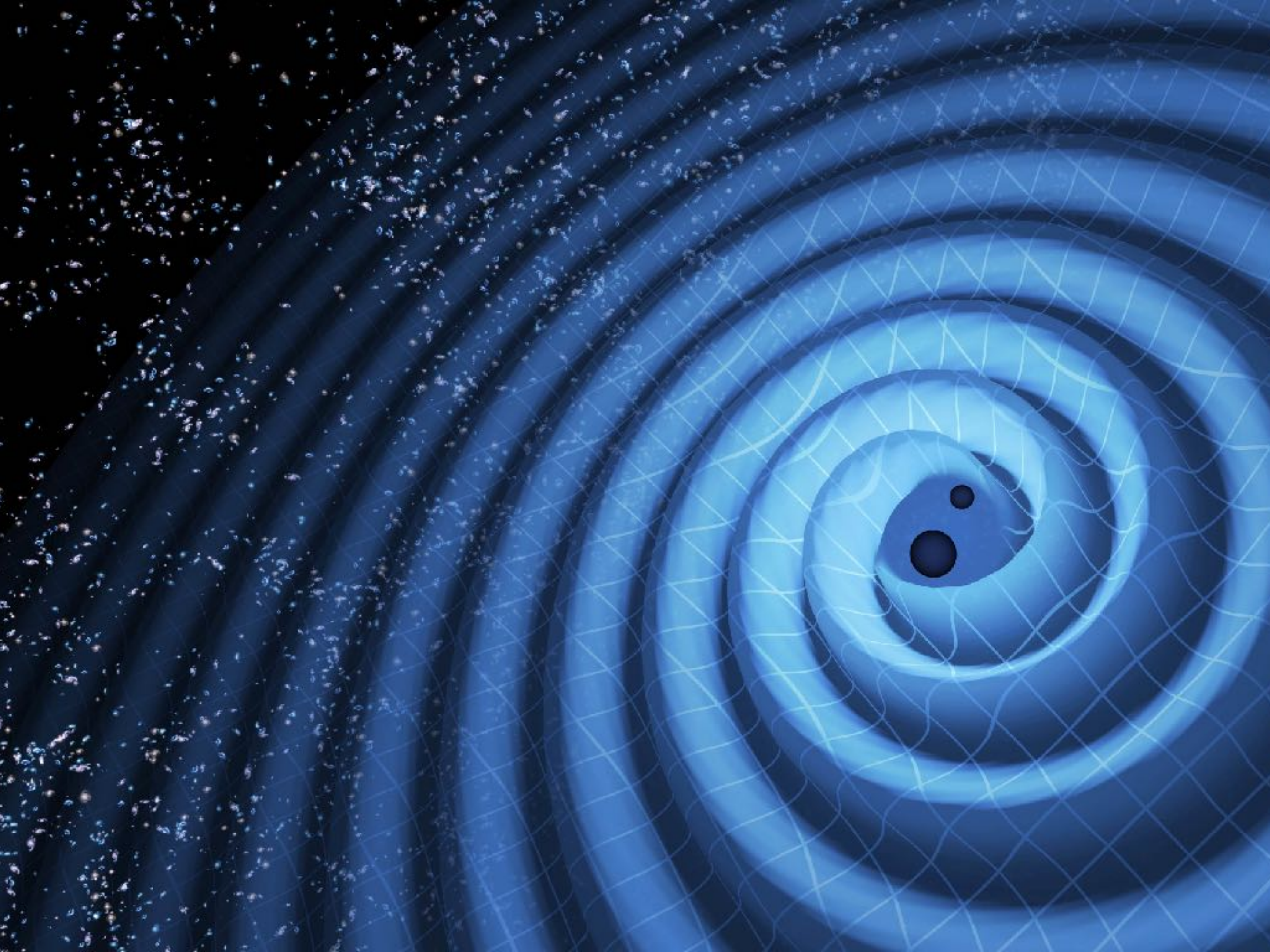
GW's amplitude scale with the inverse of the luminosity distance!

Their amplitude is sensitive to the expansion rate of the Universe!

1. Key takeaways

- Gravitational waves are *linear perturbations* of space-time that propagate across the Universe
- They propagate along *null geodesics* and carry only *two polarizations*
- Gravitational waves are sourced by the *second time derivative* of the *quadrupole moment*
- Compact binary coalescences produce sizable gravitational waves with a *chirping* waveform
- On a cosmological background, amplitude scales inversely with the *luminosity distance*

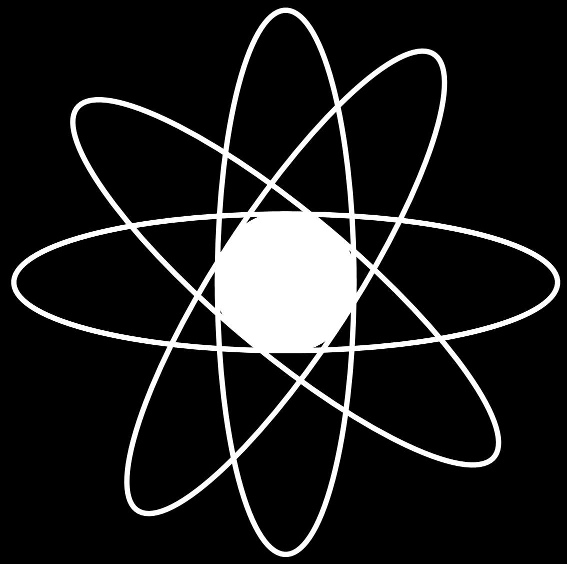
2. The new era of gravitational-wave astronomy



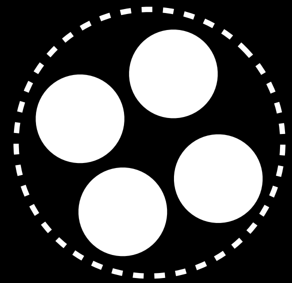
[Credit: R. Hurt, Caltech/MIT/LIGO Lab]

The variation in the distance is minuscule

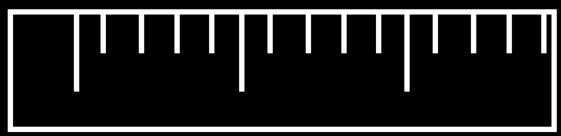
0.000000000000000001 meters



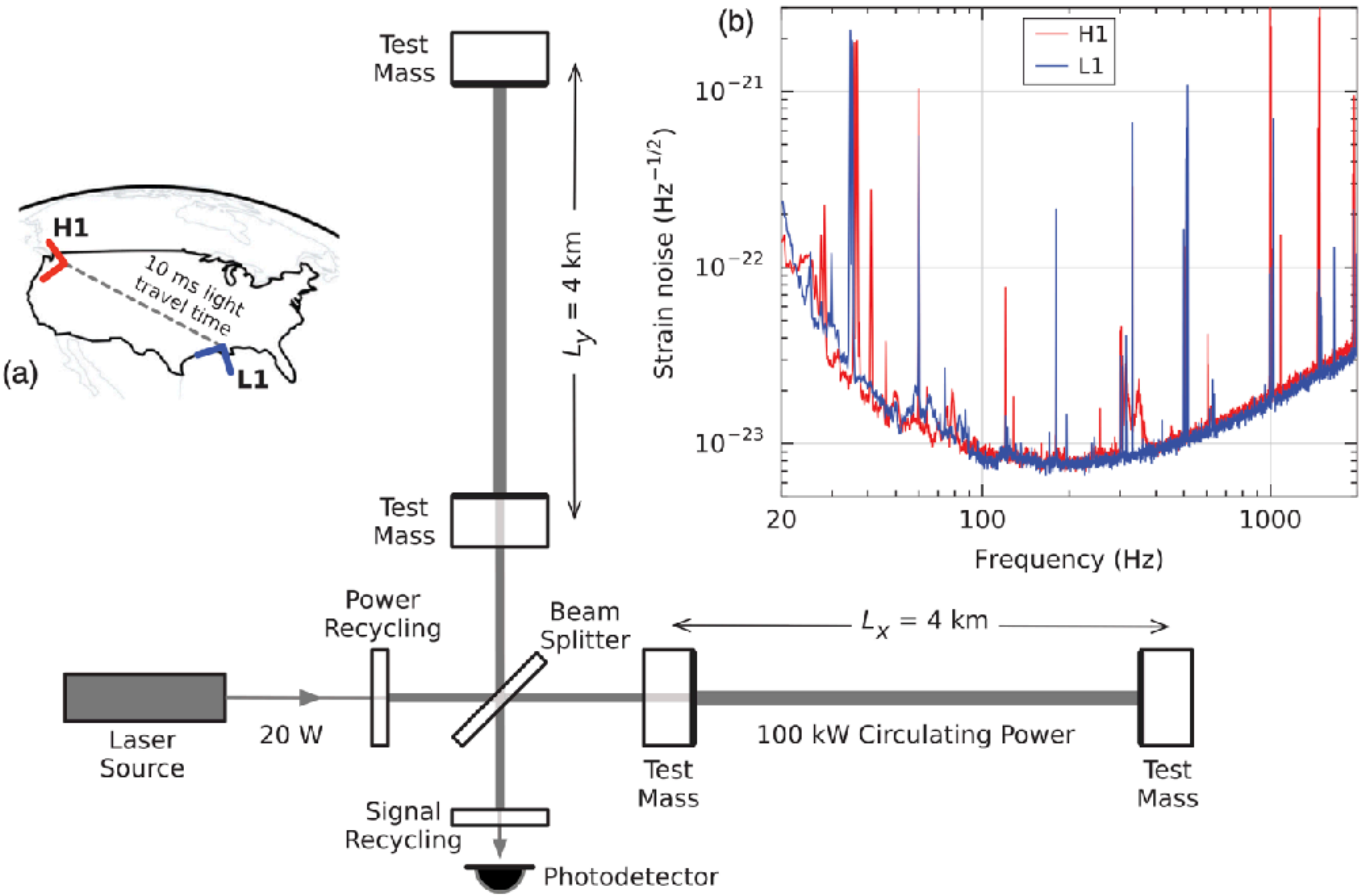
atom: 10^{-10} meters



nucleus: 10^{-15} meters



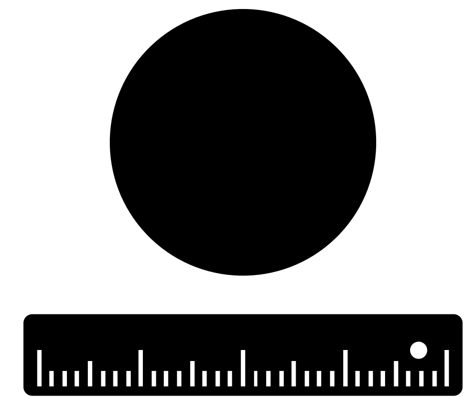
GW effect: 10^{-18} meters



[GW150914 PRL]

Tuned for detecting compact objects

$$f \sim \frac{1}{2\pi}\frac{1}{2t_{\rm Sch}} \sim 800{\rm Hz}\left(\frac{10M_\odot}{M}\right)$$

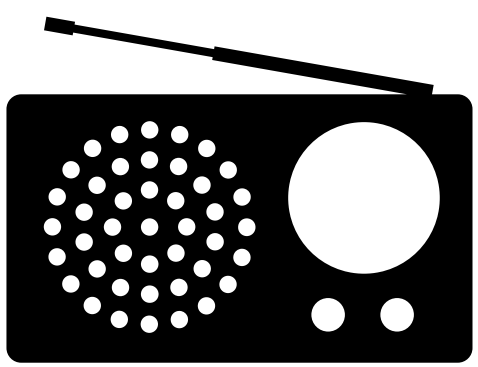


$$h \sim \mathcal{O}(1)\cdot\frac{r_{\rm Sch}}{r} \sim 10^{-23}\left(\frac{1{\rm Gpc}}{r}\right)\left(\frac{M}{10M_\odot}\right)$$

$$r_{\rm Sch} = 2GM/c^2$$

Coincides audible frequencies

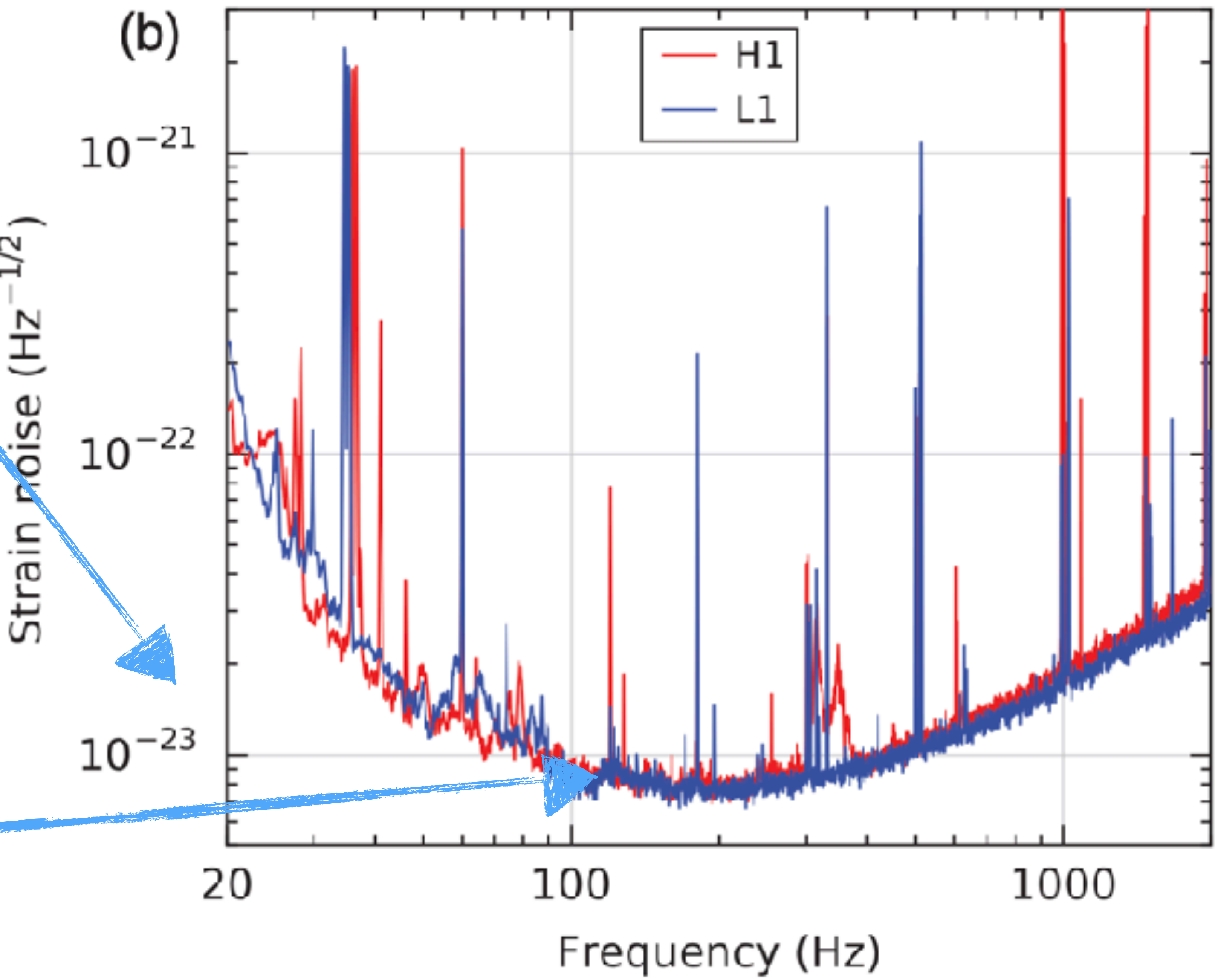
Cosmological distance



$$\frac{1{\rm Gpc}}{c} \sim 3{\rm Gyr} \sim 0.2t_{\rm Uni}$$

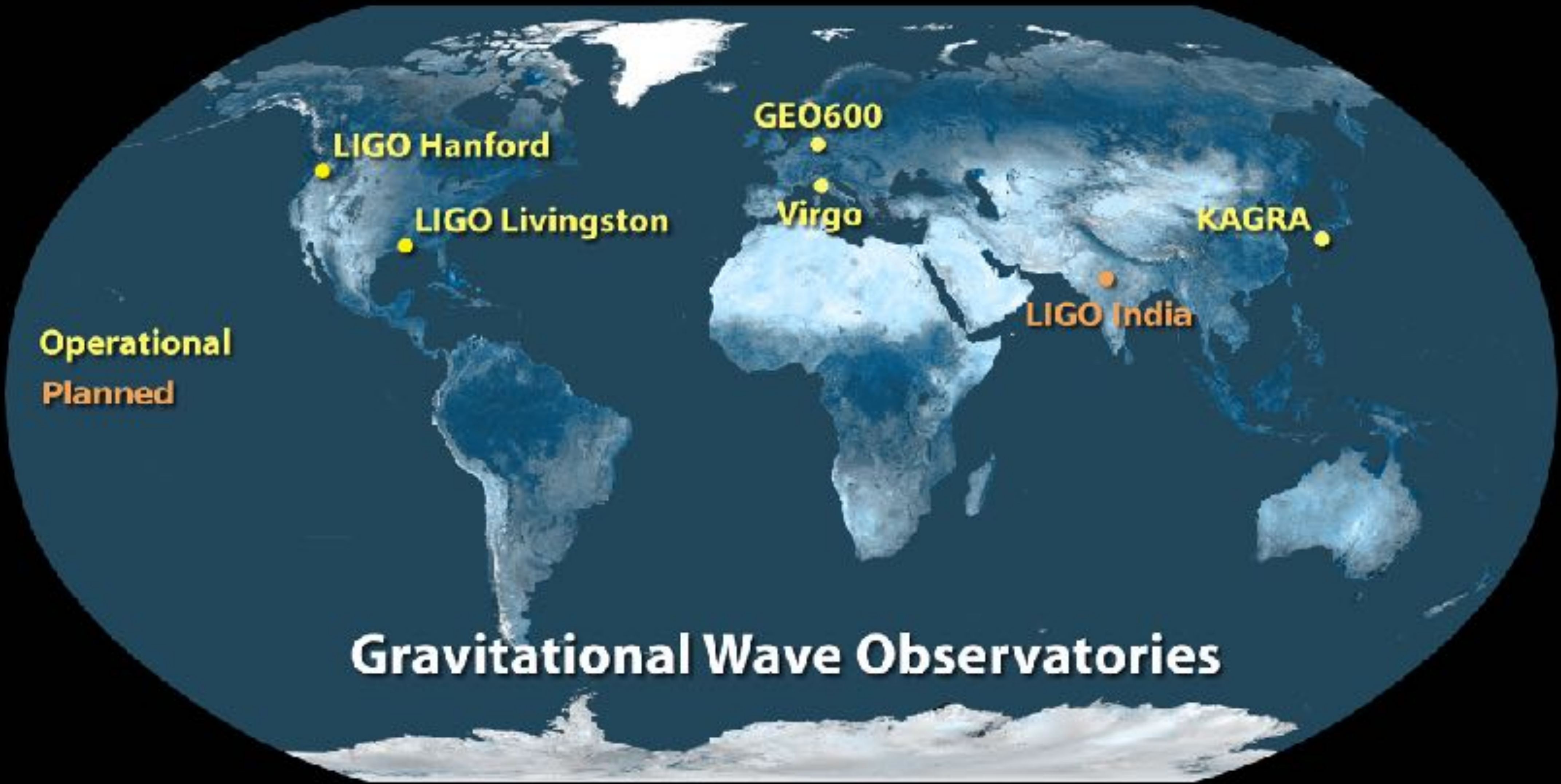
Tuned for detecting compact objects

$$h \sim 10^{-23} \left(\frac{1\mathrm{Gpc}}{r}\right)\left(\frac{M}{10M_\odot}\right)$$



$$f \sim 800\mathrm{Hz}\left(\frac{10M_\odot}{M}\right)$$

The era of gravitational wave astronomy is **here**!



[Hanford, US]

[Livingston, US]

[Virgo, Italy]

[KAGRA, Japan]

Gravitational wave detectors

- Detectors are defined by their *noise, n(t)*
- Some simplifying assumptions:

Stationary: $R(\tau) \equiv \langle n(t)n(t+\tau)\rangle$

Ergodic: $\langle n\rangle = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} n(t)dt$

Zero-mean: $\langle n(t)\rangle = 0$

Gaussian: $\langle \tilde{n}^*(f)\tilde{n}(f')\rangle = \frac{1}{2}S_n(f)\delta(f-f')$

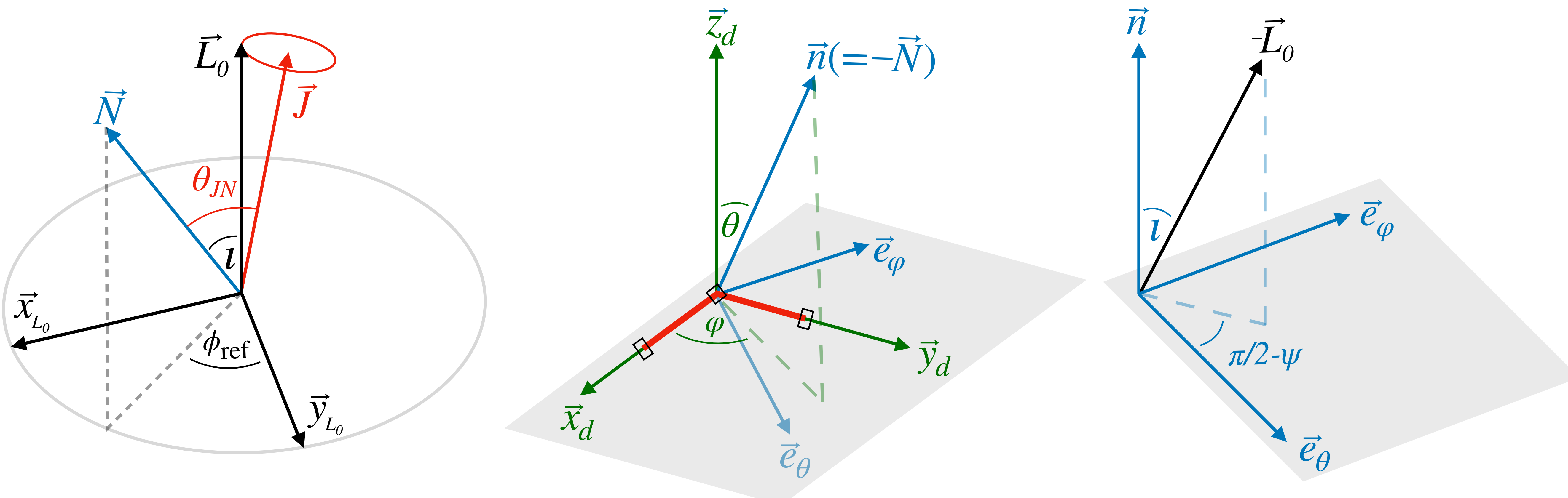
- Probability of noise realization *n(t)*

$$p_n[n(t)] \propto \exp\left[-2\int_0^\infty \frac{|\tilde{n}(f)|^2}{S_n(f)}df\right]$$

Gravitational wave detectors

- Detectors are also defined by their *antenna response*

$$h(t) = h_+(t)F_+(\hat{n}) + h_\times(t)F_\times(\hat{n})$$

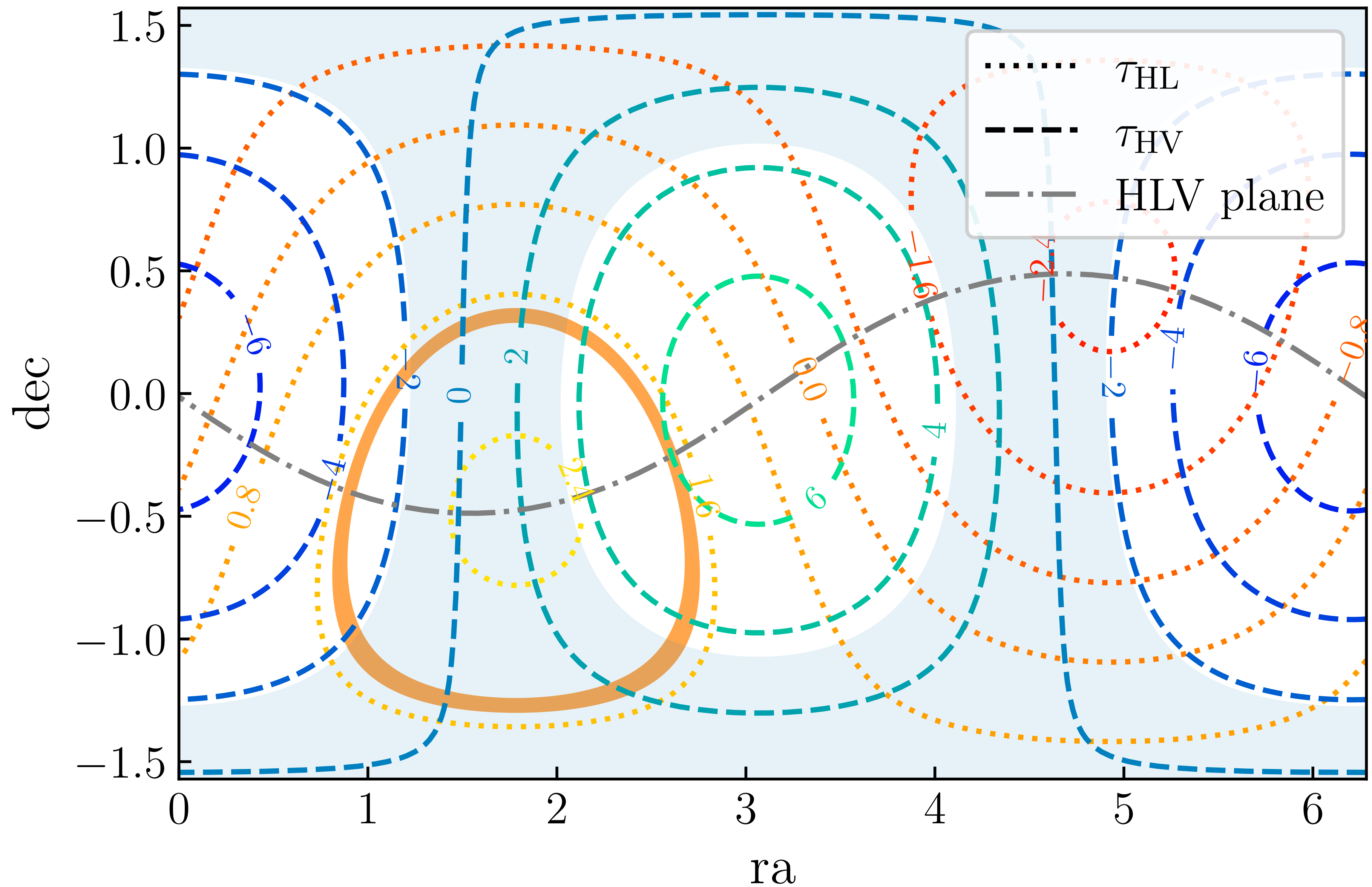


(a) Source (b) Earth detector (c) Sky

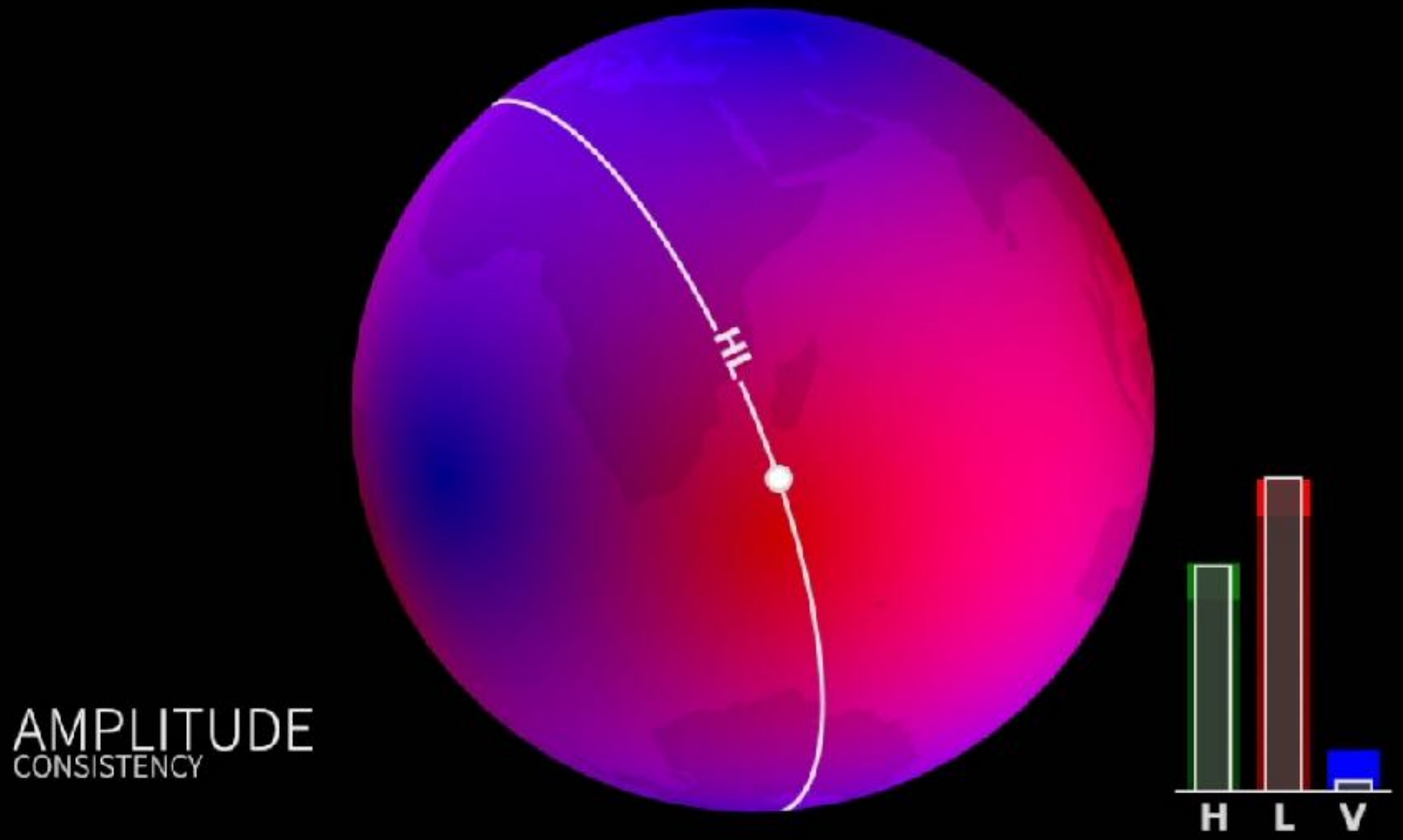
Sky localization

- The arrival time difference between two detectors defines a ring in the sky

$$\Delta t_{d_1 d_2} = \vec{n} \cdot \vec{r}_{d_1 d_2} / c$$



Sky localization



[Credit: Leo Singer]

Matched-filtering

- The data stream: $d(t) = s(t) + n(t)$
- Filter data: $\hat{d} = \int_{-\infty}^{\infty} dt\, d(t) K(t)$
- Signal to noise:

$$S/N = \frac{\int_{-\infty}^{\infty} df\, \tilde{s}(f) \tilde{K}^*(f)}{\sqrt{\int_{-\infty}^{\infty} df\, \frac{1}{2} S_n(f) |\tilde{K}(f)|^2}}$$

- Define noise weighted diner product

$$(a|b) \equiv \mathrm{Re}\left[\int_{-\infty}^{\infty} \frac{\tilde{a}^*(f)\tilde{b}(f)}{S_n(f)/2}\right] = 4\mathrm{Re}\left[\int_{0}^{\infty} \frac{\tilde{a}^*(f)\tilde{b}(f)}{S_n(f)}\right],$$

- Rewrite S/N:

$$S/N = \frac{(u|s)}{\sqrt{(u|u)}} \qquad \tilde{u}(f) = \frac{1}{2} S_n(f) \tilde{K}(f)$$

Matched-filtering

- Signal to noise: $$S/N = \frac{(u|s)}{\sqrt{(u|u)}}$$

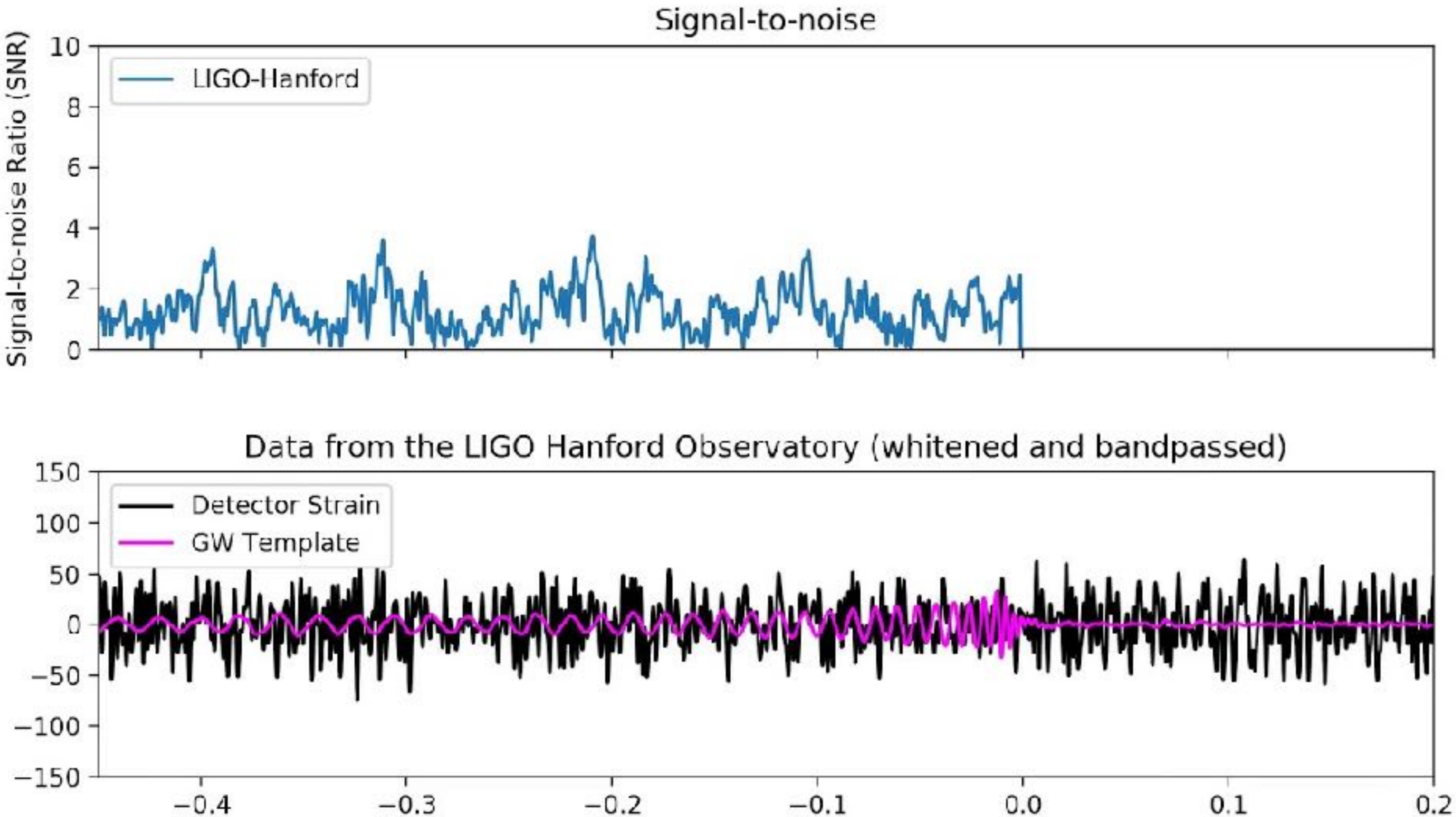
- Optimal filter when *u* is parallel to *s*

$$\tilde{K}(f) \propto \frac{\tilde{s}(f)}{S_n(f)}$$

- Optimal signal-to-noise ratio

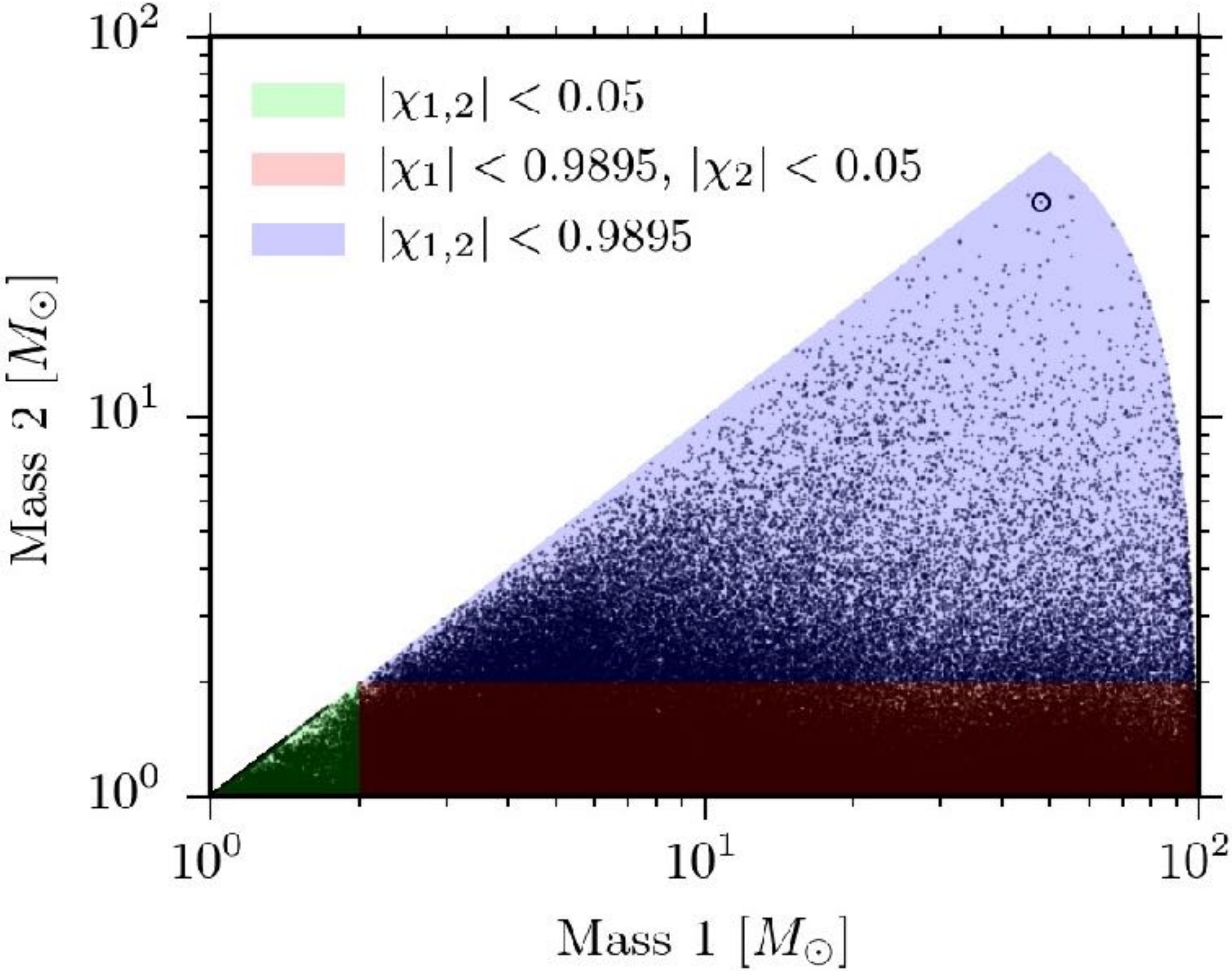
$$\rho^2_{\rm opt} = (h|h) = 4\mathrm{Re}\left[\int_0^\infty \frac{|\tilde{h}(f)|^2}{S_n(f)}\right]$$

Matched-filtering



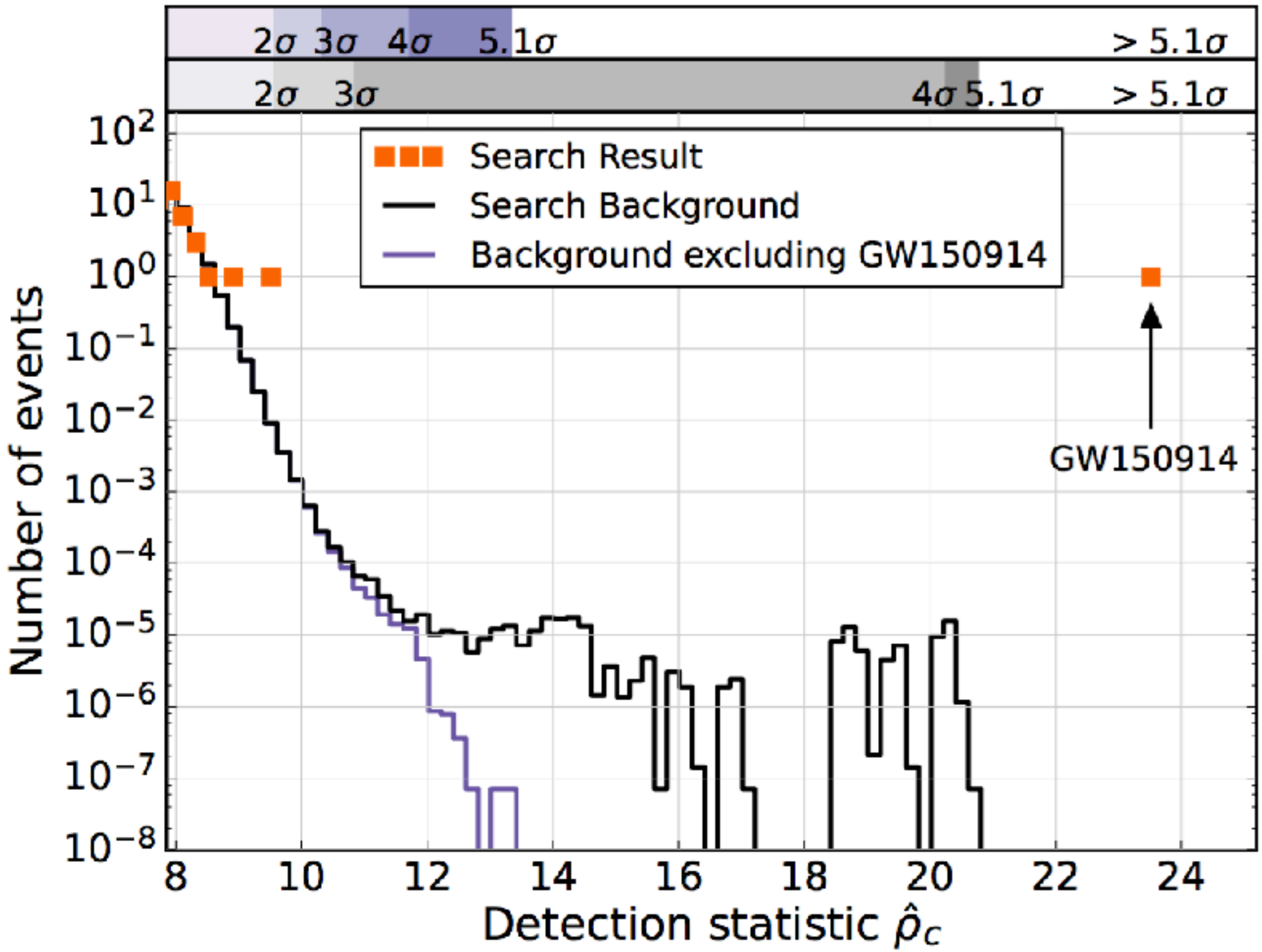
[Credit: Alex Nitz]

Matched-filtering



[Credit: LVK]

Detection statistics



[Credit: LVK]

Parameter estimation

- If we subtract the the right signal to the data, we should recover the noise

$$n(t) = d(t) - s(t)$$

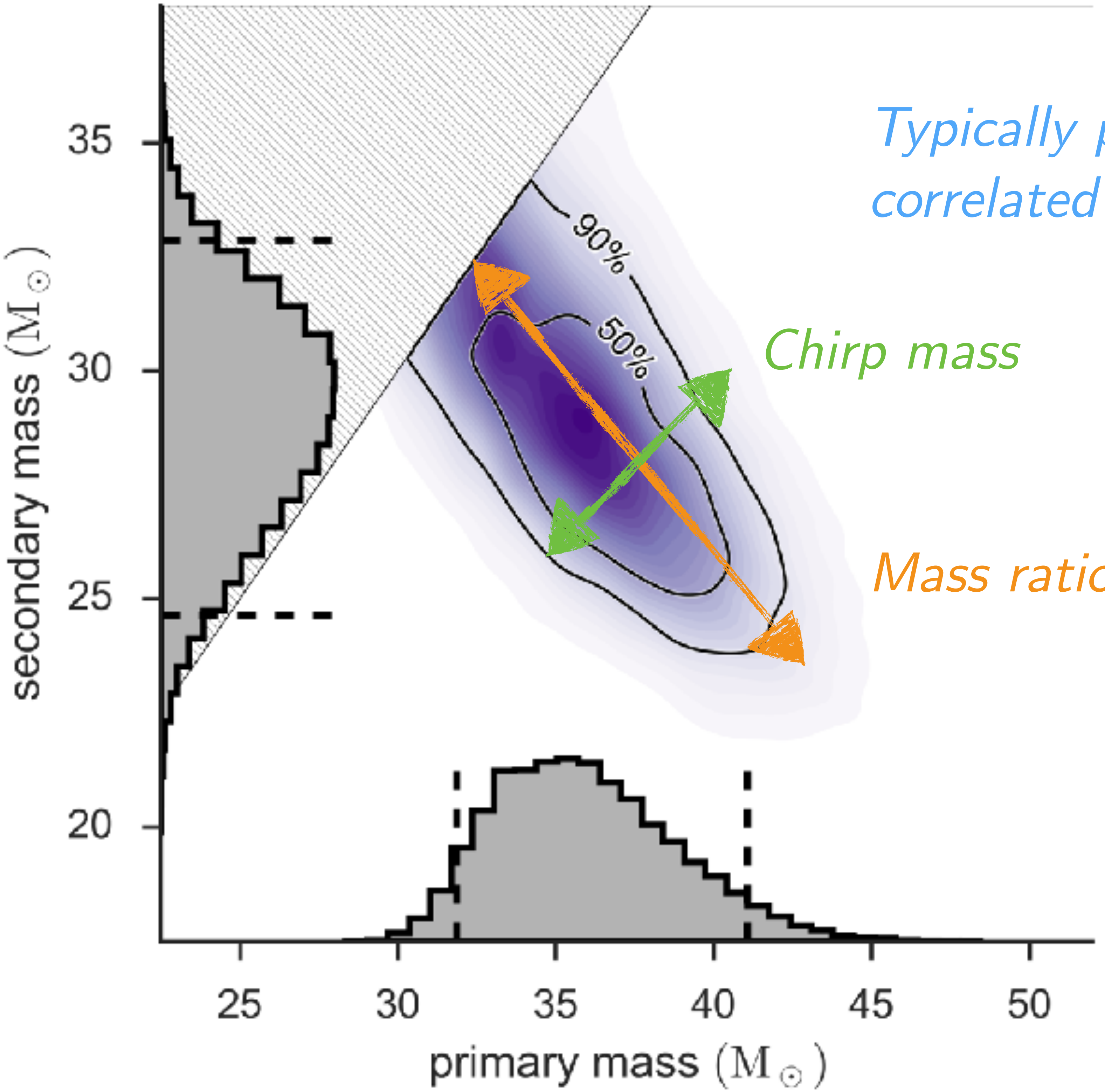
- Assuming Gaussian noise, the likelihood of the data is

$$\Lambda(d|\theta) \propto \exp\left[-\frac{1}{2}(d - h(\theta)|d - h(\theta))\right]$$
$$= \exp\left[(d|h(\theta)) - \frac{1}{2}(h(\theta)|h(\theta)) - \frac{1}{2}(d|d)\right]$$

- The posterior distribution of a parameter (Bayes theorem)

$$p(\theta|d) \propto p(\theta)\exp\left[(d|h(\theta)) - \frac{1}{2}(h(\theta)|h(\theta))\right]$$

Parameter estimation



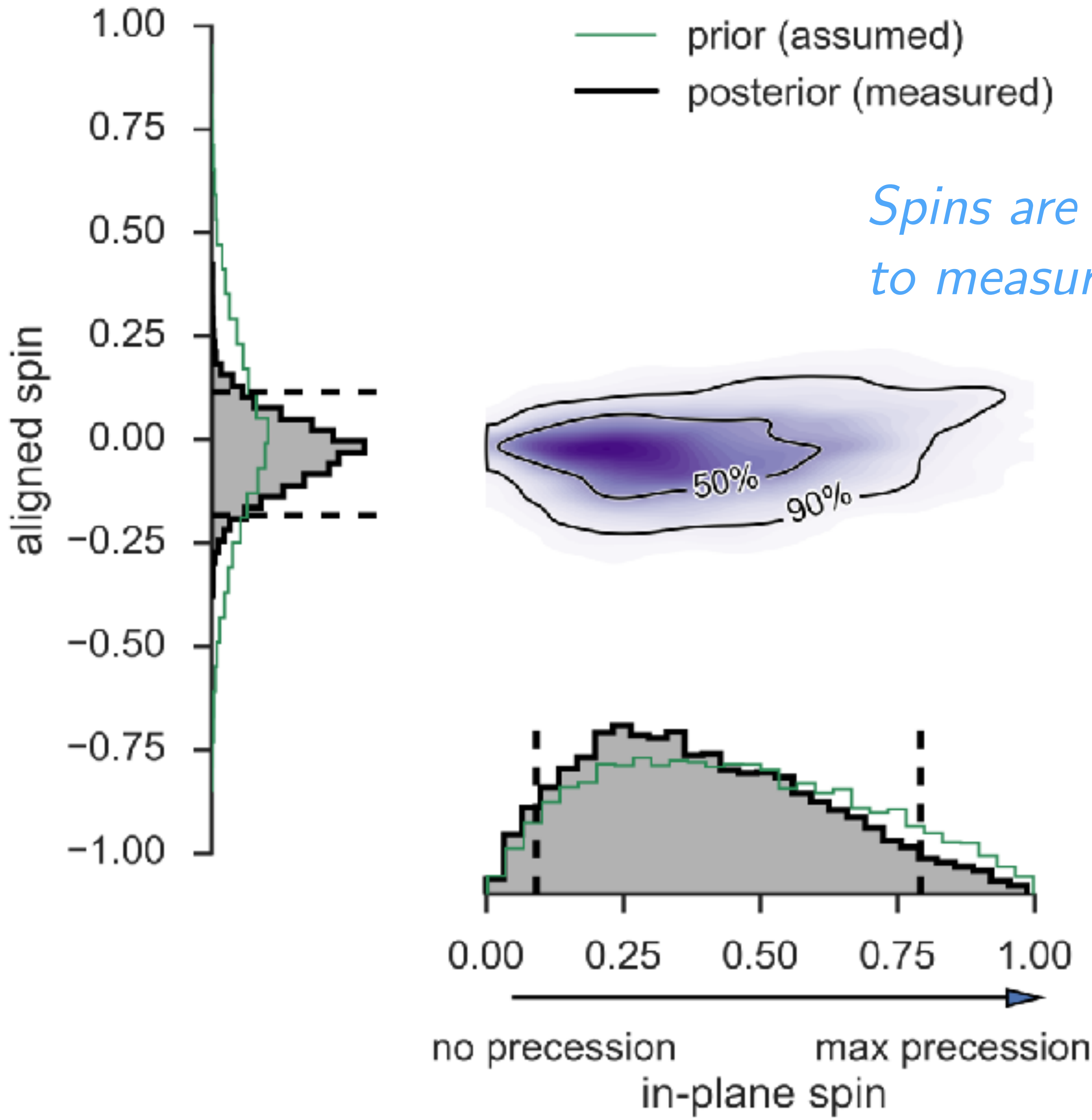
Typically parameters are correlated

Chirp mass

Mass ratio

[Credit: LVK]

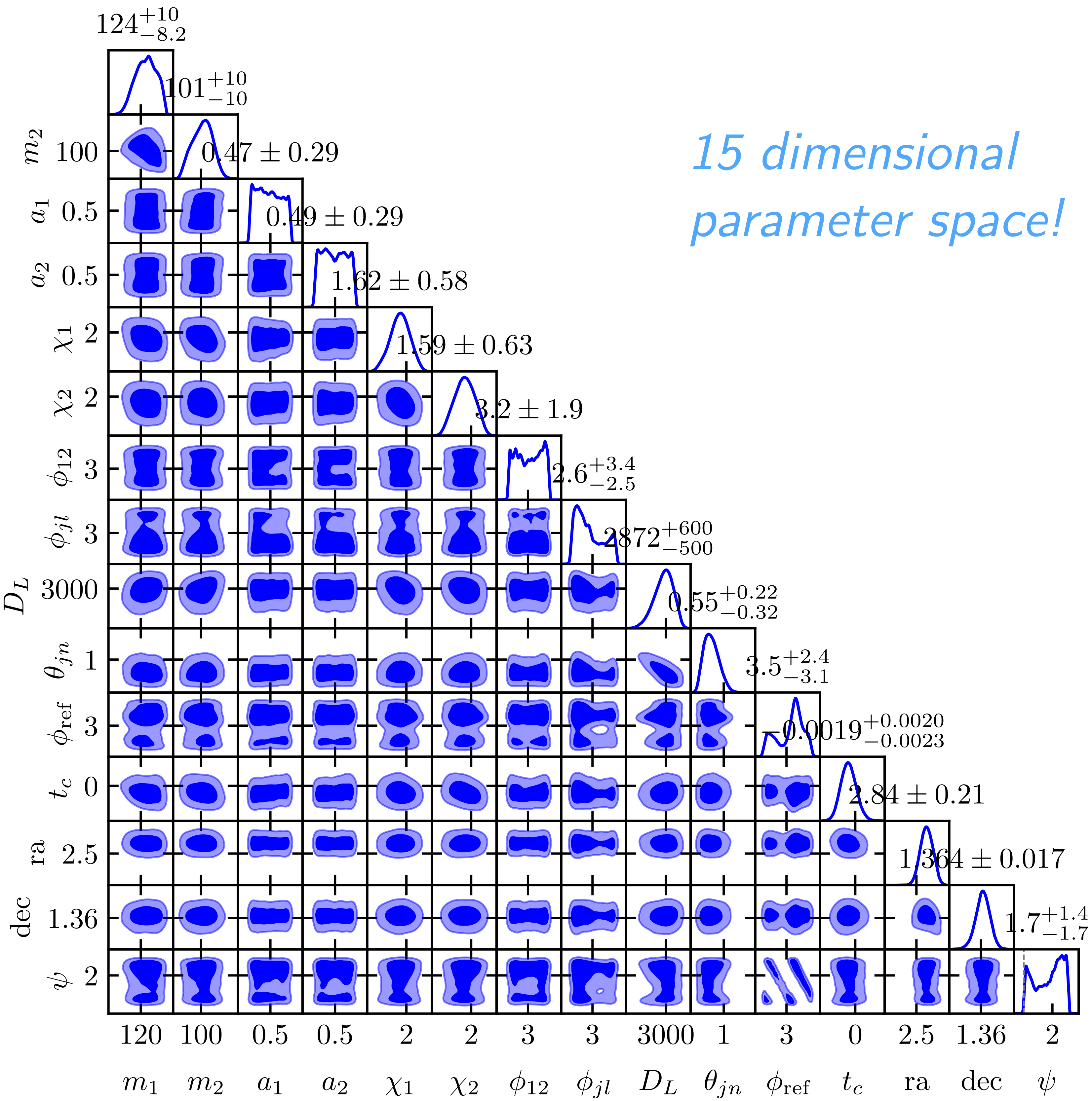
Parameter estimation



Spins are more difficult to measure

[Credit: LVK]

Parameter estimation



15 dimensional parameter space!

Measurement uncertainty

- In the high signal-to-nise limit, inferred parameters are close to the maximum likelihood (ML) value

$$\theta^i = \theta^i_{\rm ML} + \Delta\theta^i$$

- Expanding the likelihood around this value (first contribution quadratic)

$$p(\theta|d) \propto \exp\left[-\frac{1}{2}\Gamma_{ij}\Delta\theta^i\Delta\theta^j\right]$$

$$\boxed{\Gamma_{ij} = (\partial_i\partial_j h|h-s) + (\partial_i h|\partial_j h) \approx (\partial_i h|\partial_j h)}$$

- E.g. $\tilde{h}(f) = Ae^{i\phi}$

$$\sigma_{\ln A} = \sigma_\phi = 1/\rho$$

Population inference

- The posterior distribution of the hyper-parameters

$$p(\lambda|\{d_i\}) \propto p(\lambda)p(\{d_i\}|\lambda) = p(\lambda)\prod_{i=1}^{N_{\rm obs}} \frac{p_{\rm pop}(\theta_i|\lambda)}{\int d\theta p_{\rm pop}(\theta|\lambda)}$$

- Including selection effects

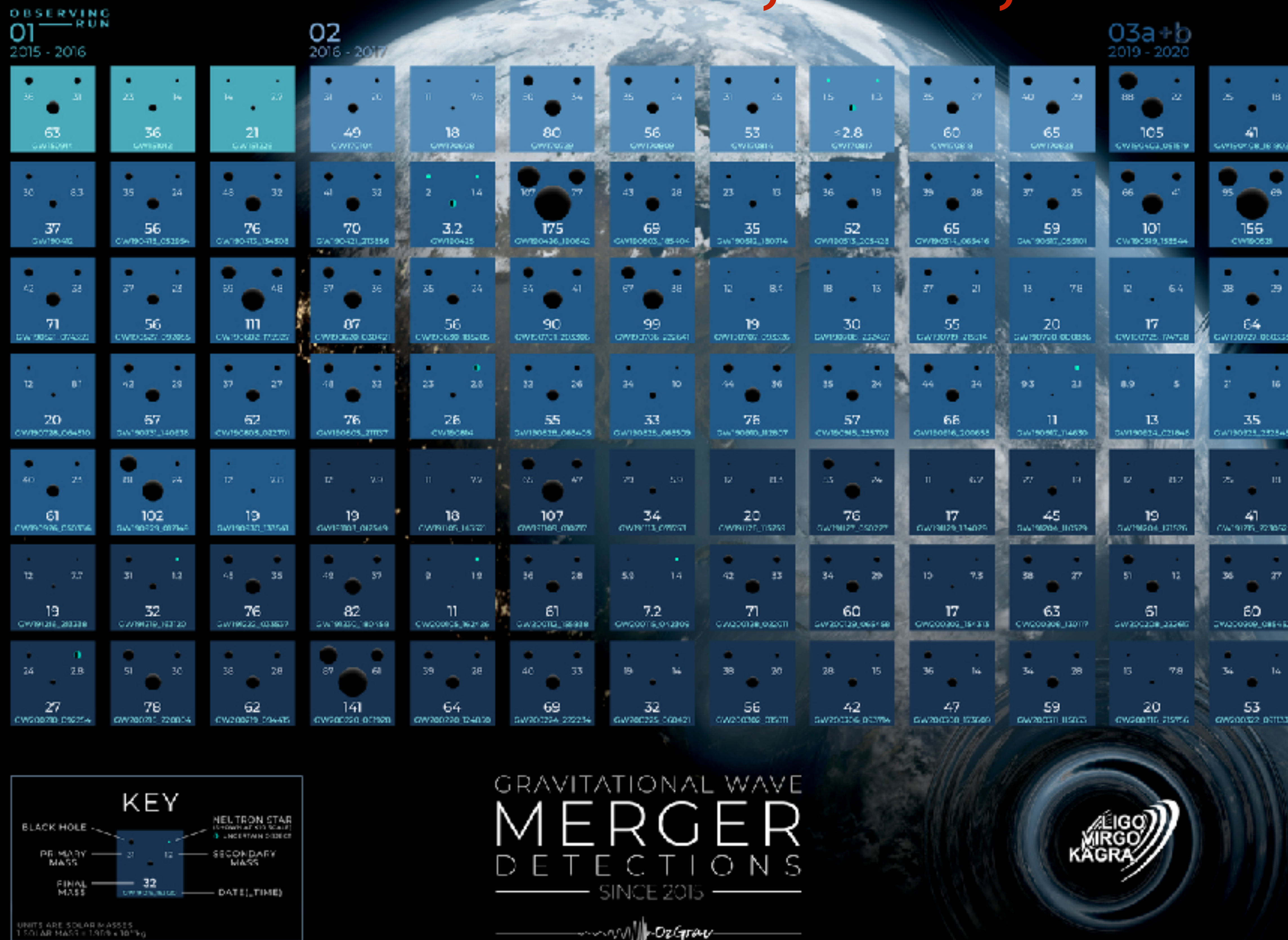
$$p(\{d_i\}|\lambda) = \prod_{i=1}^{N_{\rm obs}} \frac{p_{\rm pop}(\theta_i|\lambda)p_{\rm det}(\theta_i)}{\int d\theta p_{\rm pop}(\theta|\lambda)p_{\rm det}(\theta)} = \prod_{i=1}^{N_{\rm obs}} \frac{p_{\rm pop}(\theta_i|\lambda)}{\int d\theta p_{\rm pop}(\theta|\lambda)p_{\rm det}(\theta)}$$

- Including measurement uncertainties

$$p(\{d_i\}|\lambda) = \prod_{i=1}^{N_{\rm obs}} \frac{\int d\theta p(\theta|d_i)p_{\rm pop}(\theta|\lambda)}{\int d\theta p_{\rm pop}(\theta|\lambda)p_{\rm det}(\theta)}$$

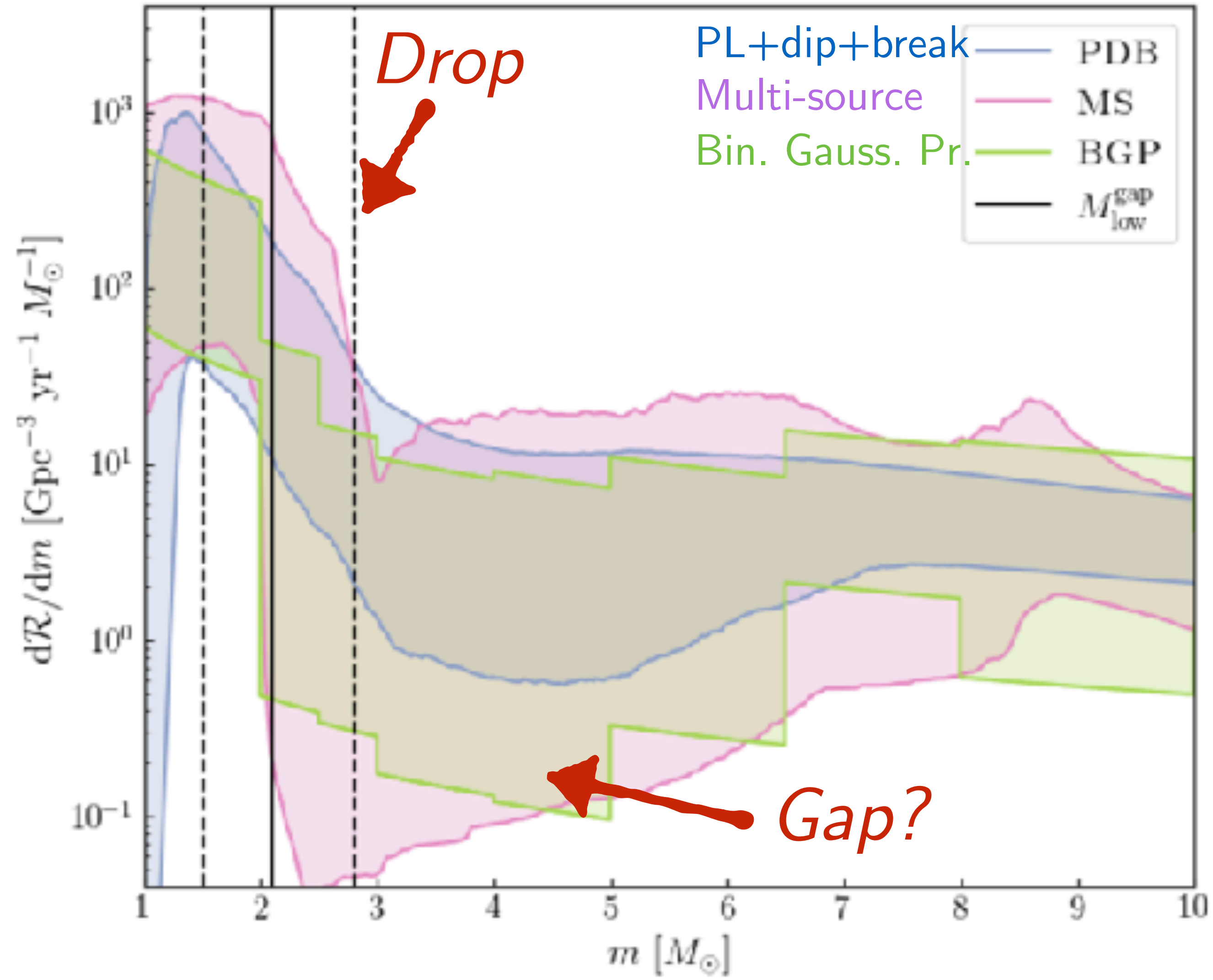
The era of gravitational wave astronomy is **here**!

~100 events: BBH, BNS, NSBH

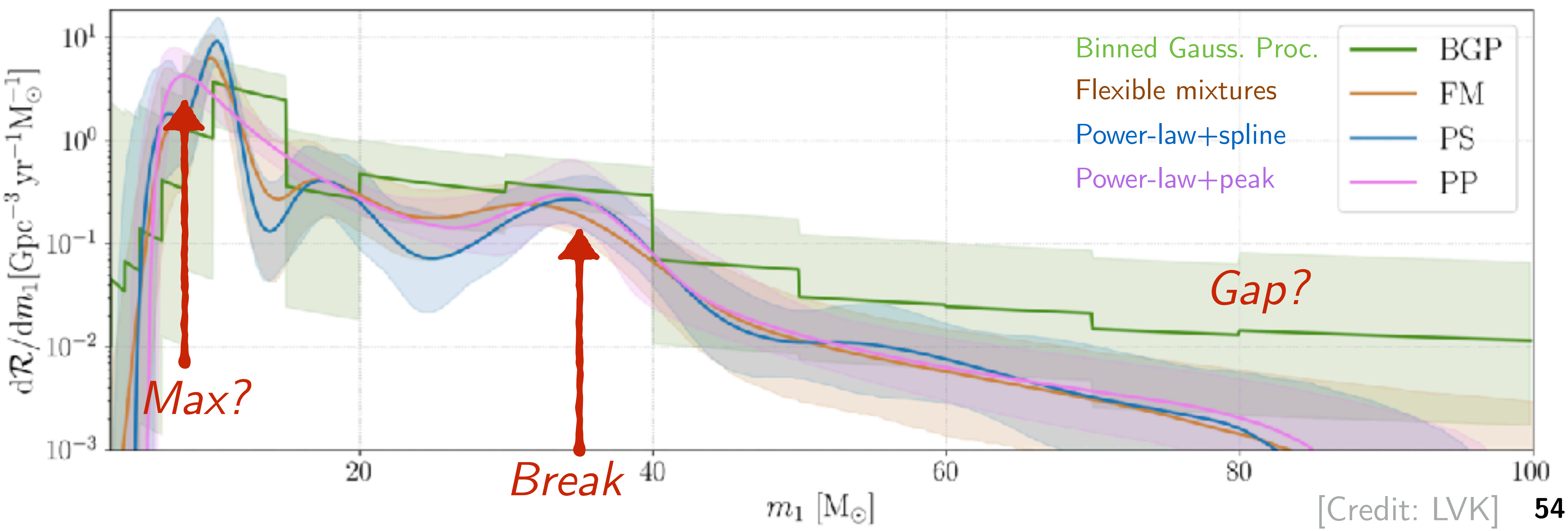


GWTC-3 popul

Low-mass mass distribution

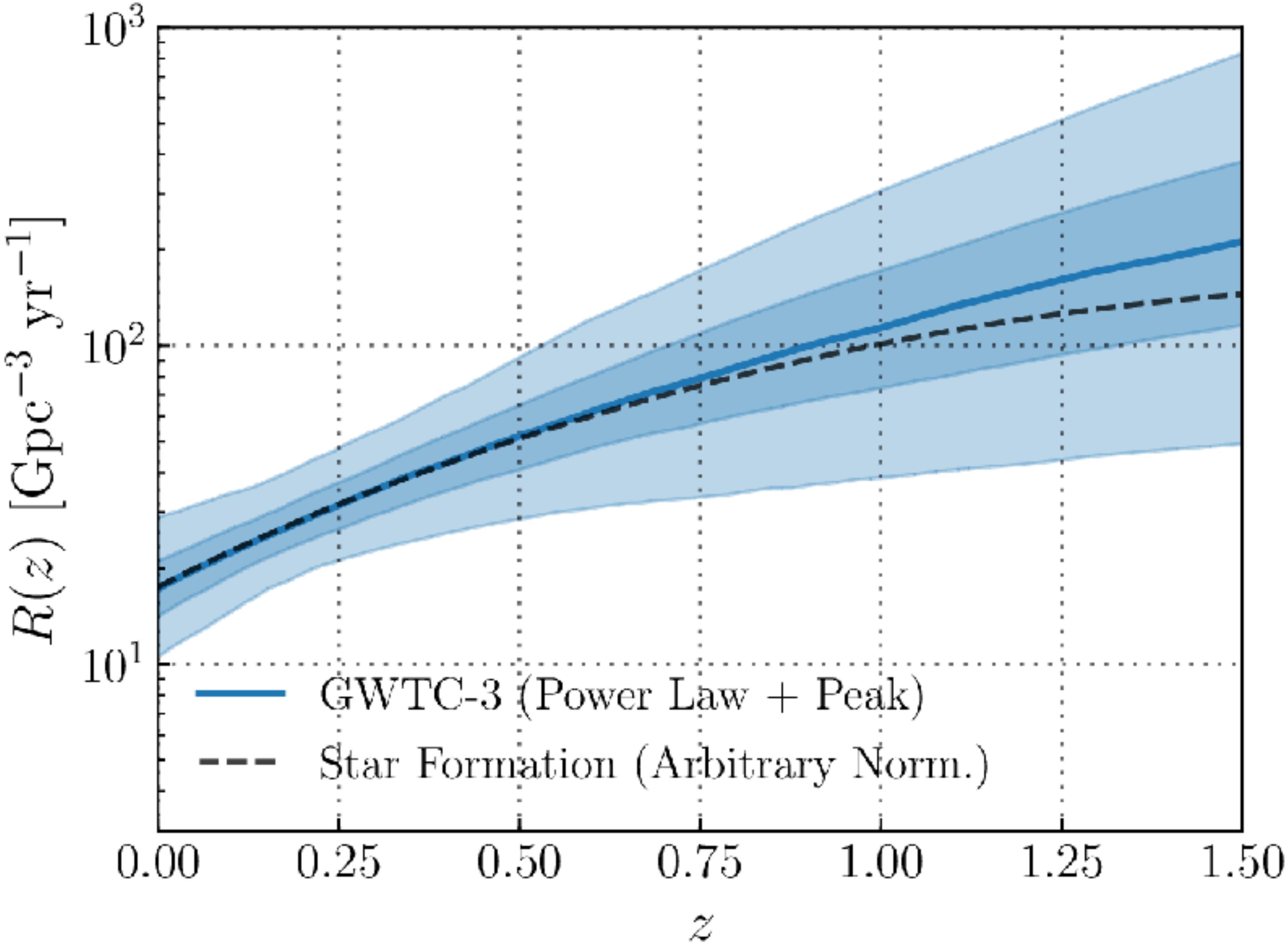


BBH mass distribution



[Credit: LVK]

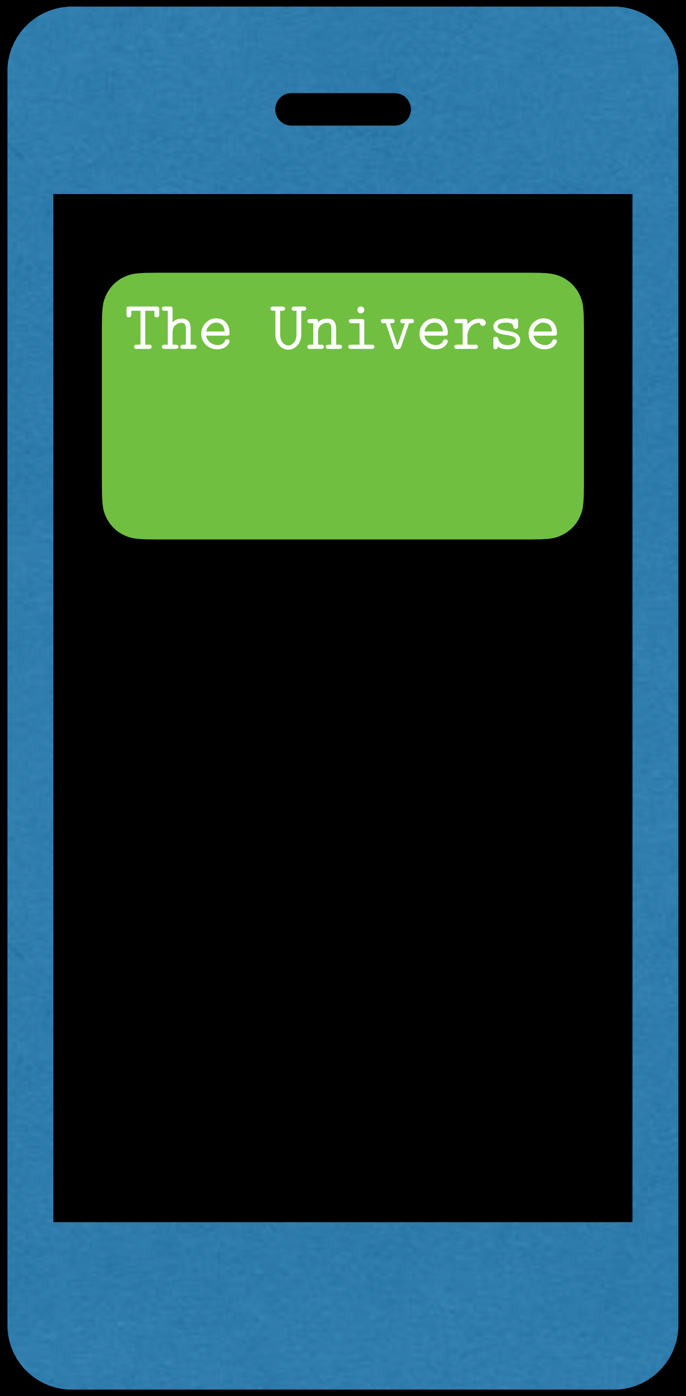
GWTC-3 population



[Credit: LVK]

O4 is happening!

https://gracedb.ligo.org/superevents/public/O4/#



Gravitational Waves

DAY 2

Jose María Ezquiaga
Niels Bohr Institute
jose.ezquiaga@nbi.ku.dk
ezquiaga.github.io

[Diego Rivera]

GraceDB Public Alerts Latest Search Documentation Login

Please log in to view full database contents.

LIGO/Virgo/KAGRA Public Alerts

- More details about public alerts are provided in the LIGO/Virgo/KAGRA Alerts User Guide.
- Retractions are marked in red. Retraction means that the candidate was manually vetted and is no longer considered a candidate of interest.
- Less-significant events are marked in grey, and are not manually vetted. Consult the LVK Alerts User Guide for more information on significance in O4.
- Less-significant events are not shown by default. Press "**Show All Public Events**" to show significant and less-significant events.

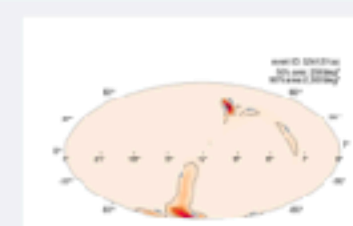
O4 Significant Detection Candidates: **167** (186 Total - 19 Retracted)

O4 Low Significance Detection Candidates: **2839** (Total)

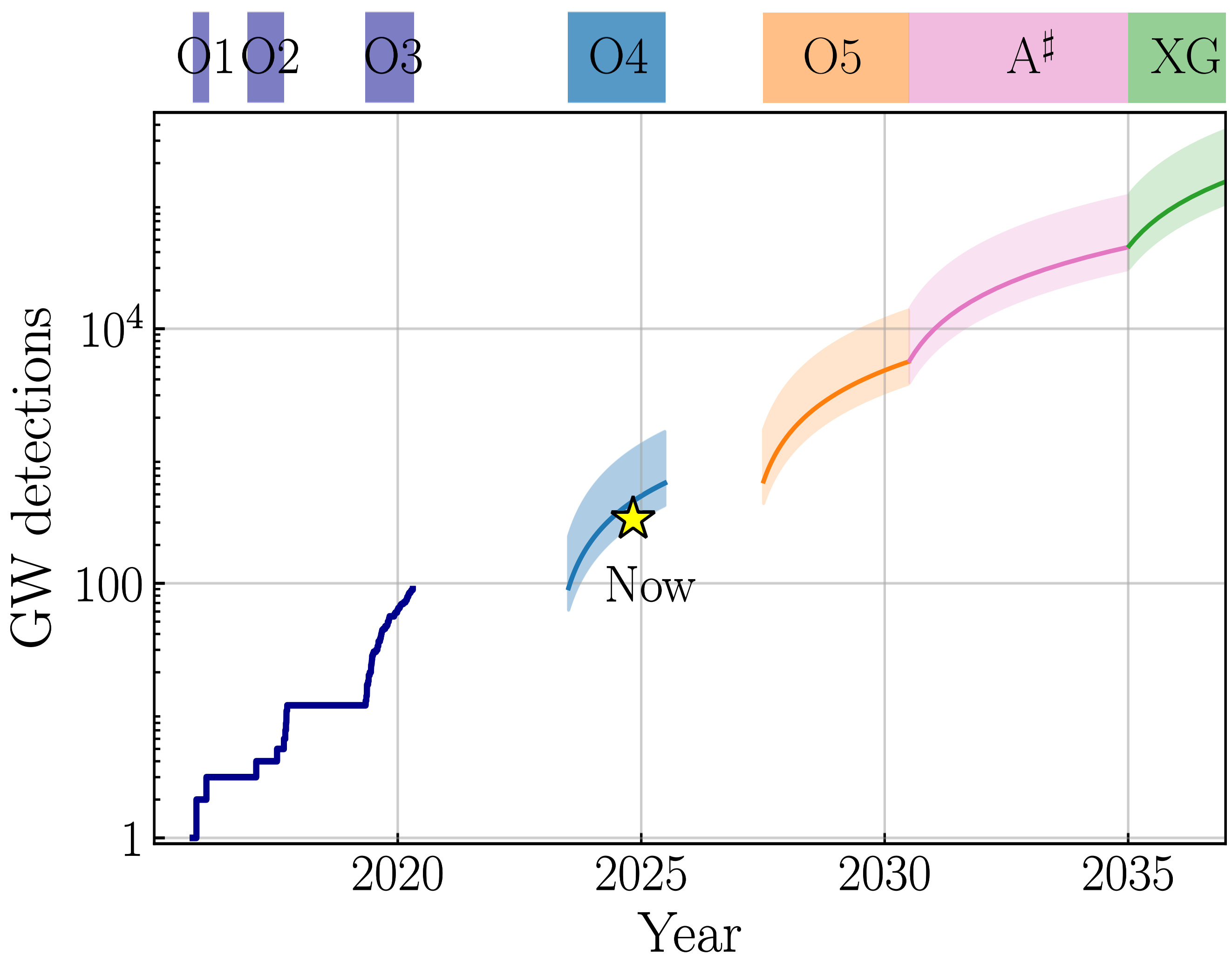
Show All Public Events

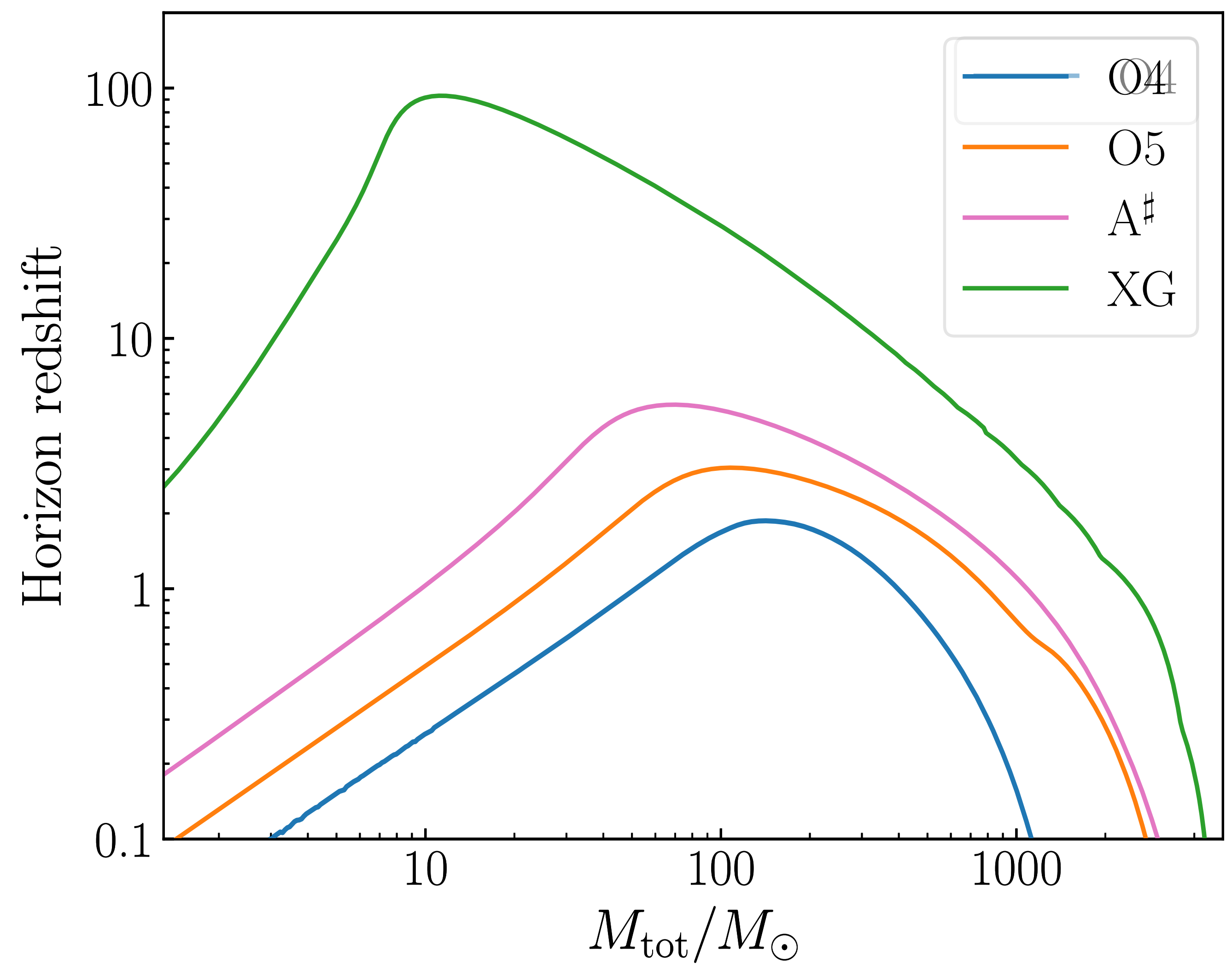
Page 1 of 13. next last »

SORT: EVENT ID (A-Z) ▼

Event ID	Possible Source (Probability)	Significant	UTC	GCN	Location
S241201ac	BBH (97%), Terrestrial (3%)	Yes	Dec. 1, 2024 05:57:58 UTC	GCN Circular Query Notices | VOE	

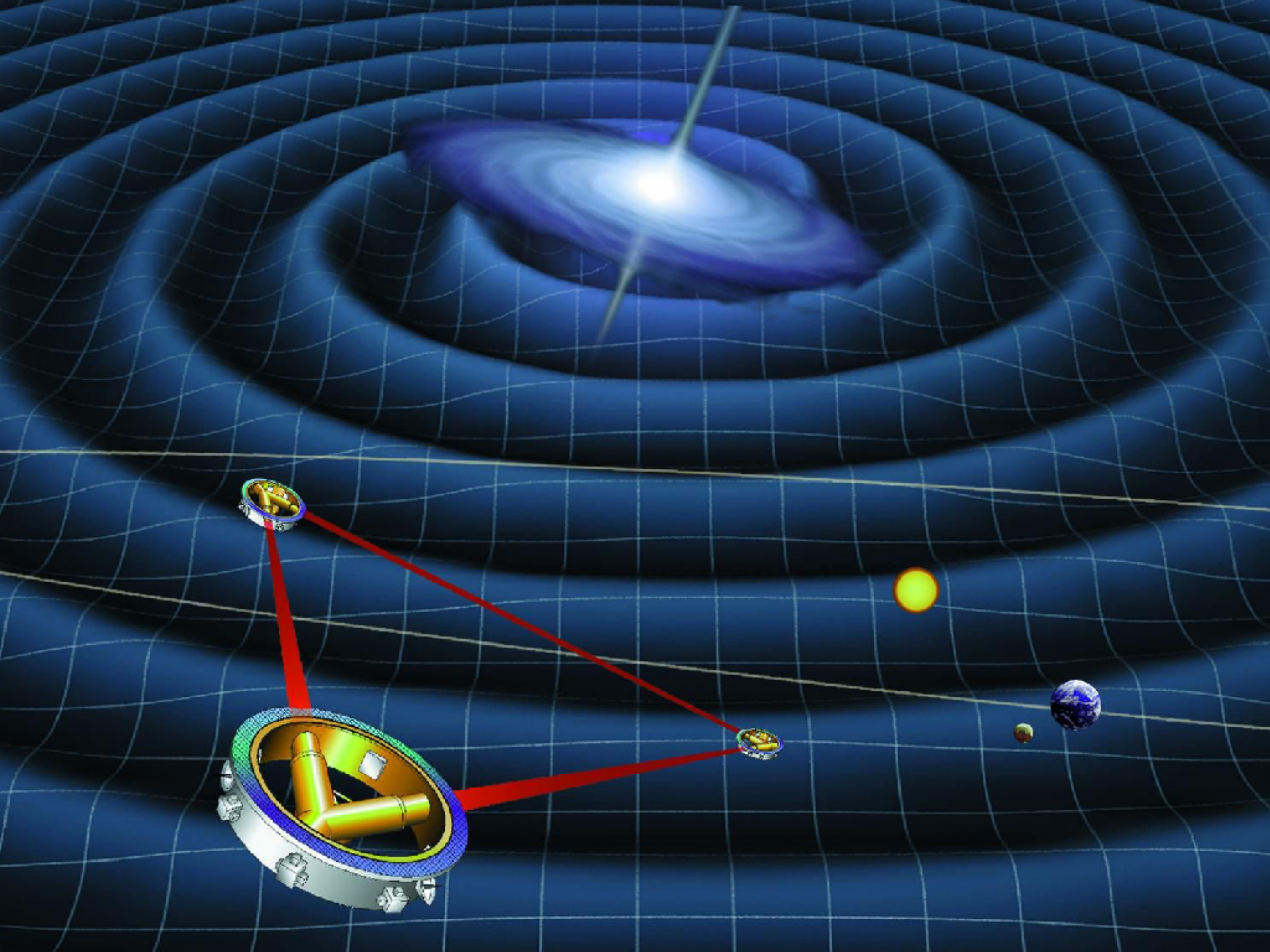
The **future**: "big data" & distant Universe

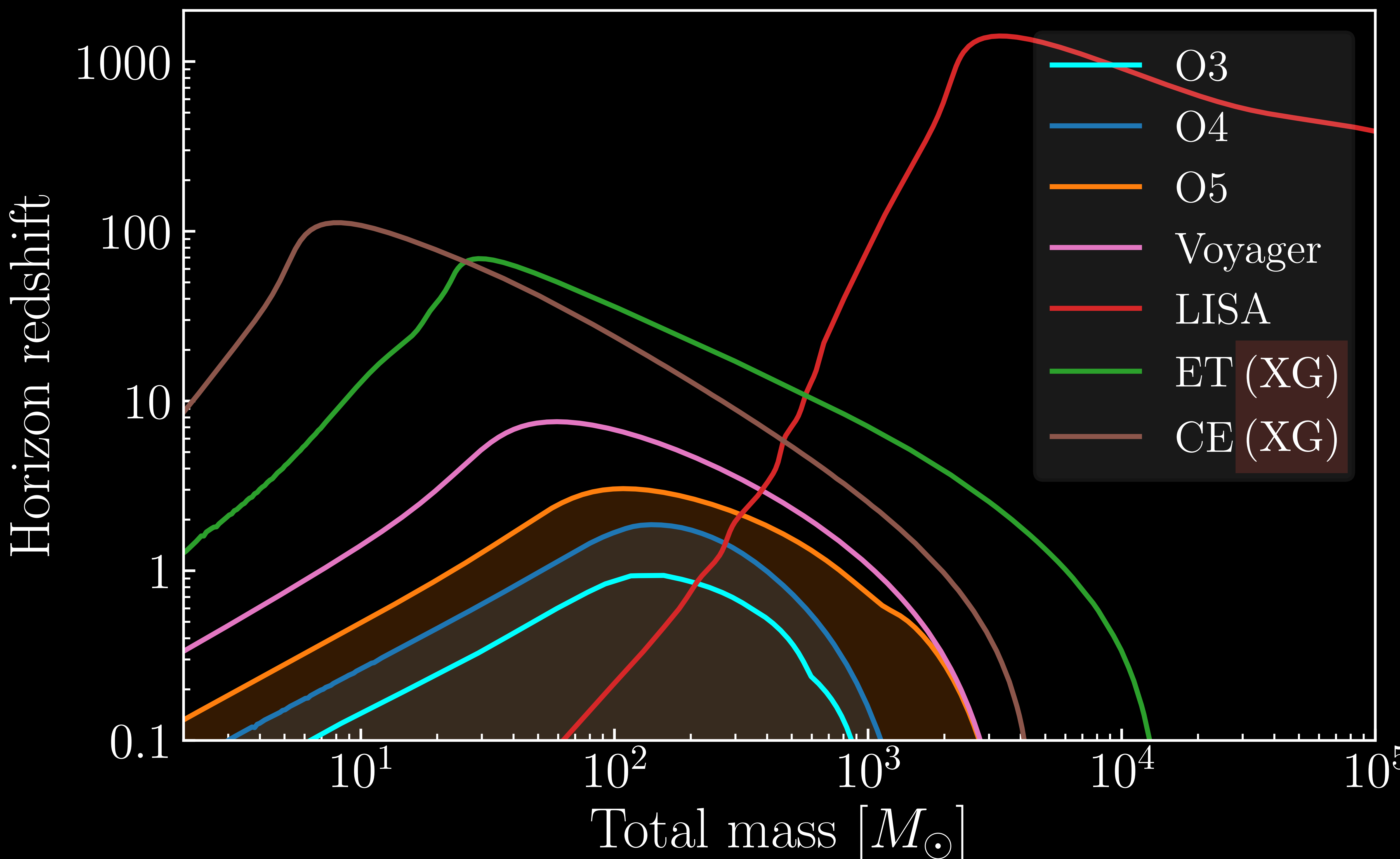




[XG = next-generation detector = Cosmic Explorer / Einstein Telescope]

[Chen, **Ezquiaga** & Gupta (CQG'24)]




Horizon redshift
1000
100
10
1
0.1
10^1
10^2
10^3
10^4
10^5
Total mass $[M_\odot]$
O3
O4
O5
Voyager
LISA
ET (XG)
CE (XG)

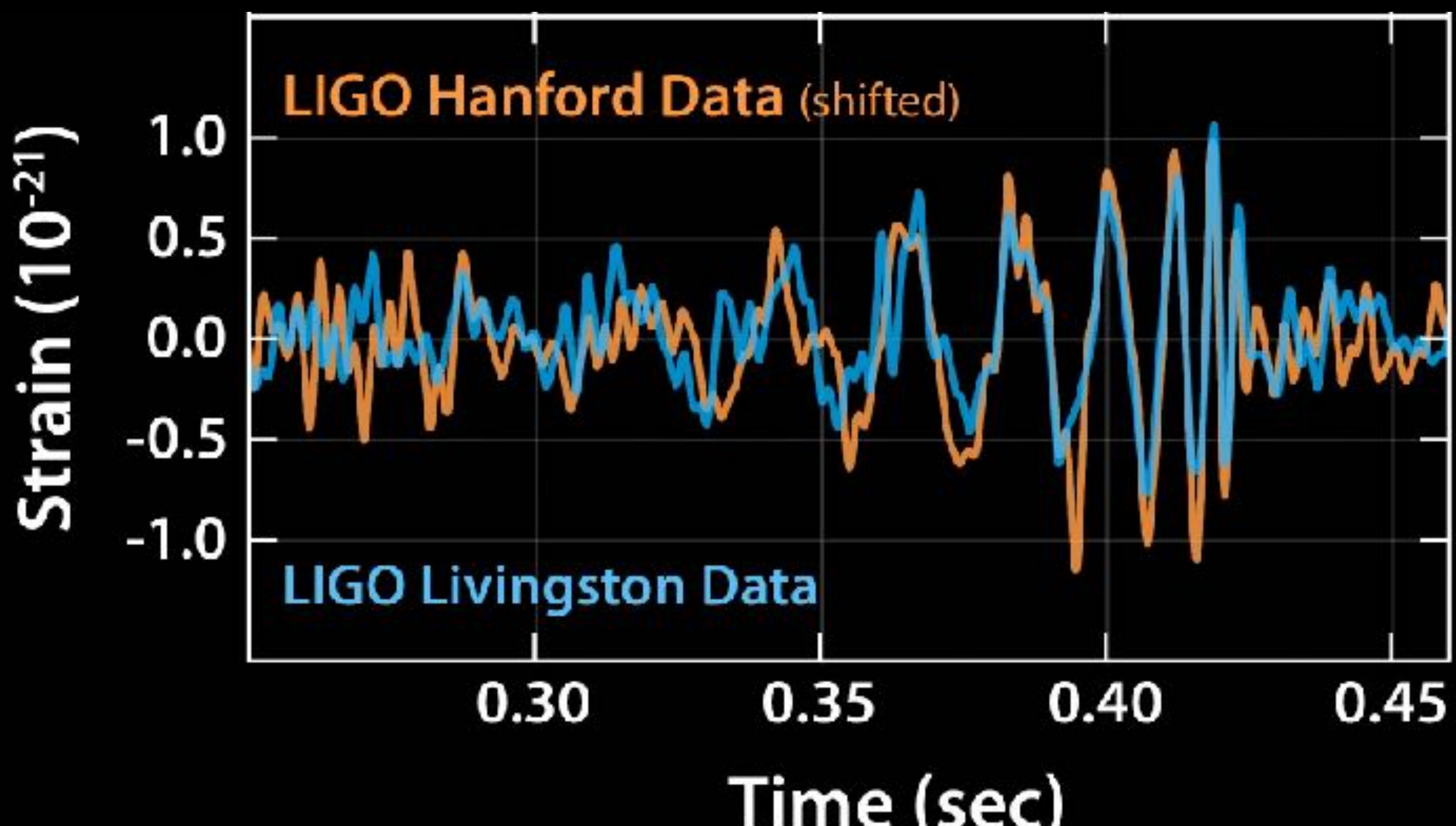
2. Key takeaways

- Gravitational waves detectors are describes by their *noise* and *antenna pattern* function
- The *optimal signal to noise* is given when the filter matches the signal
- Data stream can be *matched filtered* using a template bank. An event is found when it cannot be explained by noise background
- Once an event is detected, we can infer the parameters. This is a *15D* parameter space
- Almost *300* significant candidates since the first observation. *Many* more to come in the *future*!

3. Standard siren cosmology

Gravitational waves are standard sirens

[general relativity predicts waveform]

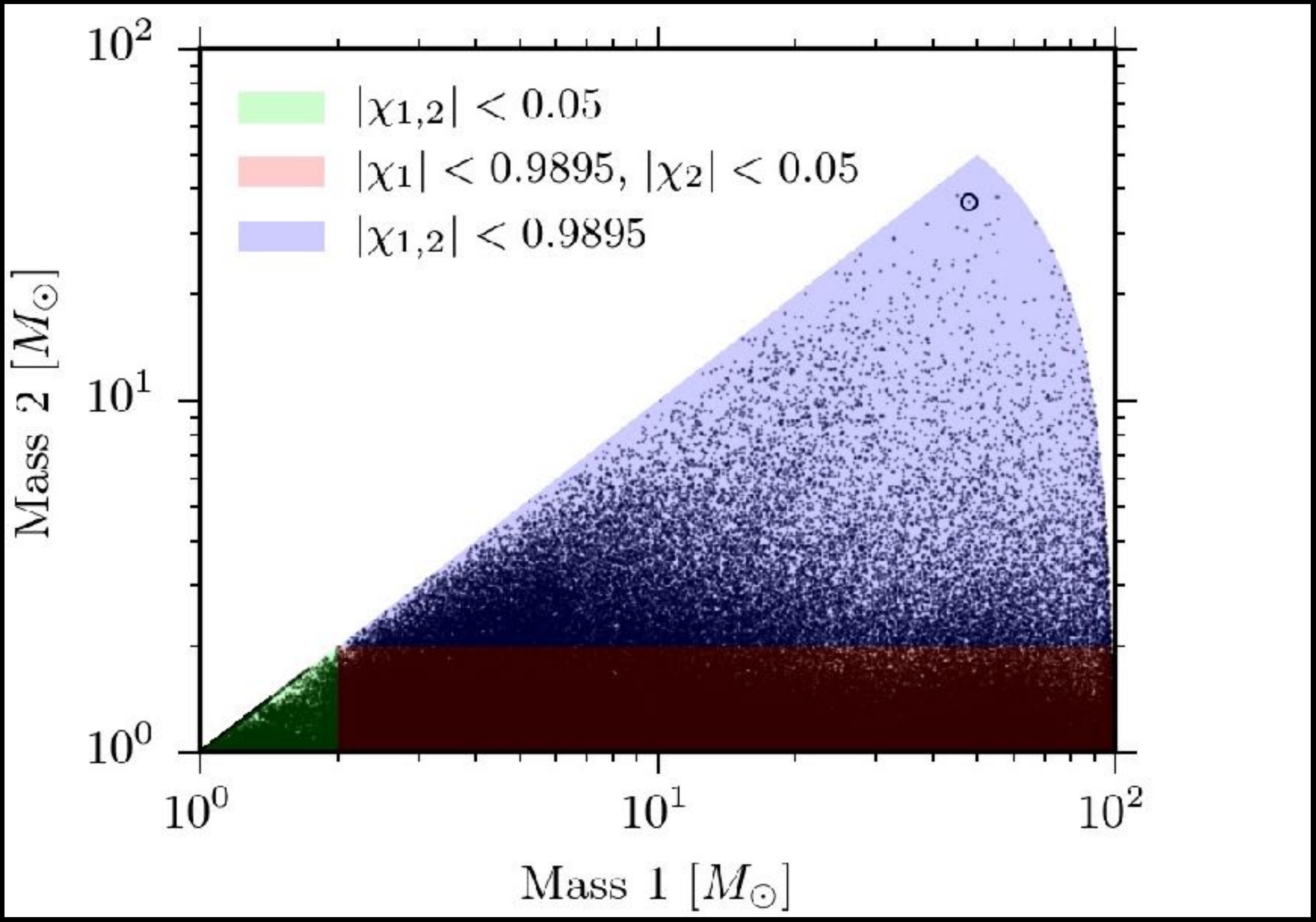


Gravitational waves are standard sirens

[general relativity predicts waveform]

$$h_c(t_{\rm obs}) \sim \frac{\mathcal{M}_z^{5/3}\, f_{\rm obs}^{2/3}}{d_{\rm L}^{\rm gw}}$$

Gravitational waves are standard sirens



Gravitational waves are standard sirens

[well understood detectability]

[cosmo propagation]

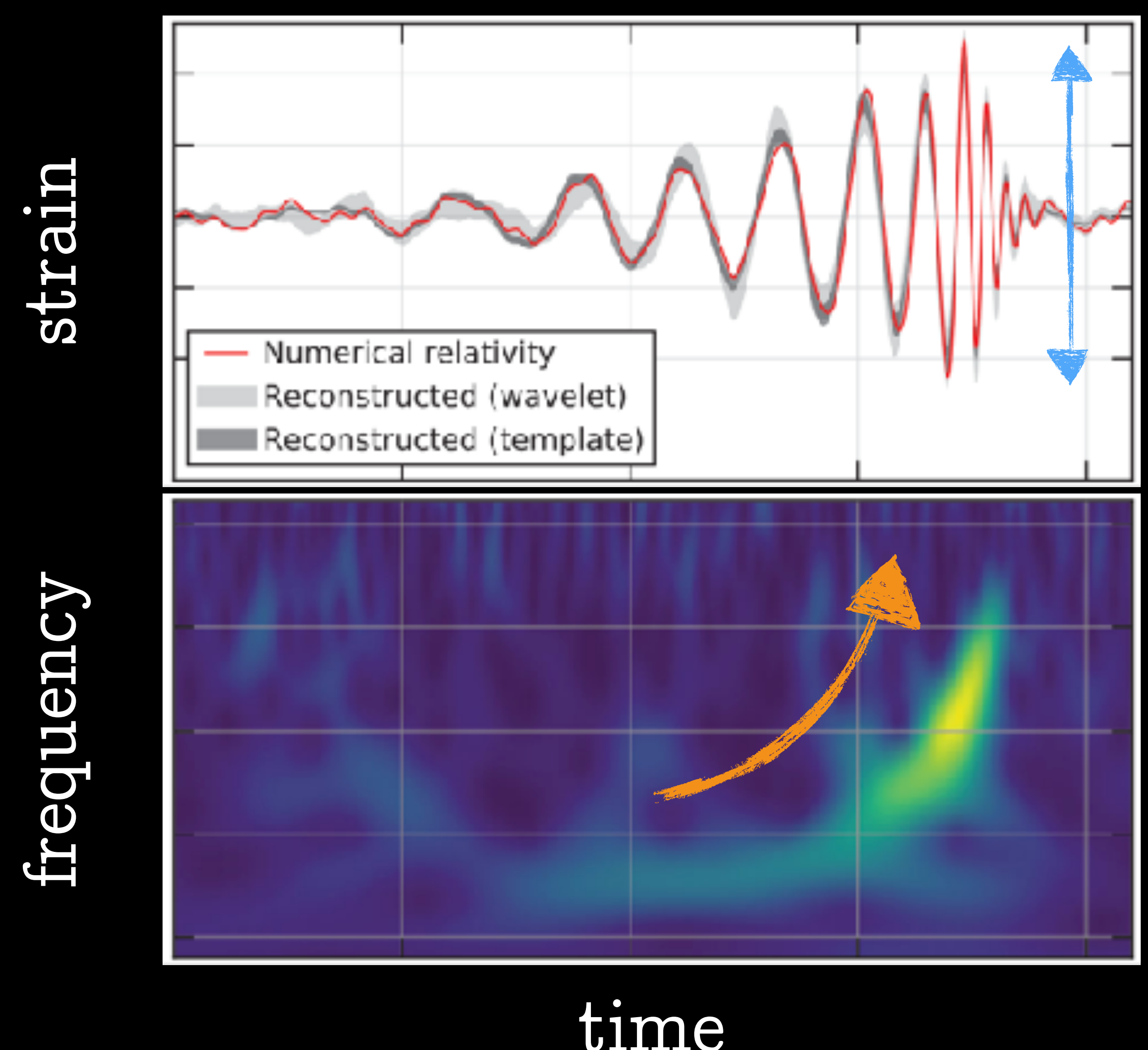
[general relativity predicts waveform]

$$d_L(z)$$

[GW Hubble diagram]

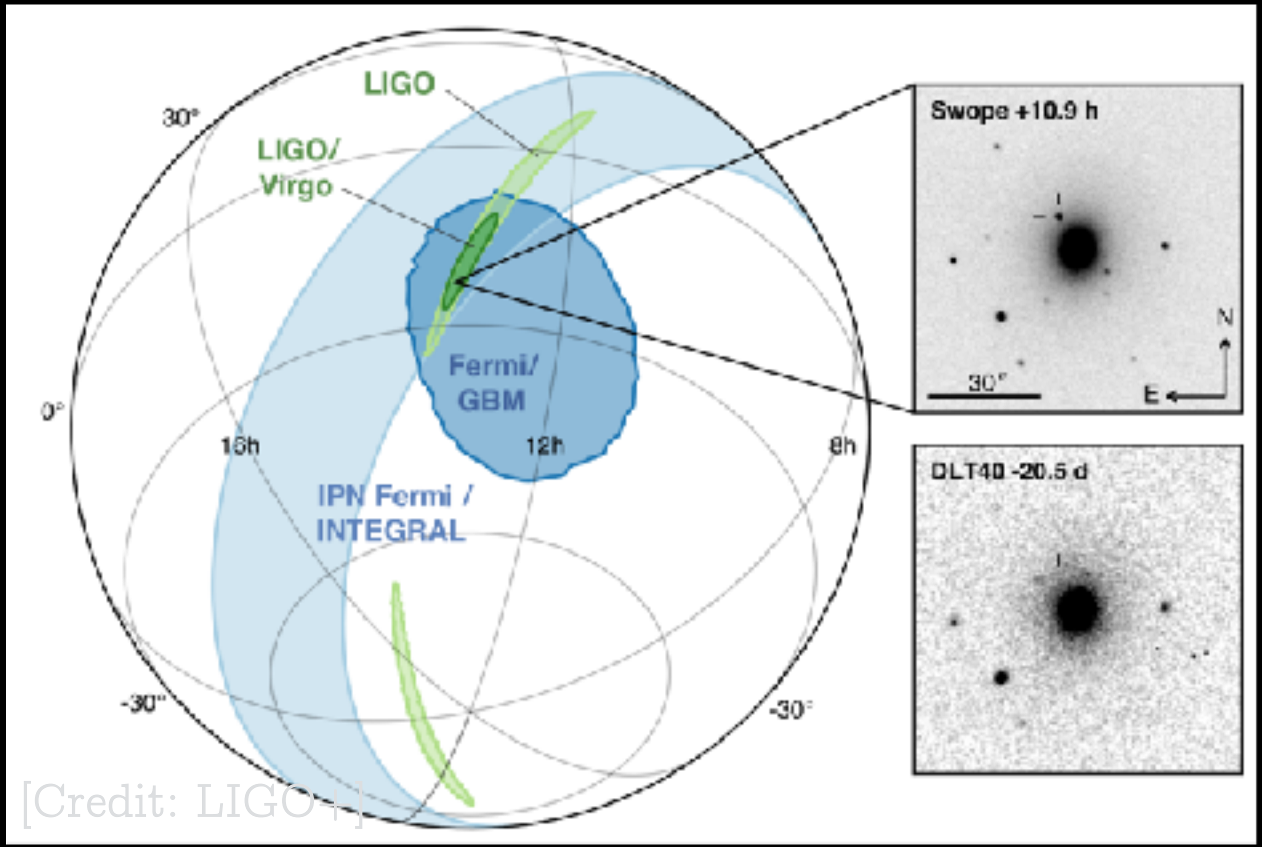
$$m_{\rm det} = (1+z)m$$

[Interplay with astrophysics]

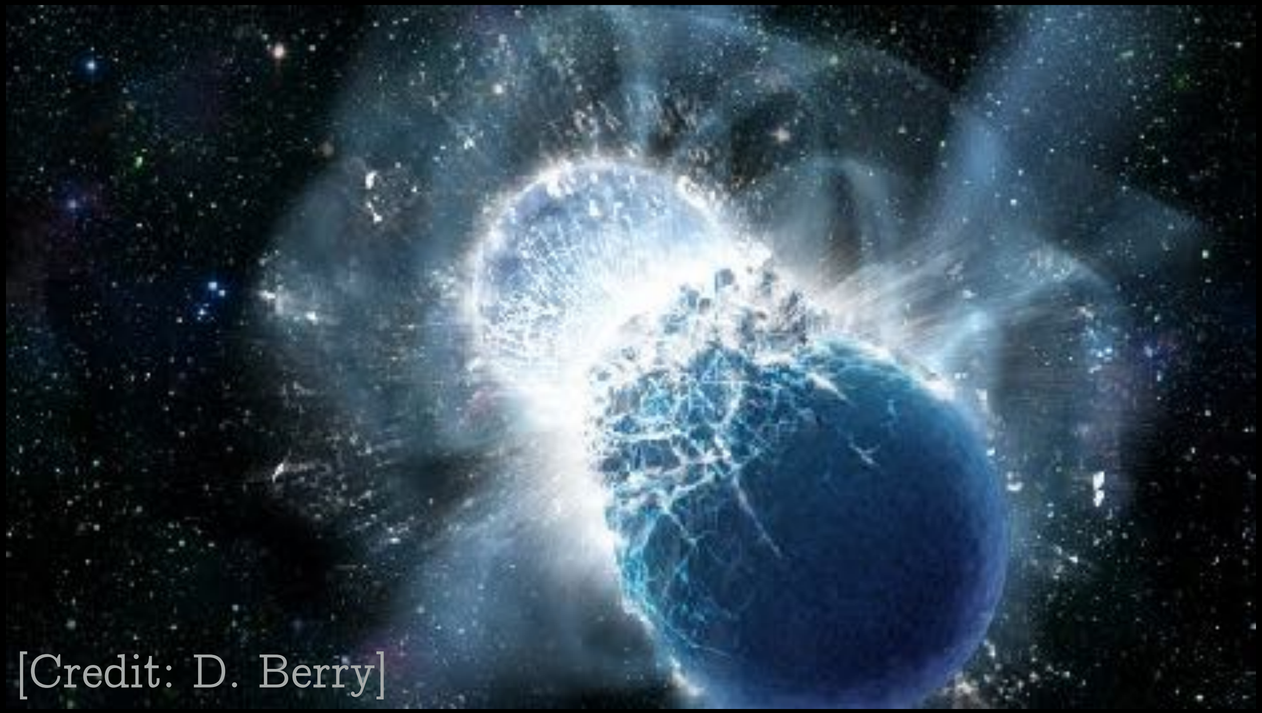


BRIGHT SIRENS

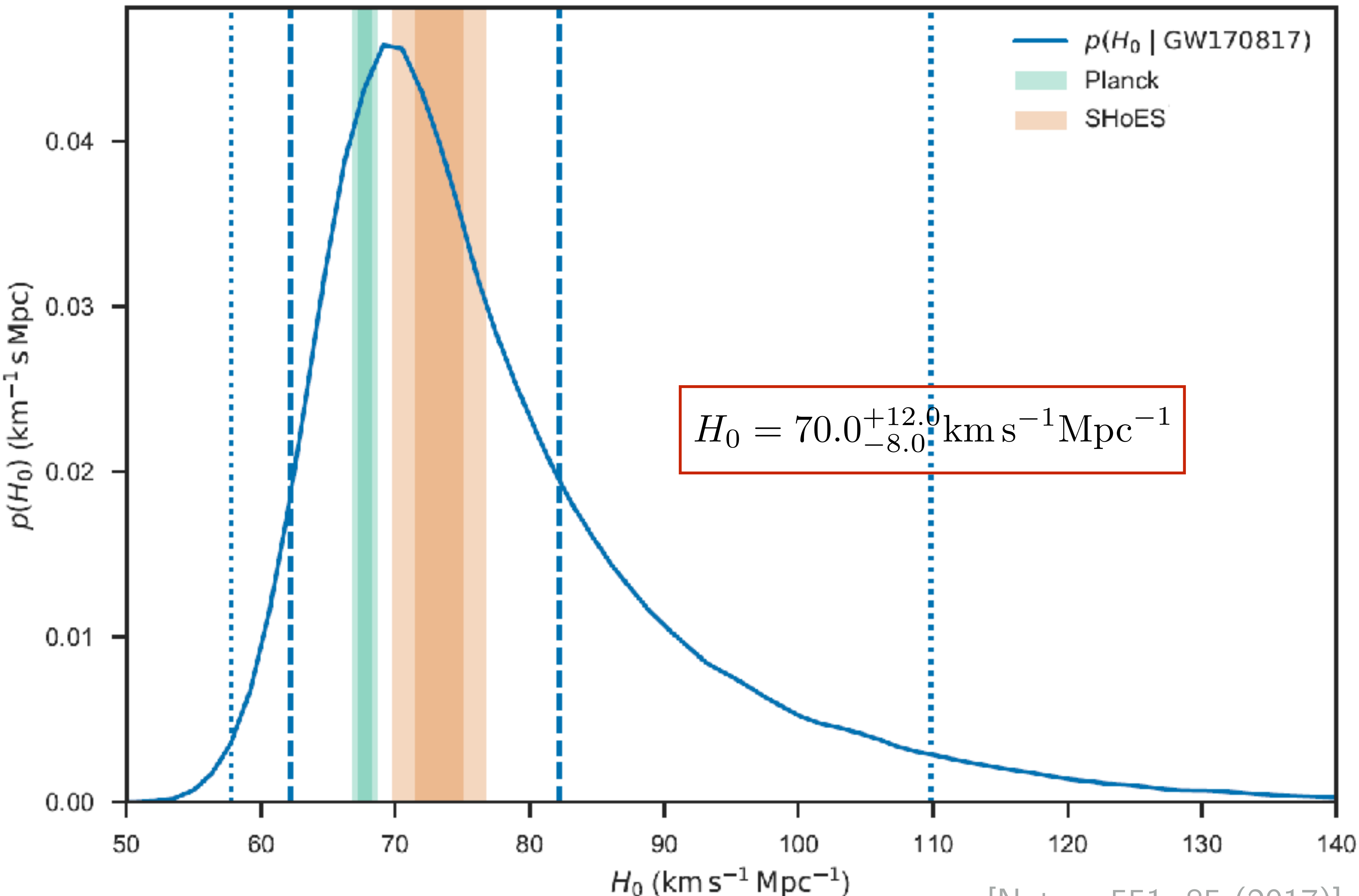
- Redshift from electromagnetic counterpart (e.g. identifying host galaxy)
- GW170817
- Need matter around merger: **neutron stars**!, AGN?
- Bright counterpart at high-z?



[Credit: LIGO+]


[Credit: D. Berry]

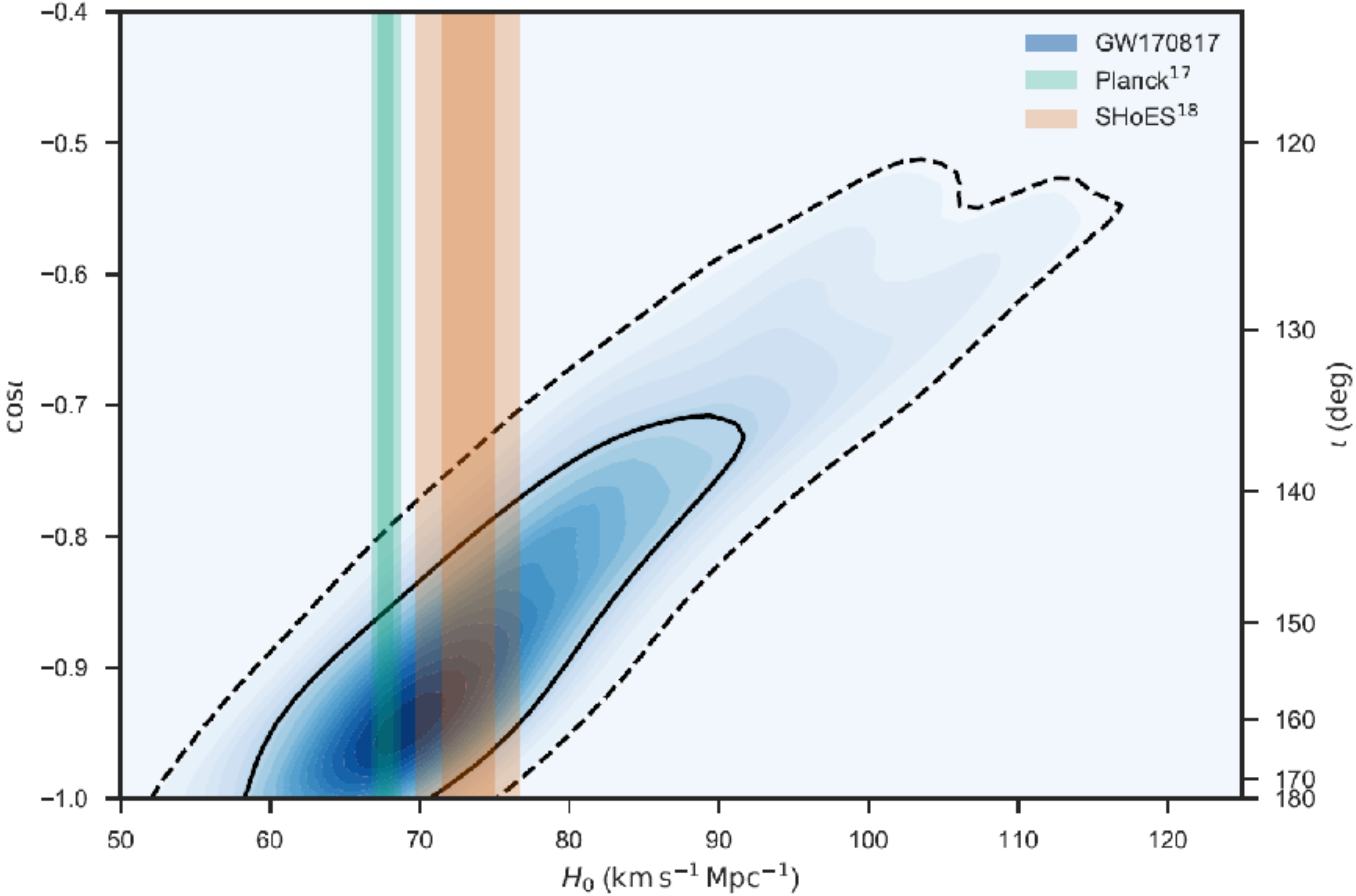
Bright sirens



$$H_0 = 70.0^{+12.0}_{-8.0}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

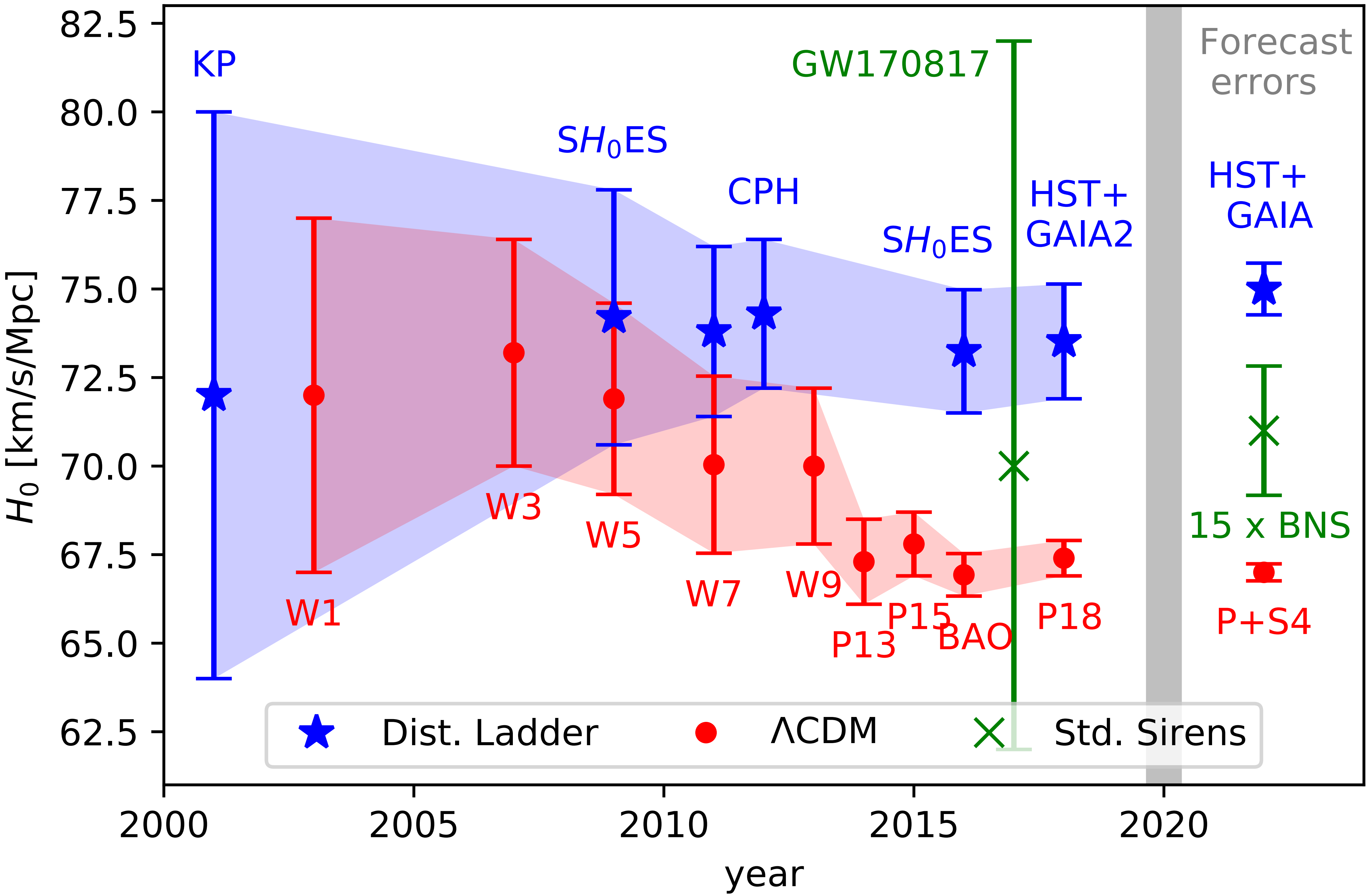
[Nature 551, 85 (2017)]

Inclination matters! [recall Enrico's talk]



[Nature 551, 85 (2017)]

Solve Hubble tension?



$$\sigma_{H_0}^{\text{tot}} \sim \frac{\sigma_{H_0}}{\sqrt{N}}$$

[Ezquiaga & Zumalacárregui '18]

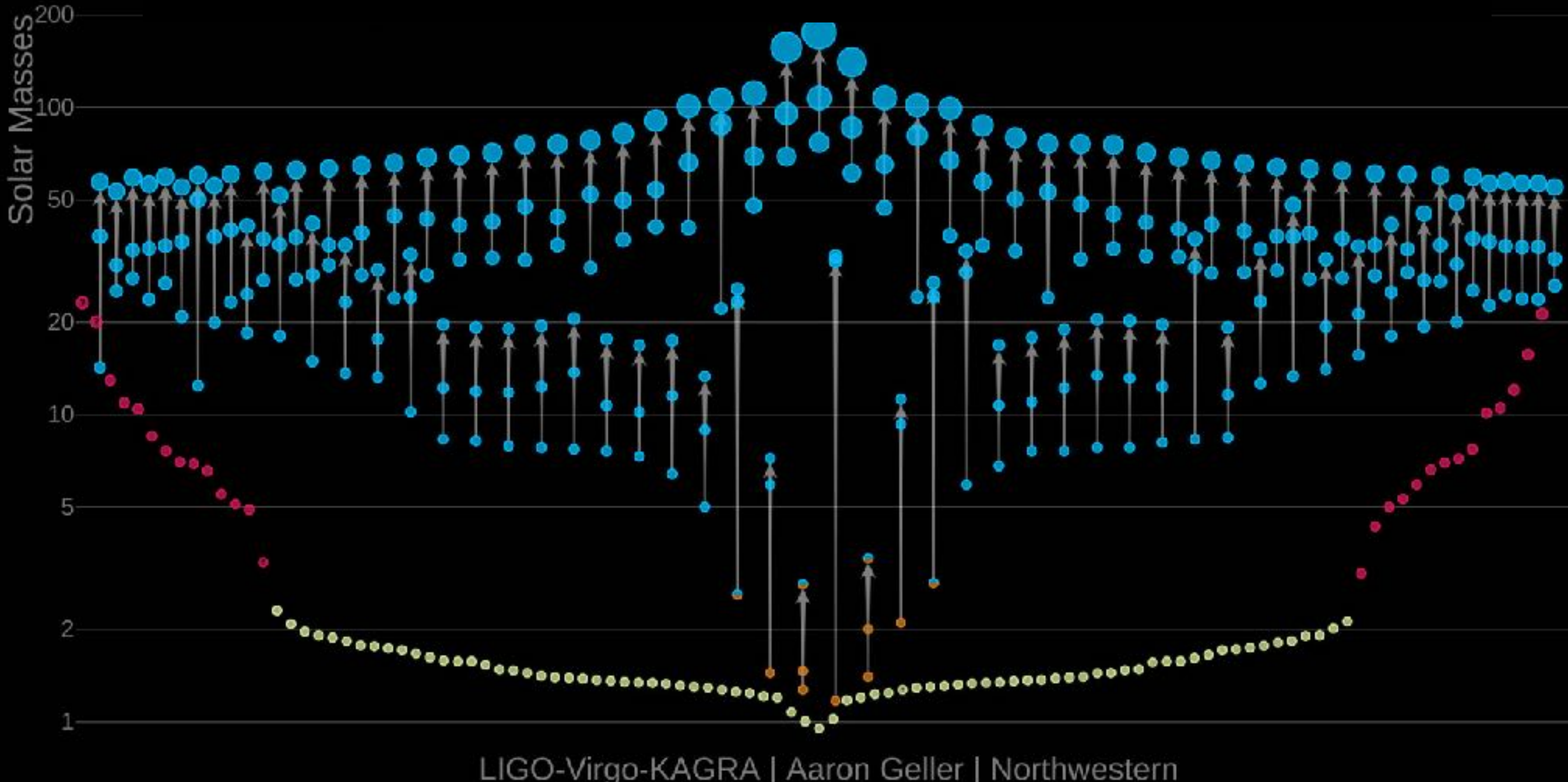
Where are the binary neutron stars?

- O1-O2 BNS rate: **110** — 3840 / Gpc³ / yr (90% CI for 1 model)
  - Confident BNS: GW170817
- O3a BNS rate: **80** — 810 / Gpc³ / yr (90% CI for 1 model)
  - Confident BNS: GW190425
- O3a BNS rate: **10** — 1700 / Gpc³ / yr (90% CI across 3 models)
  - Confident BNS: None
- O4 significant BNS candidates so far: **0?**

Predictions for O4: $7.7^{+11.9}_{-5.7}\,\mathrm{yr}^{-1}$ BNS

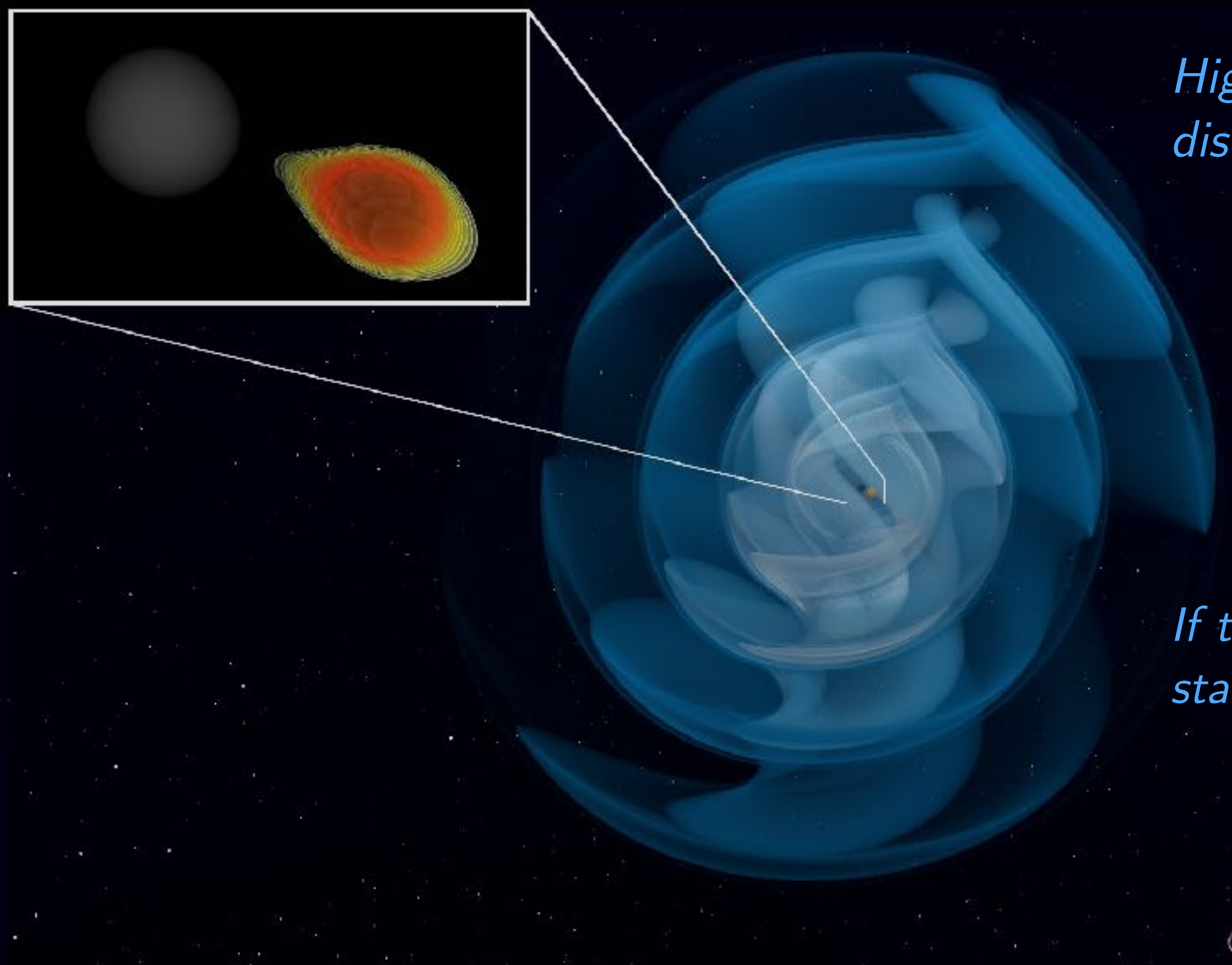
[2204.07592] 74% kilonova, 2% GRB

Neutron star - black hole mergers to the rescue?



LVK Black Holes LVK Neutron Stars EM Black Holes EM Neutron Stars

Neutron star - black hole mergers to the rescue?

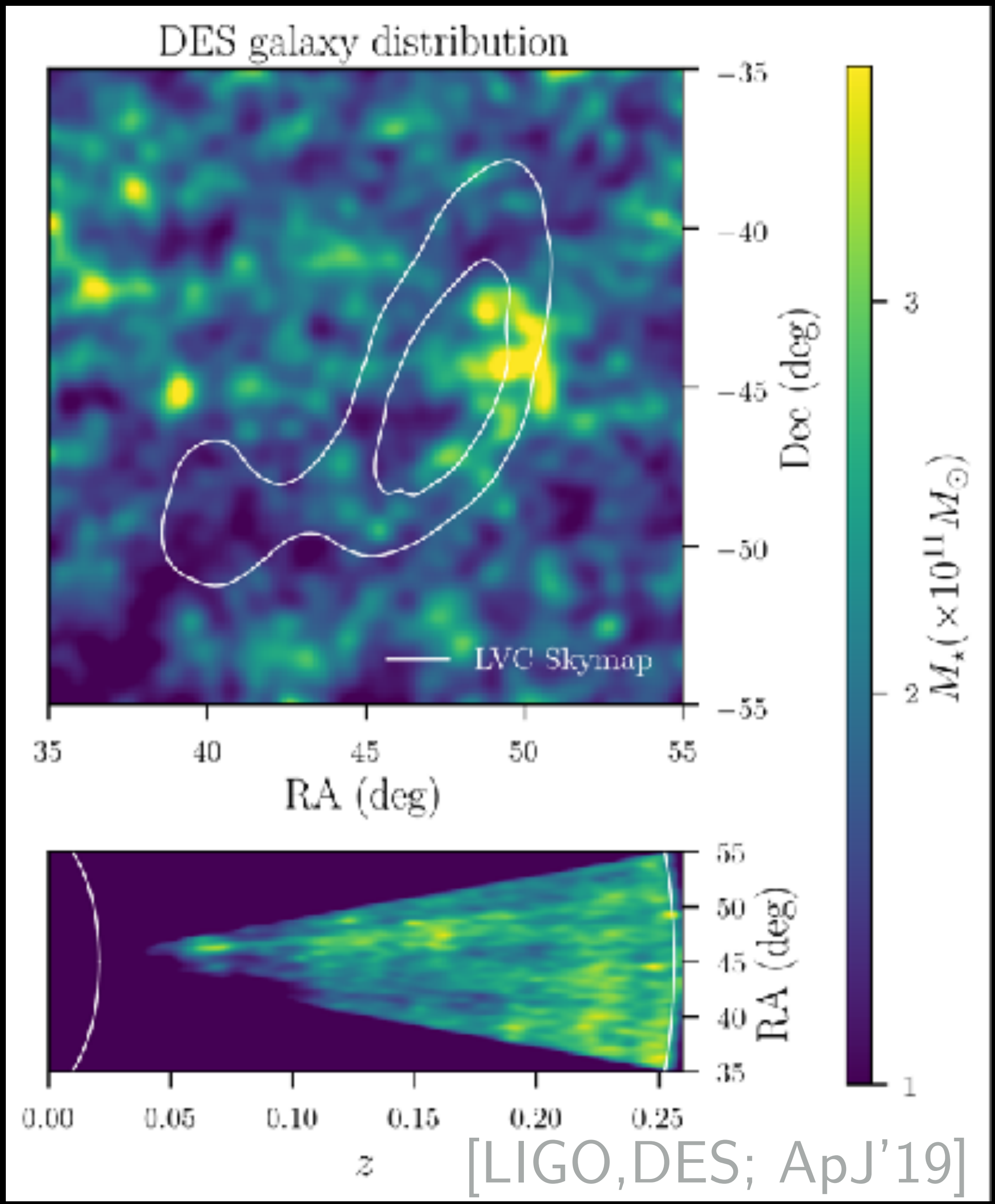


Higher modes to break distance degeneracies!

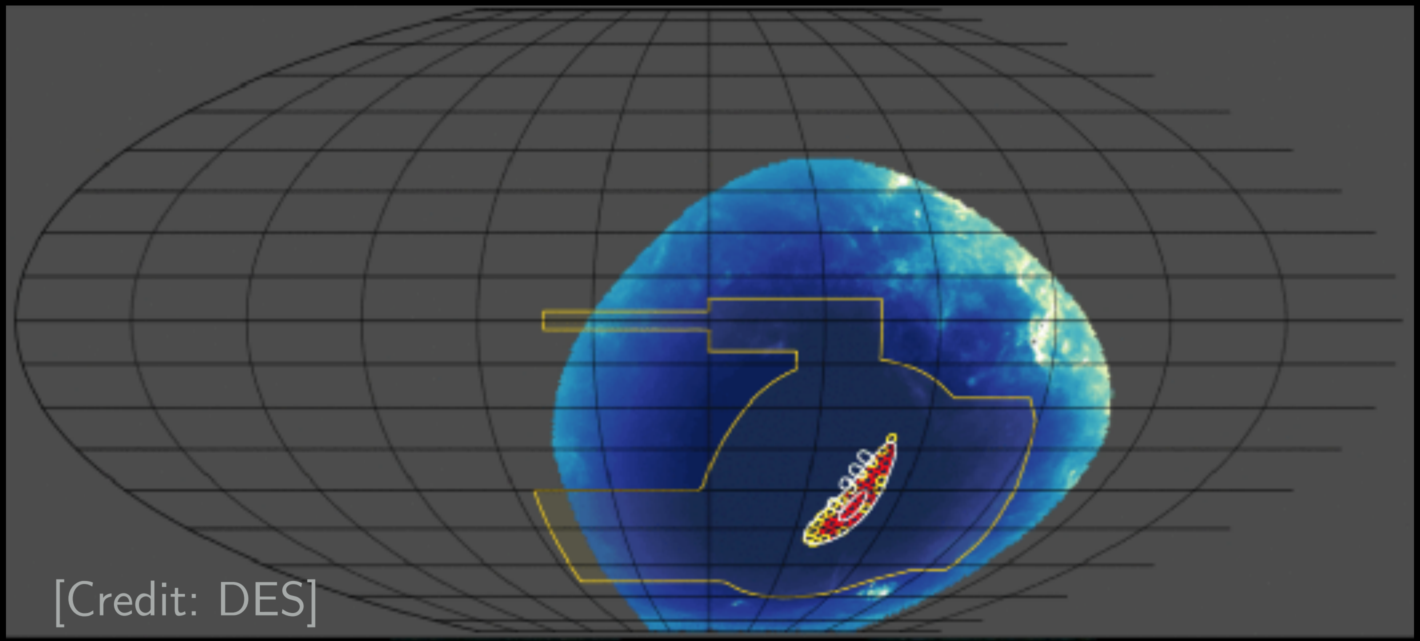
If too asymmetric neutron star is quickly eaten...

DARK SIRENS

- Statistically infer *z* from galaxies in localization volume
- E.g. GW170814
- Need good localization and **complete** galaxy catalogs!



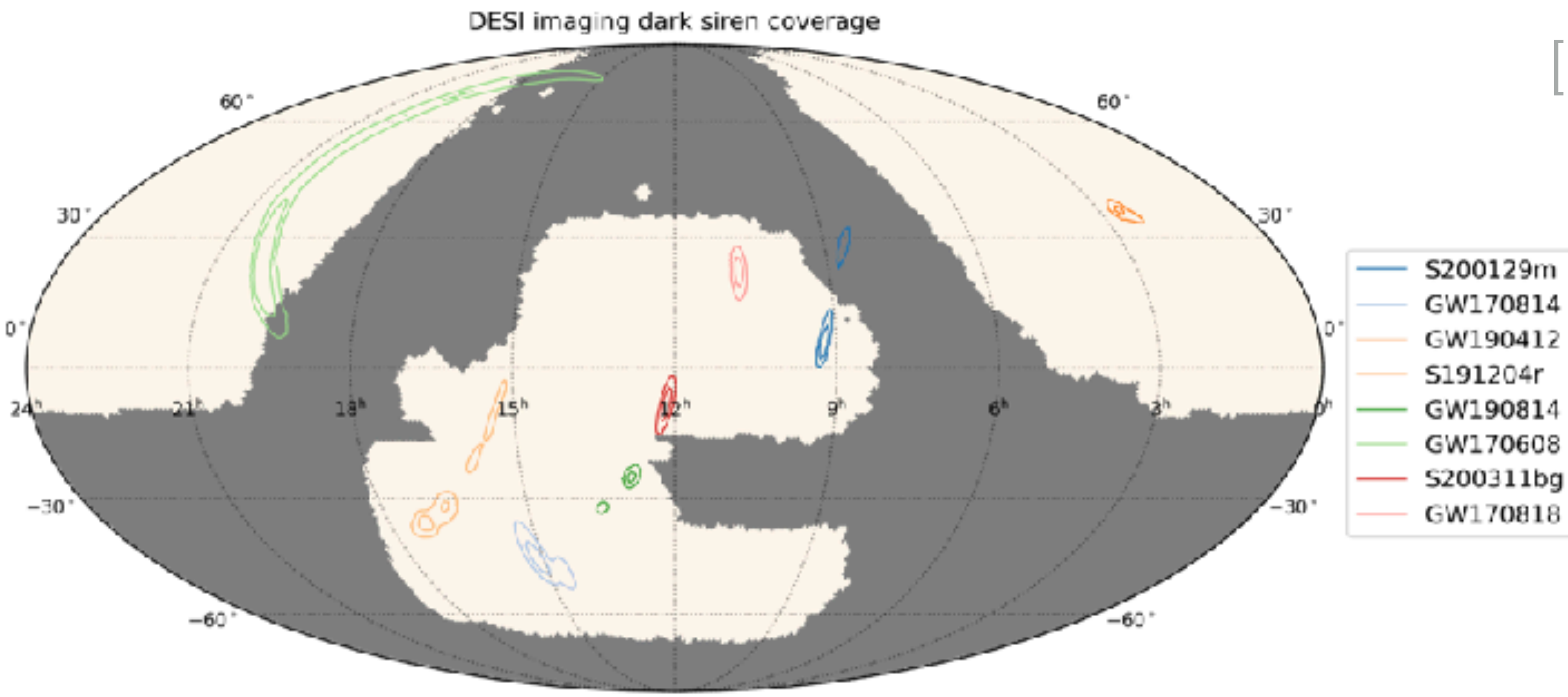
[LIGO,DES; ApJ'19]

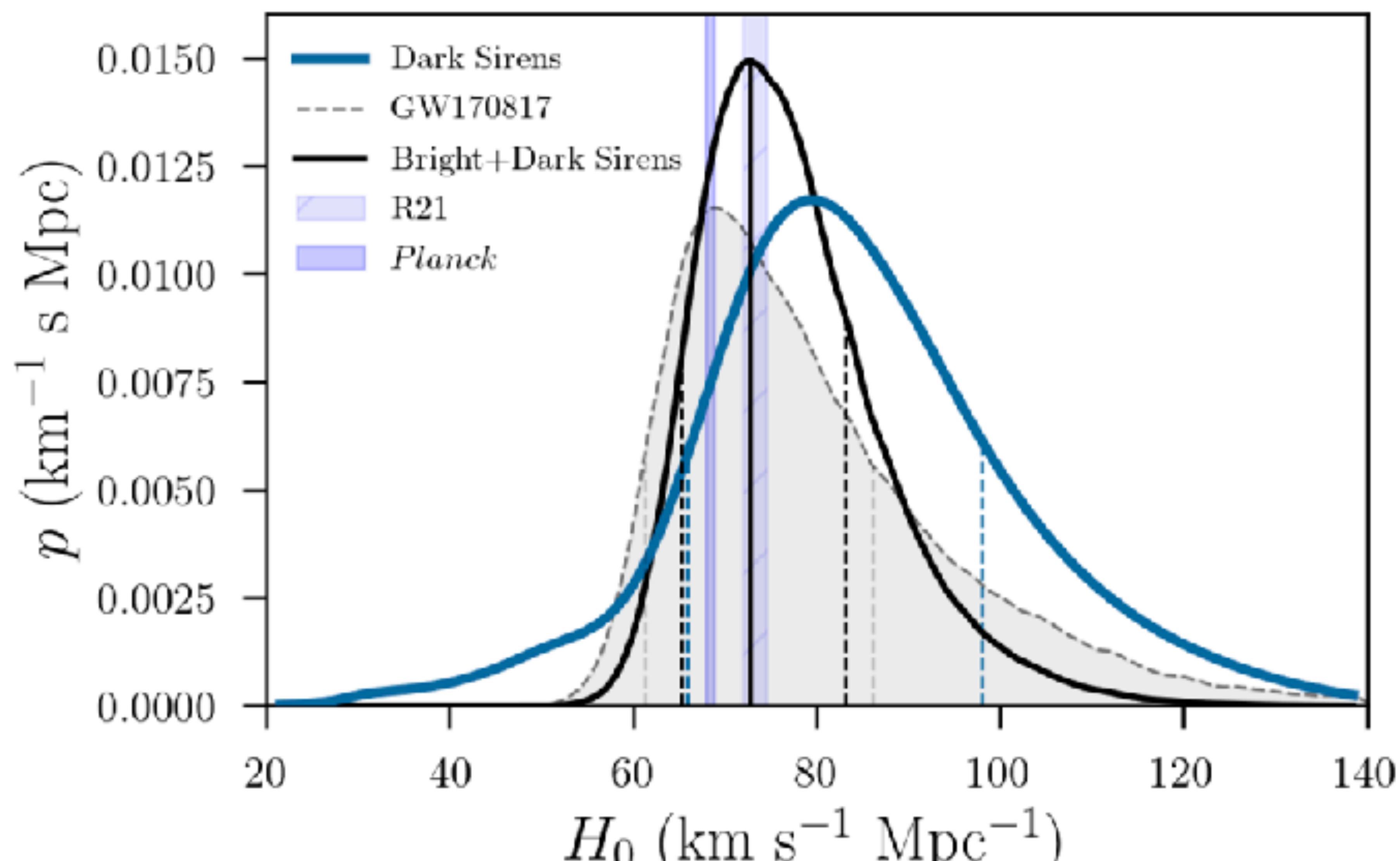


[Credit: DES]

For more details see:
Hitchhiker guide GW galaxy catalog cosmo (arXiv 2212.08694)

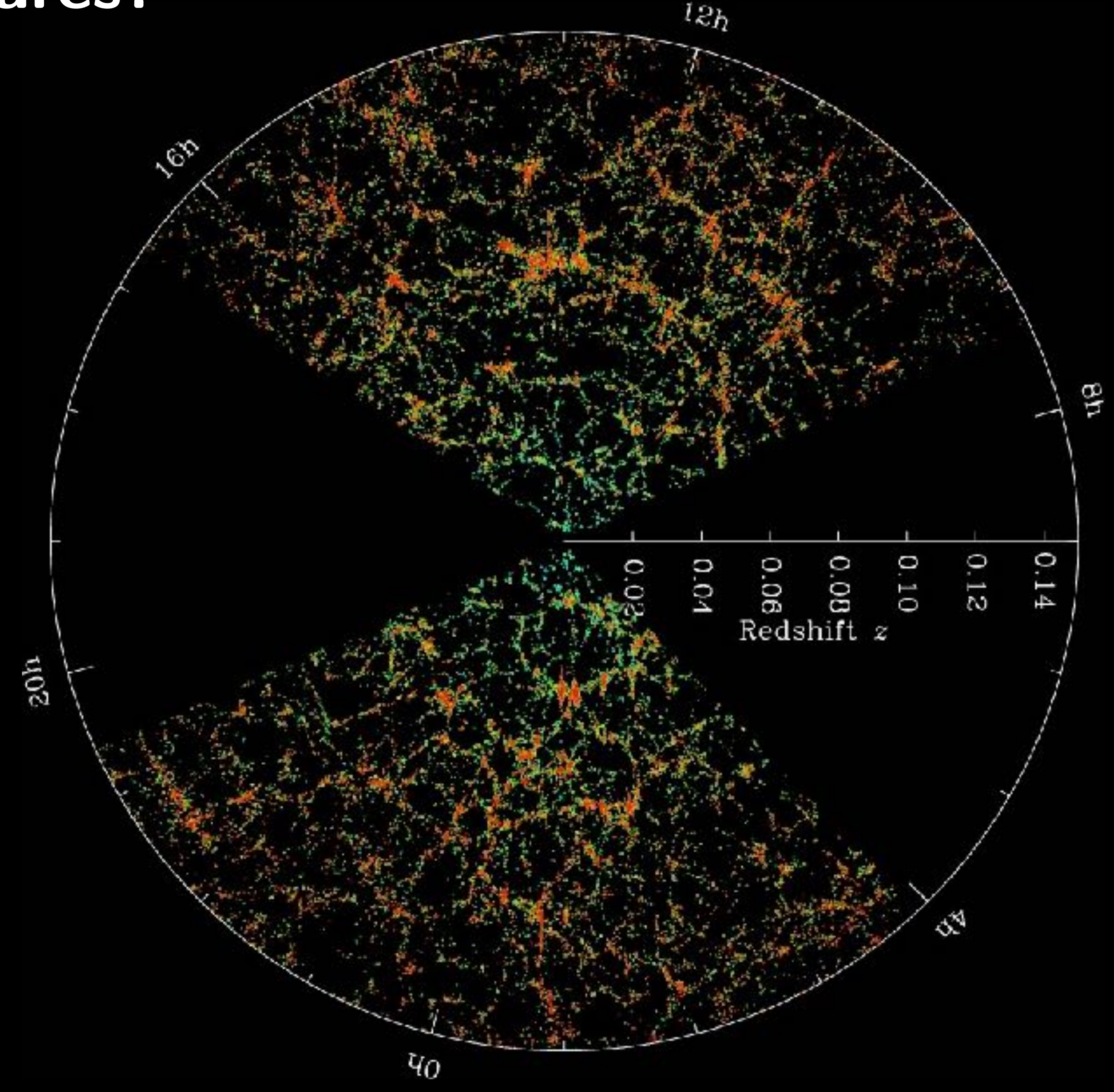
[Palmese et al., ApJ'23]





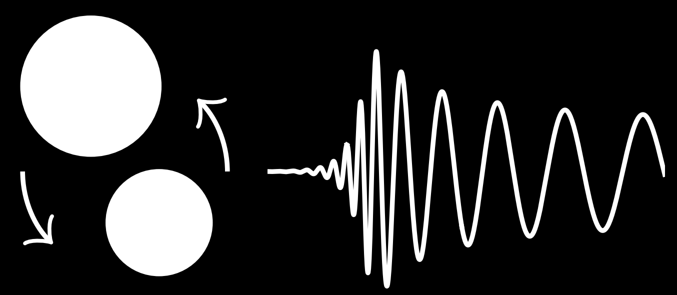
Dark sirens

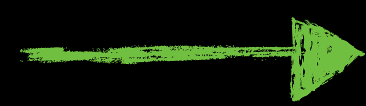
Do binary black holes trace the large scale structures?

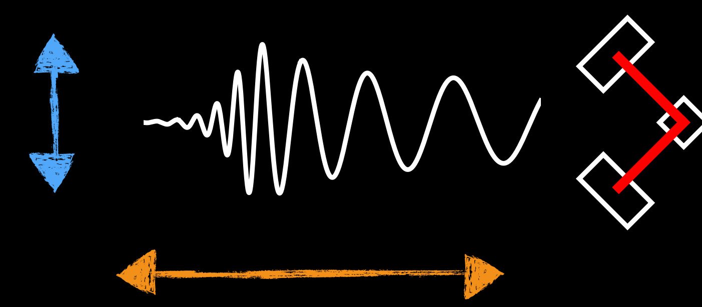


Gravitational waves are standard sirens

[well understood detectability]







[cosmo propagation]

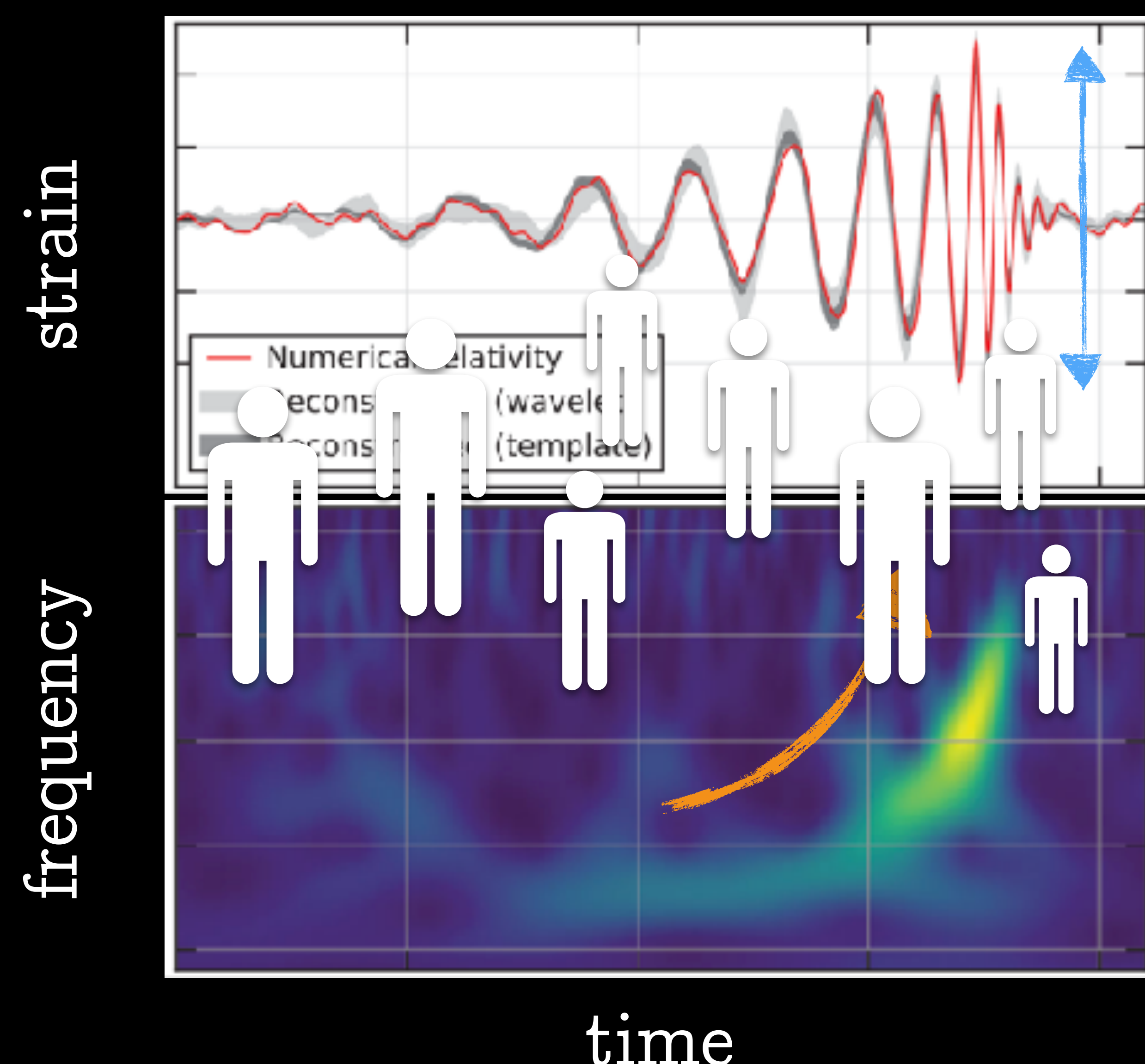
[general relativity predicts waveform]

$$d_L(z)$$

[GW Hubble diagram]

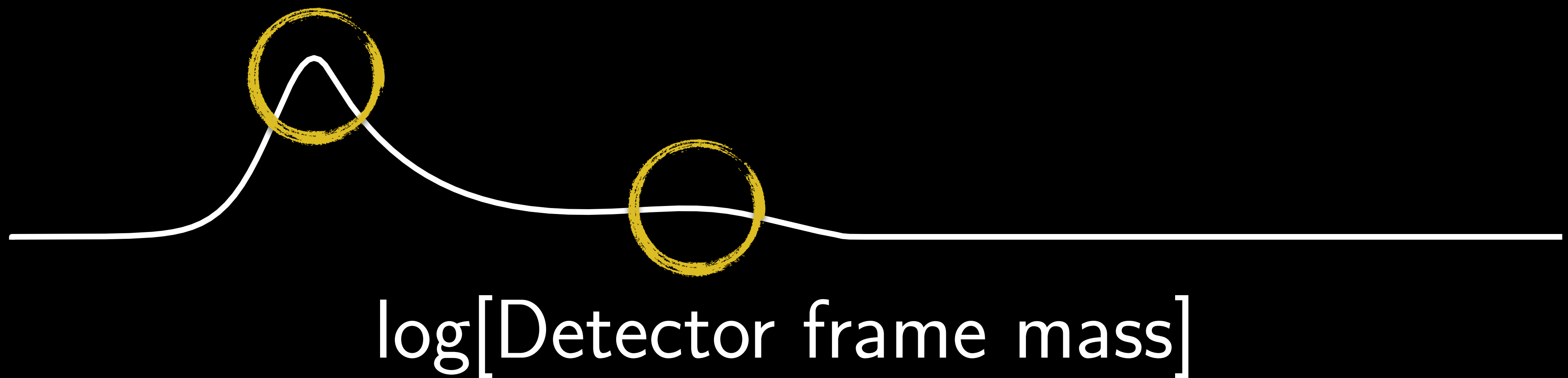
$$m_{\rm det} = (1+z)m$$

[Interplay with astrophysics]

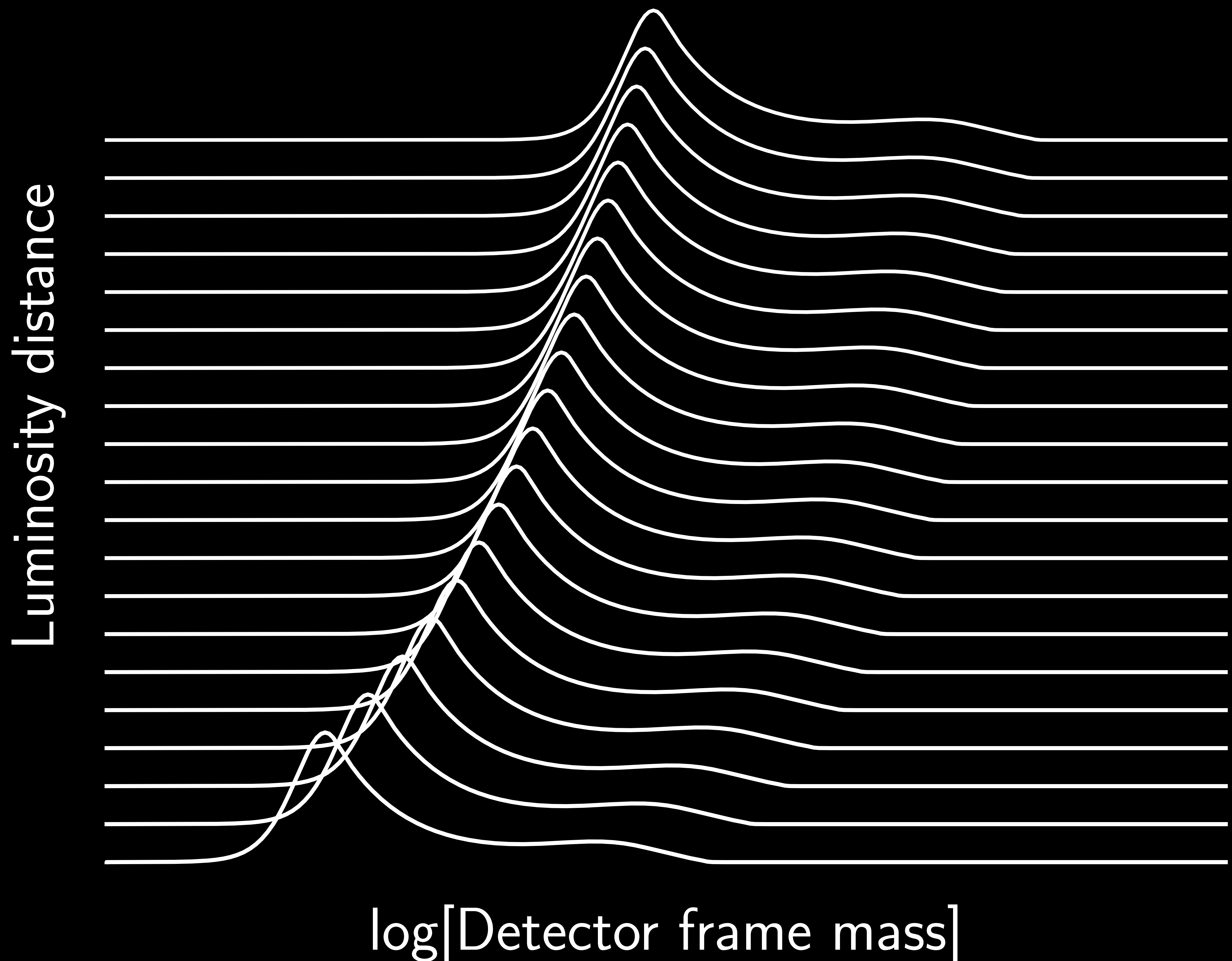


SPECTRAL SIRENS

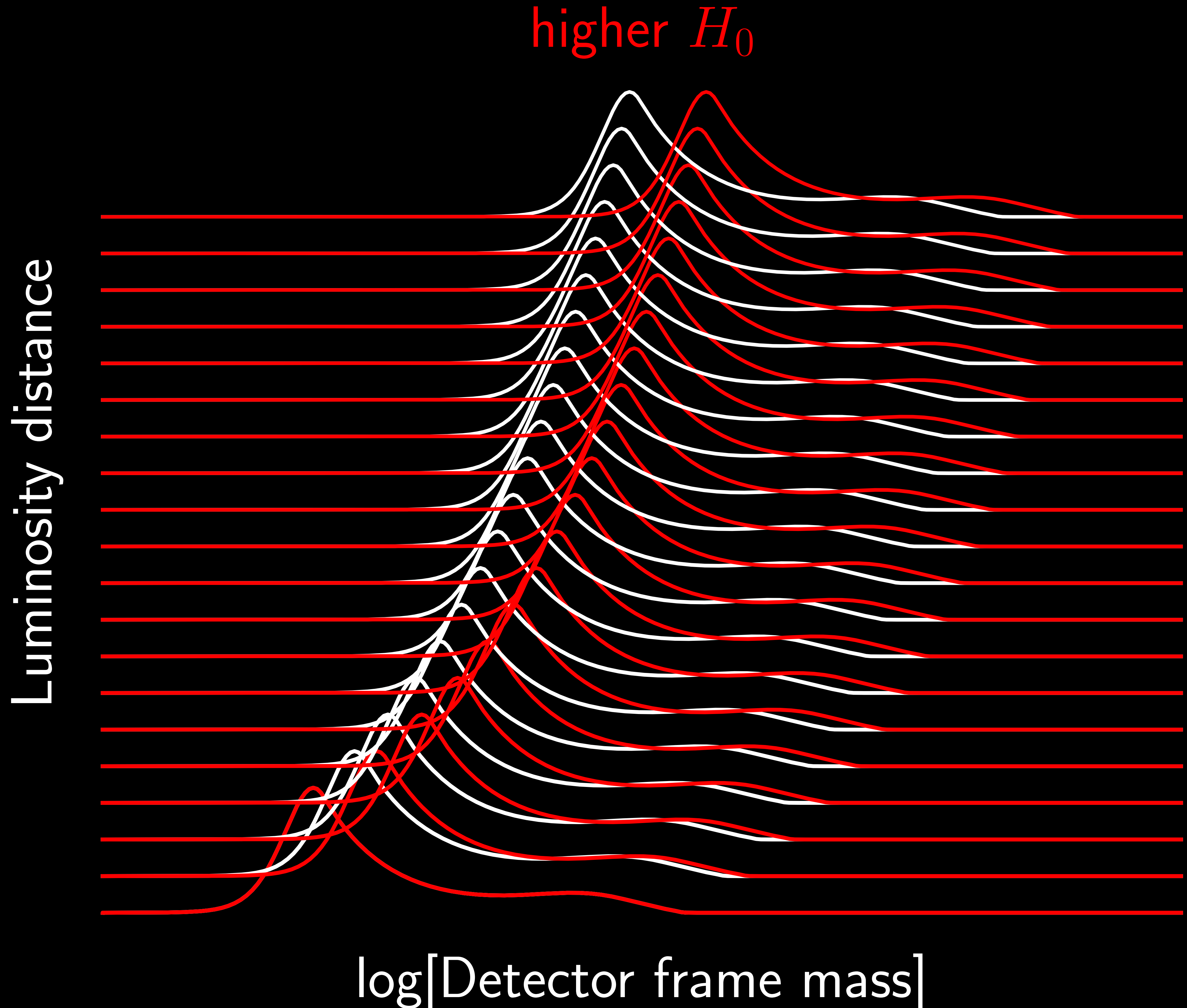
$$\{d_L(z), m_{\rm det} = (1+z)m\}$$



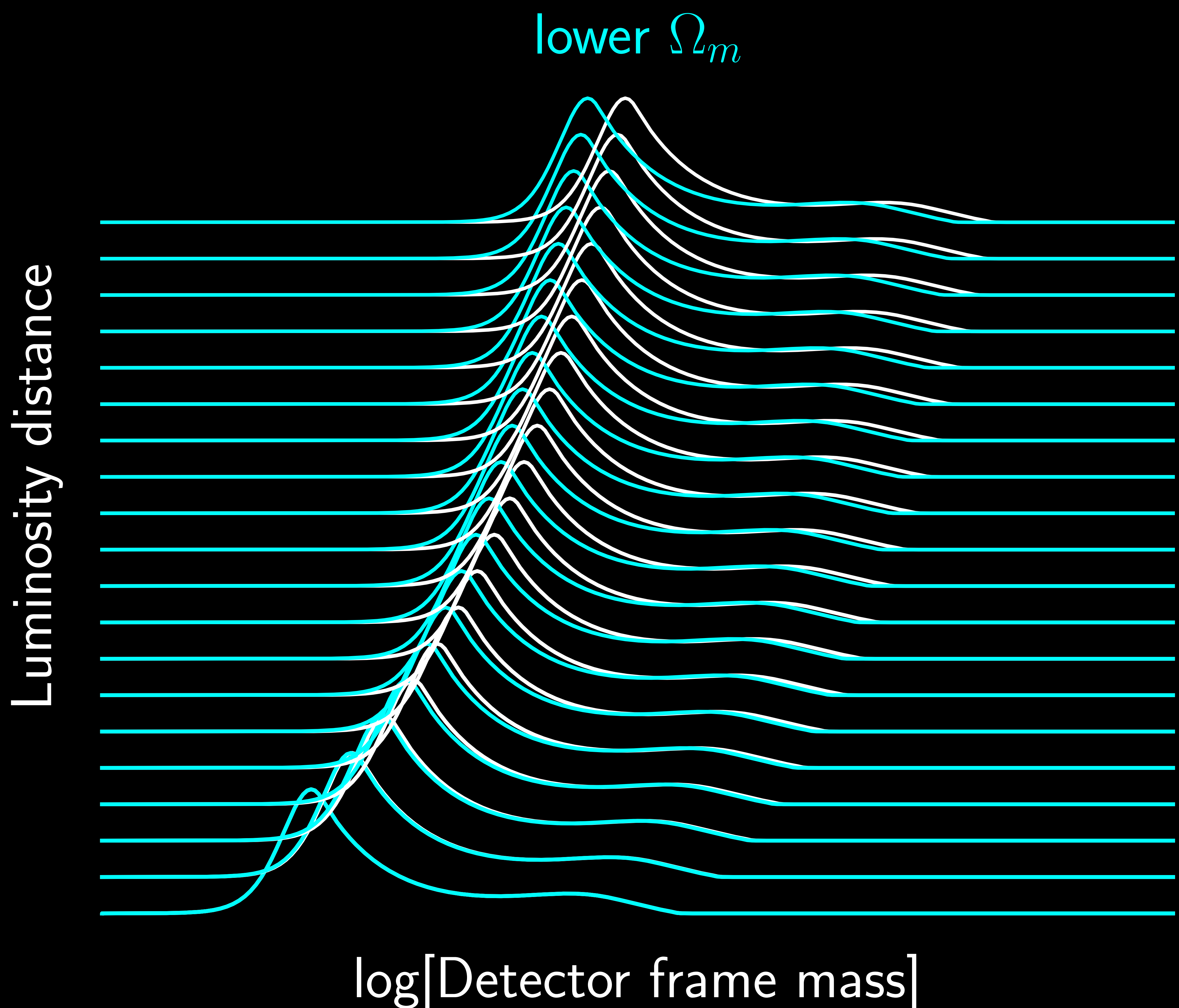
SPECTRAL SIRENS



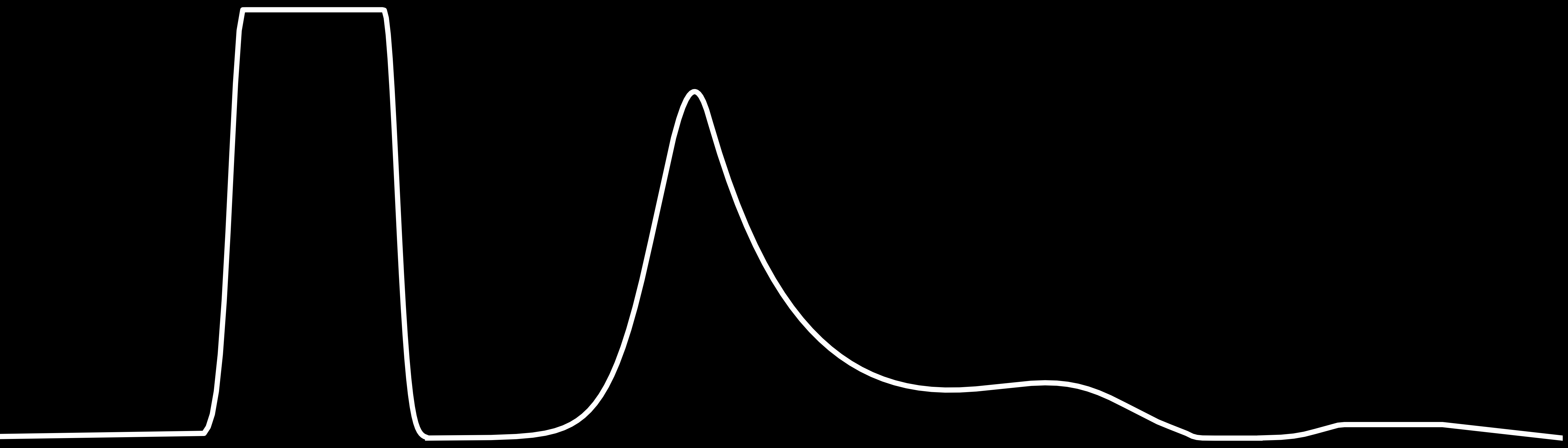
SPECTRAL SIRENS



SPECTRAL SIRENS



All compact binaries are standard sirens, no electromagnetic information is necessary



Binary neutron stars ***“Stellar-mass” binary black holes*** ***“Far-side” binaries***

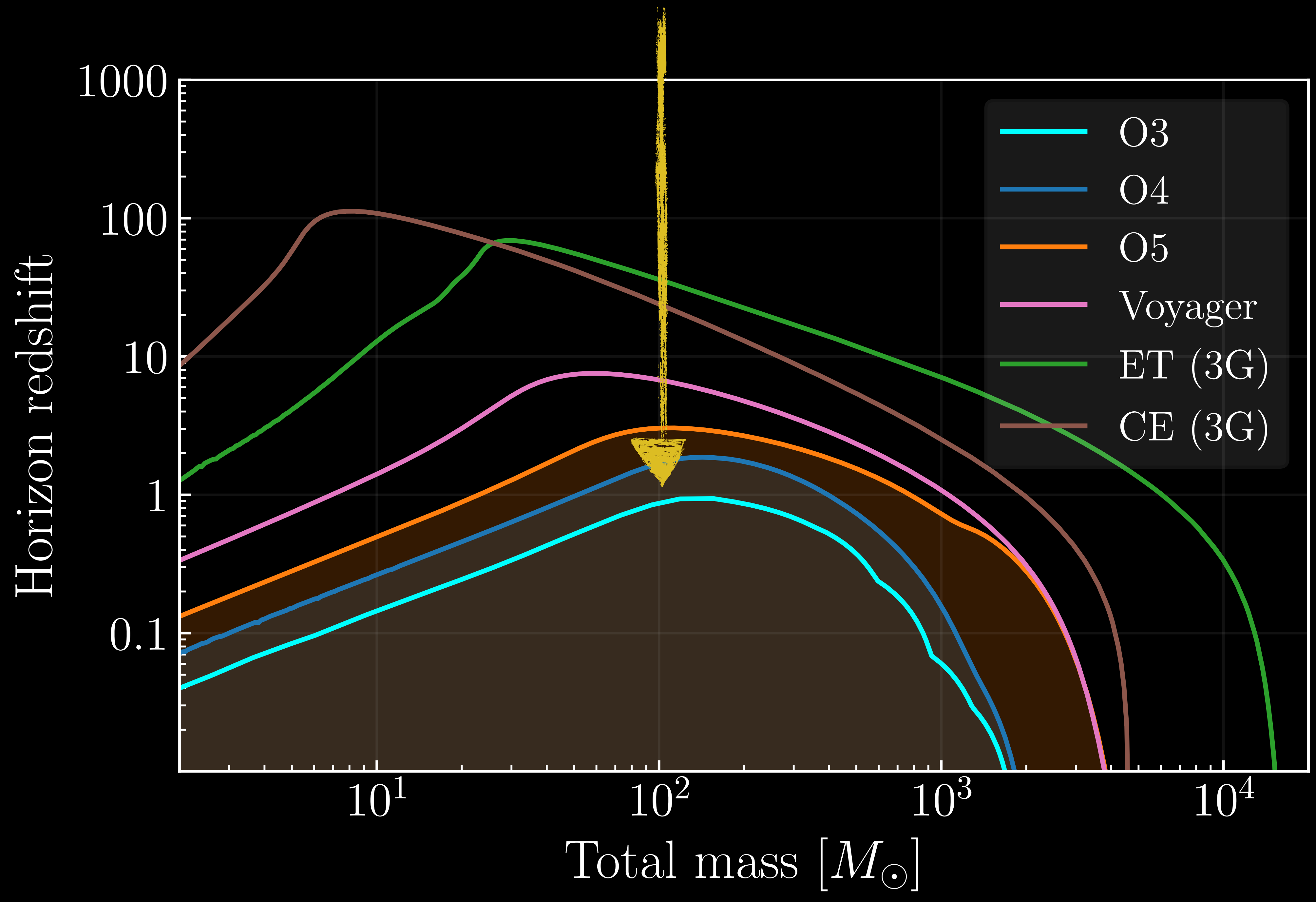
Ezquiaga & Holz; *Spectral sirens: Cosmology from full mass distribution of compact binaries* (PRL’22, arXiv 2202.08240)

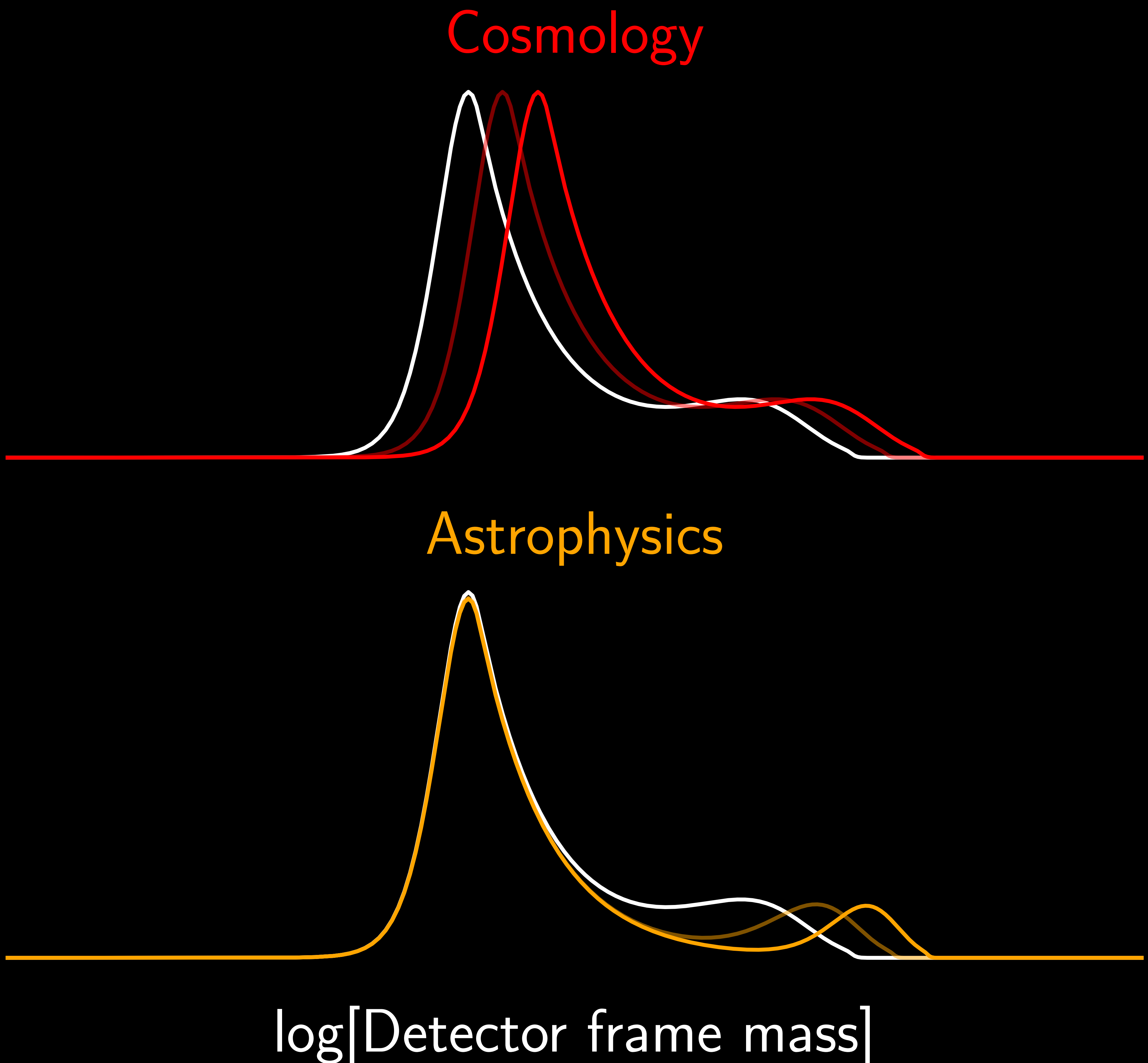
[Chernoff&Finn'93] [Roy+'24] [Farr+’19] [Mastrogiovanni+’21] [Ezquiaga&Holz’20]

[Taylor+'11] [Mali&Esscik’24] [You+'20] [LVC Cosmo GWTC-3 ’21]

All compact binaries are spectral sirens, no electromagnetic information is necessary

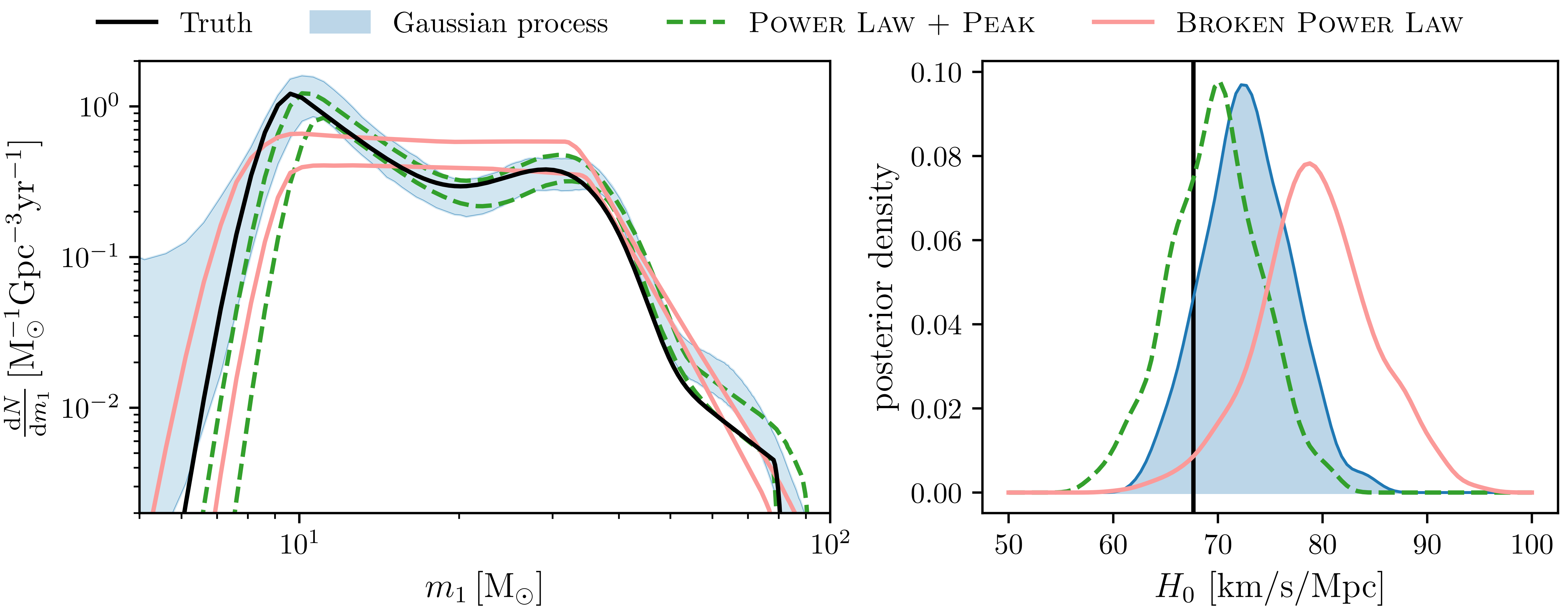
Currently, binary black holes most promising




Cosmology
Astrophysics
log[Detector frame mass]

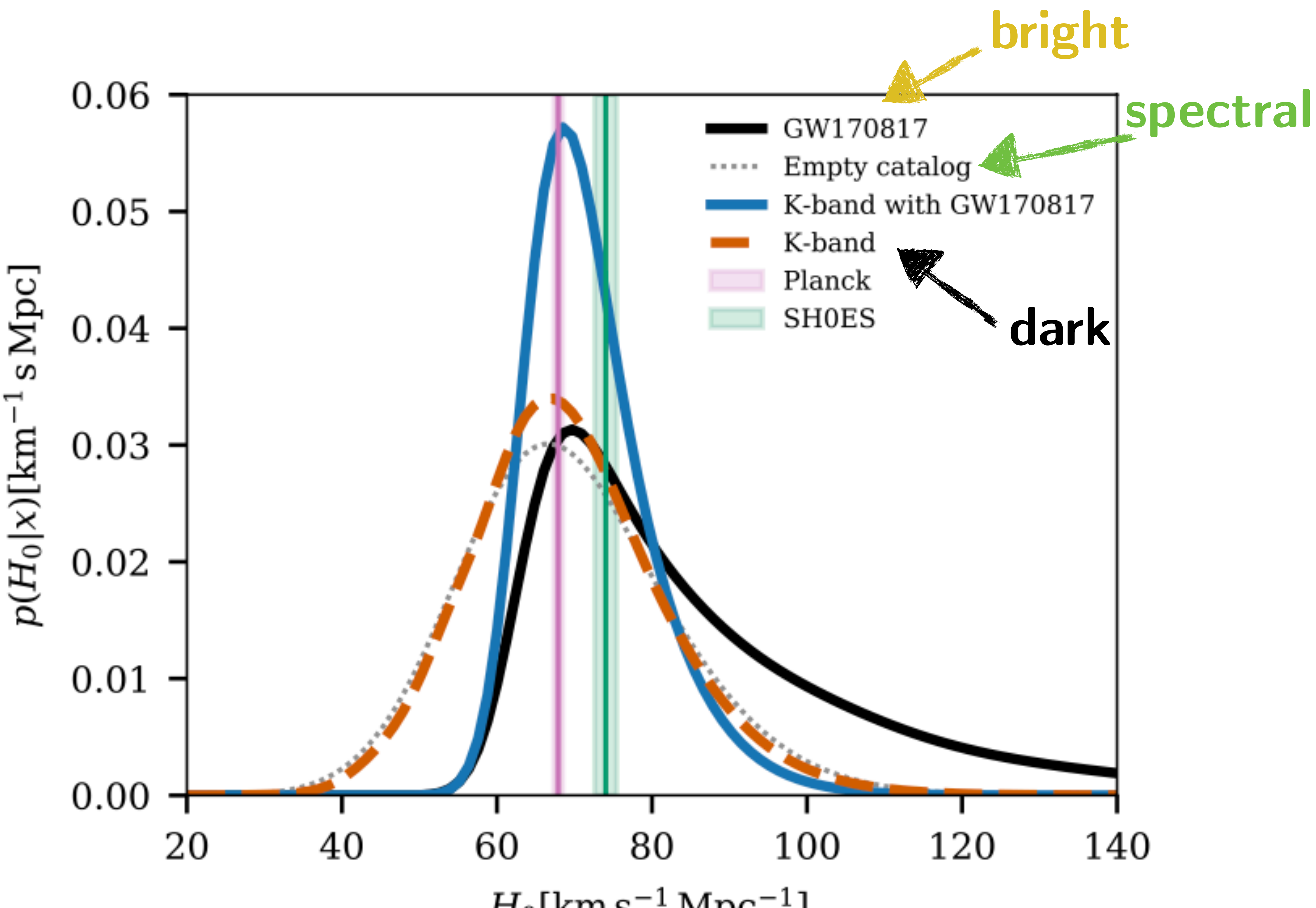
Non-parametric reconstruction

Unbiased cosmological inference *without* prior assumptions



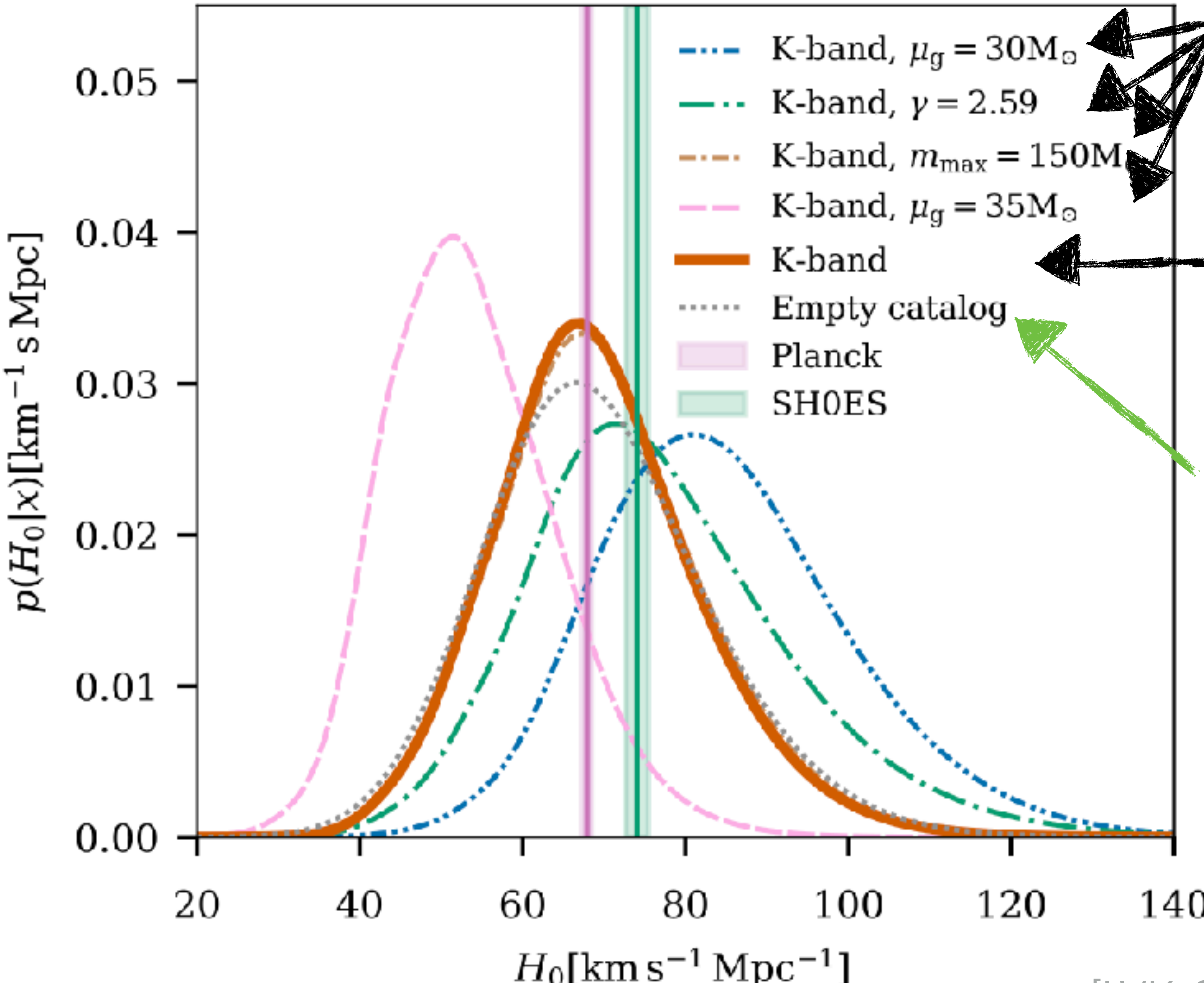
[Farah et al.; ApJ'24]

Standard sirens: *current results*



[LVK GWTC-3]

Standard sirens: *current results*



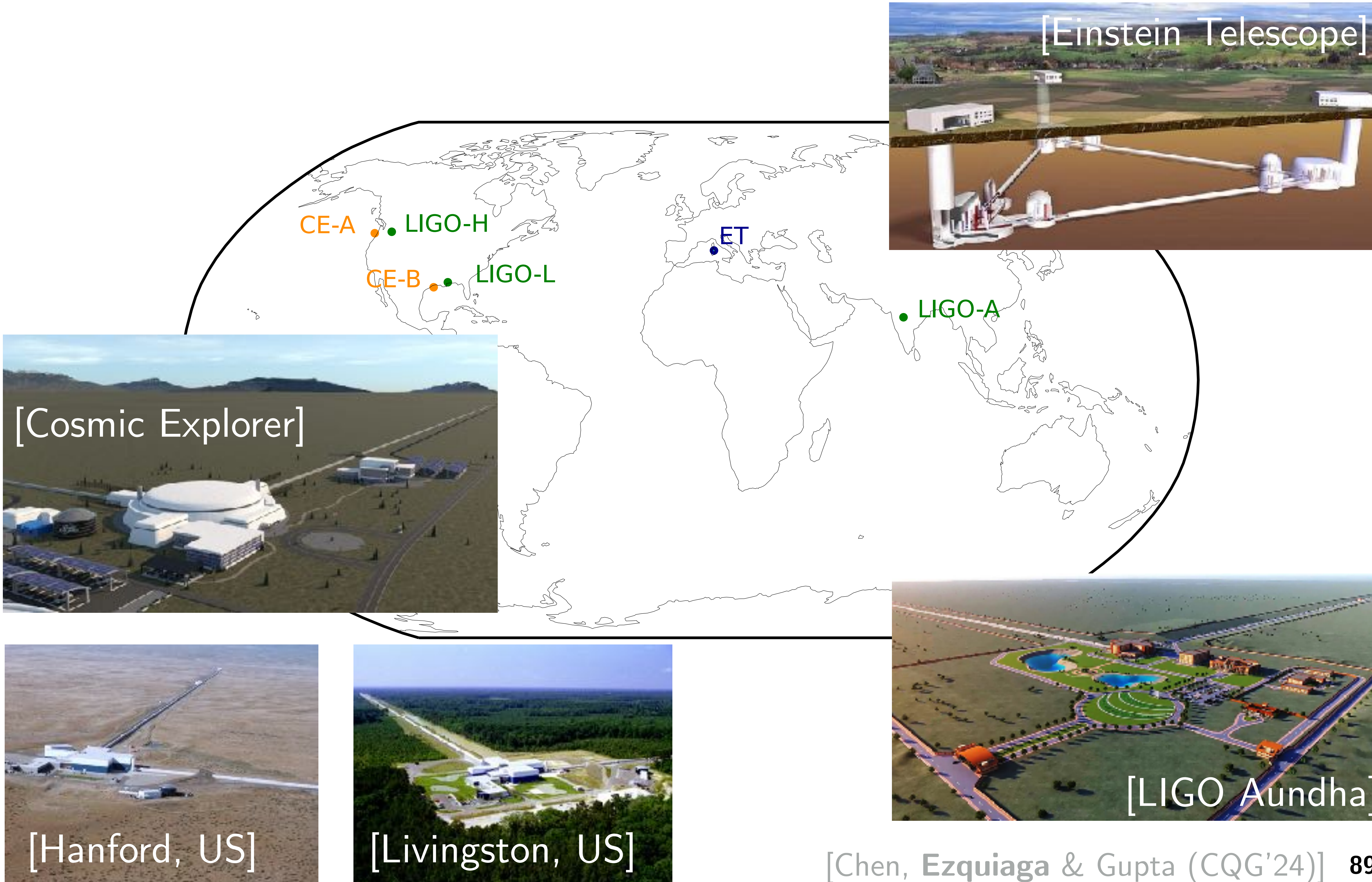
dark
(*other priors*)

dark
(*default*)

spectral

[LVK GWTC-3]

Standard sirens: *forecasts*



[Chen, **Ezquiaga** & Gupta (CQG'24)]

Bright sirens at higher distances

- If the Dark Energy equation of states evolves in time

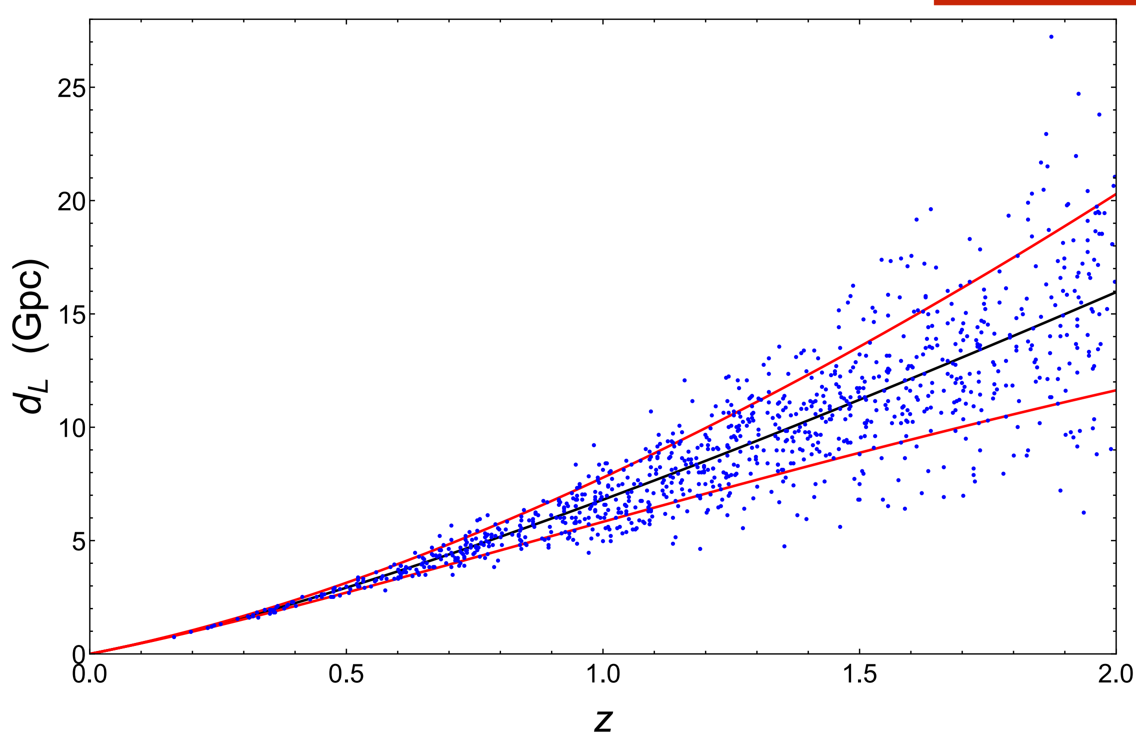
$$\Omega_{\rm DE}(z) = \Omega_\Lambda (1+z)^{3(1+w_0)} e^{-3\int_0^z \frac{w(z')-w_0}{1+z'} dz'}$$

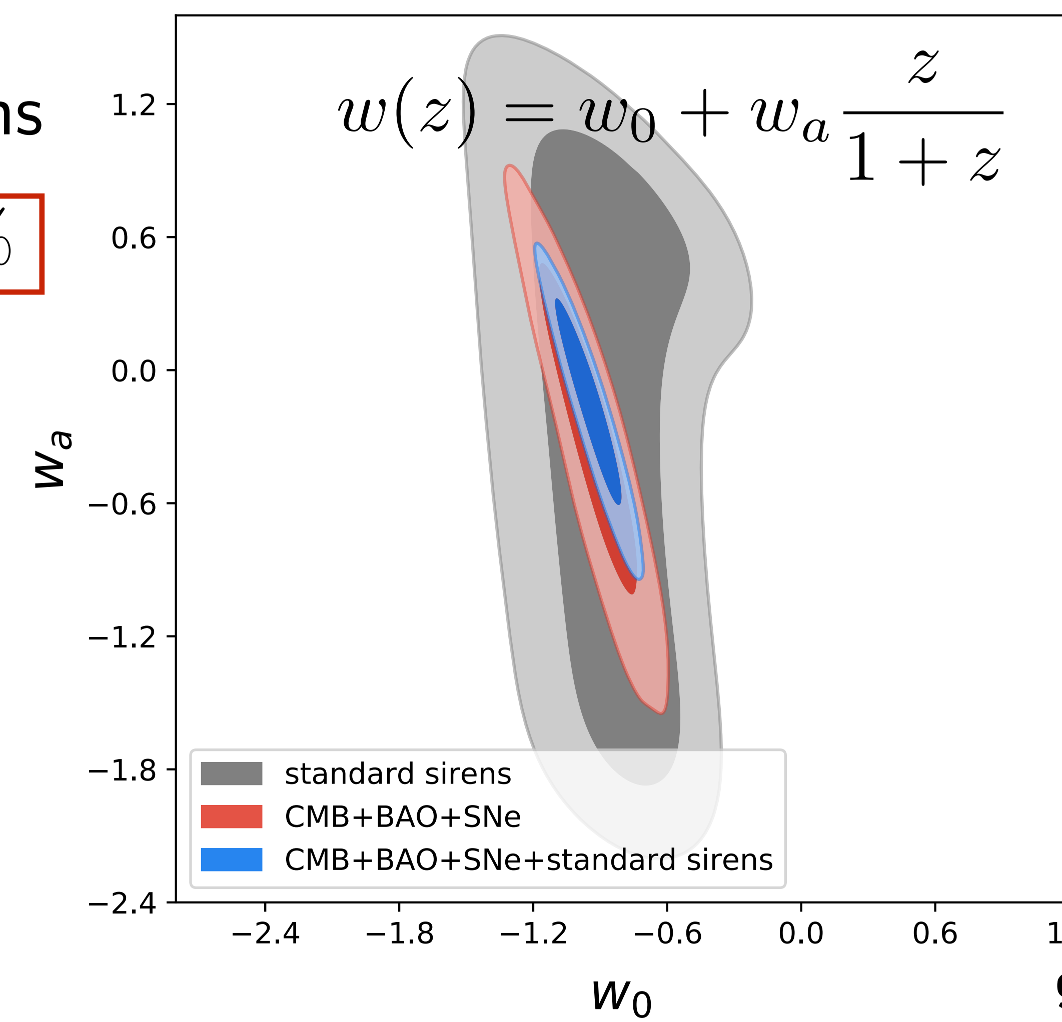
$$d_{\rm L}^{\rm gw} = a_0(1+z)\int_0^z \frac{dz'}{H_0\sqrt{\Omega_{M,0}(1+z')^3+\Omega_{\rm DE}(z')}}$$

- Einstein Telescope with 1000 standard sirens

[Belgacem et al.'18]

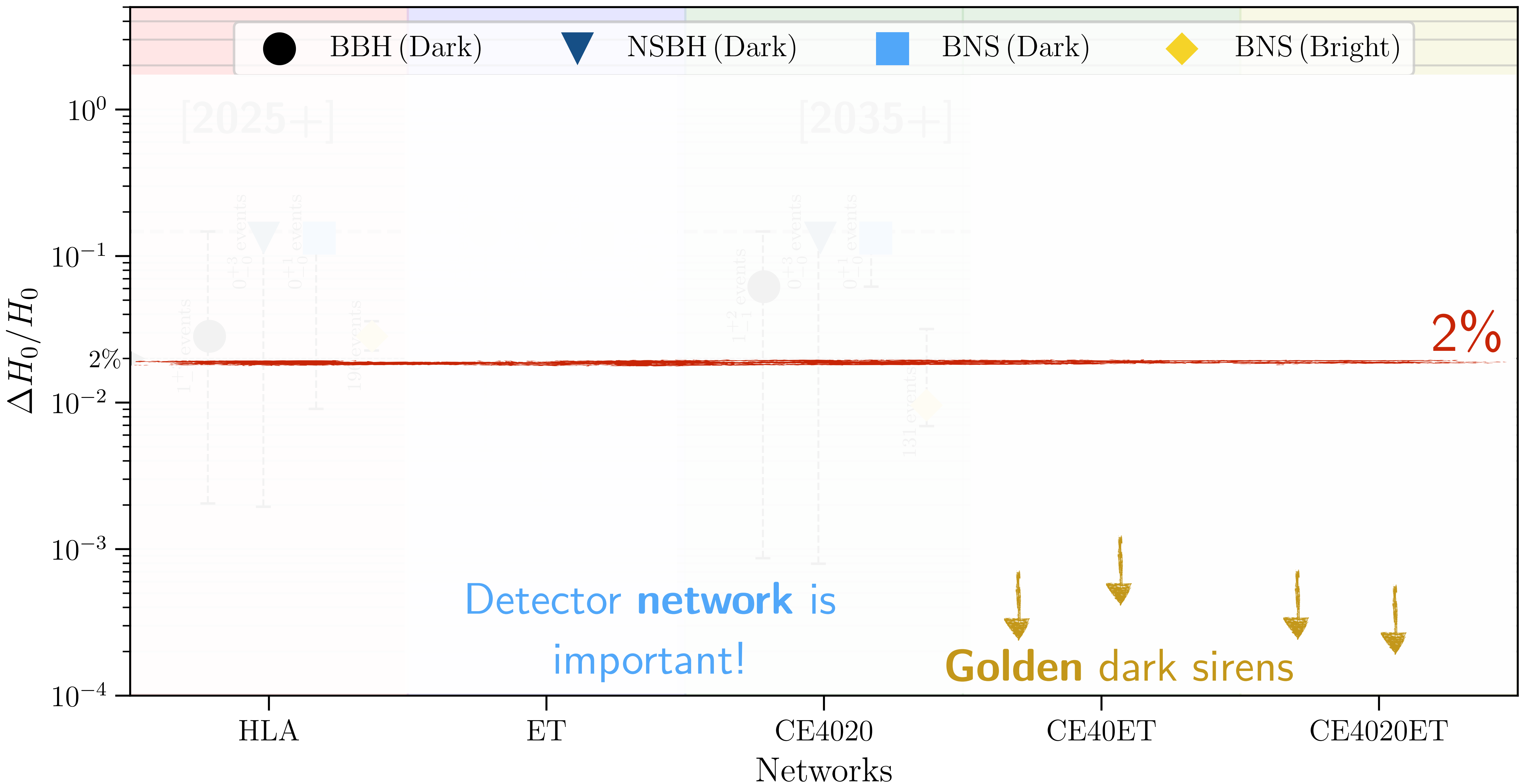
$\sigma_{w_0}/w_0 \sim 20\%$





H_0 (also) with dark sirens

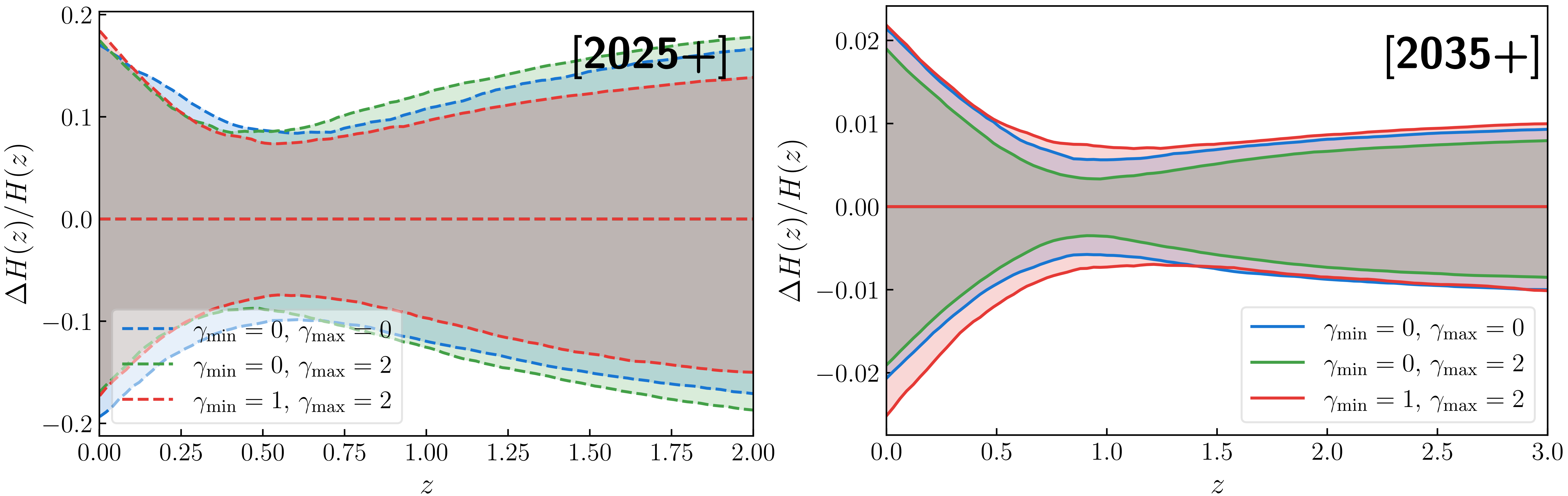
H: Hanford (US)
L: Livingston (US)
A: Aundha (India)
ET: Einstein Telescope (EU)
CE: Cosmic Explorer (US)



[Chen, **Ezquiaga** & Gupta (CQG'24)]

Spectral sirens: *forecasts*

[**BBHs** between NSBH and PISN gap]



2G: **<10%** within 1 year at approx. z=0.7

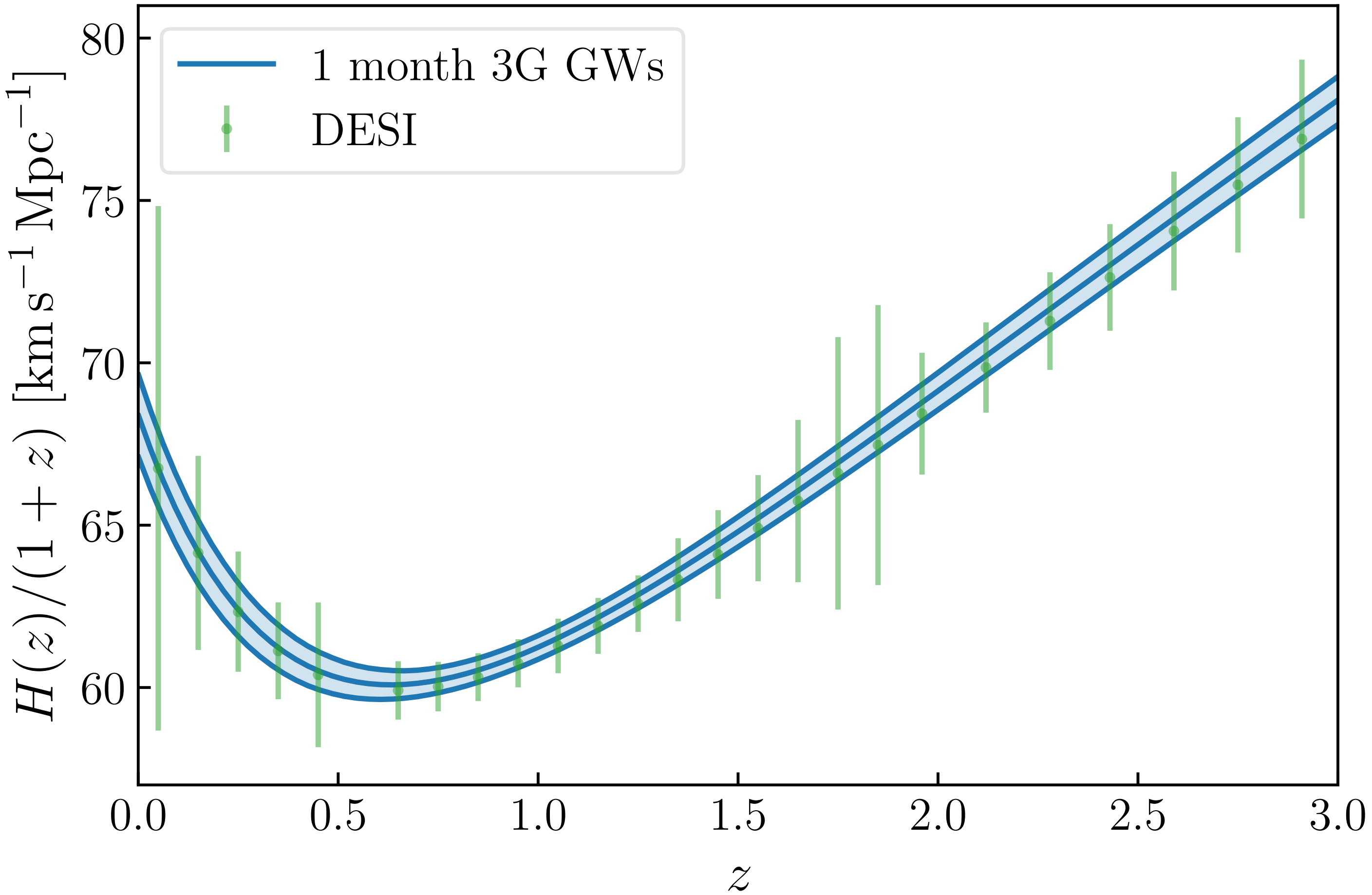
3G: **Sub-percent** within 1 month. High-redshift!

[**Ezquiaga** & Holz (PRL'22)]

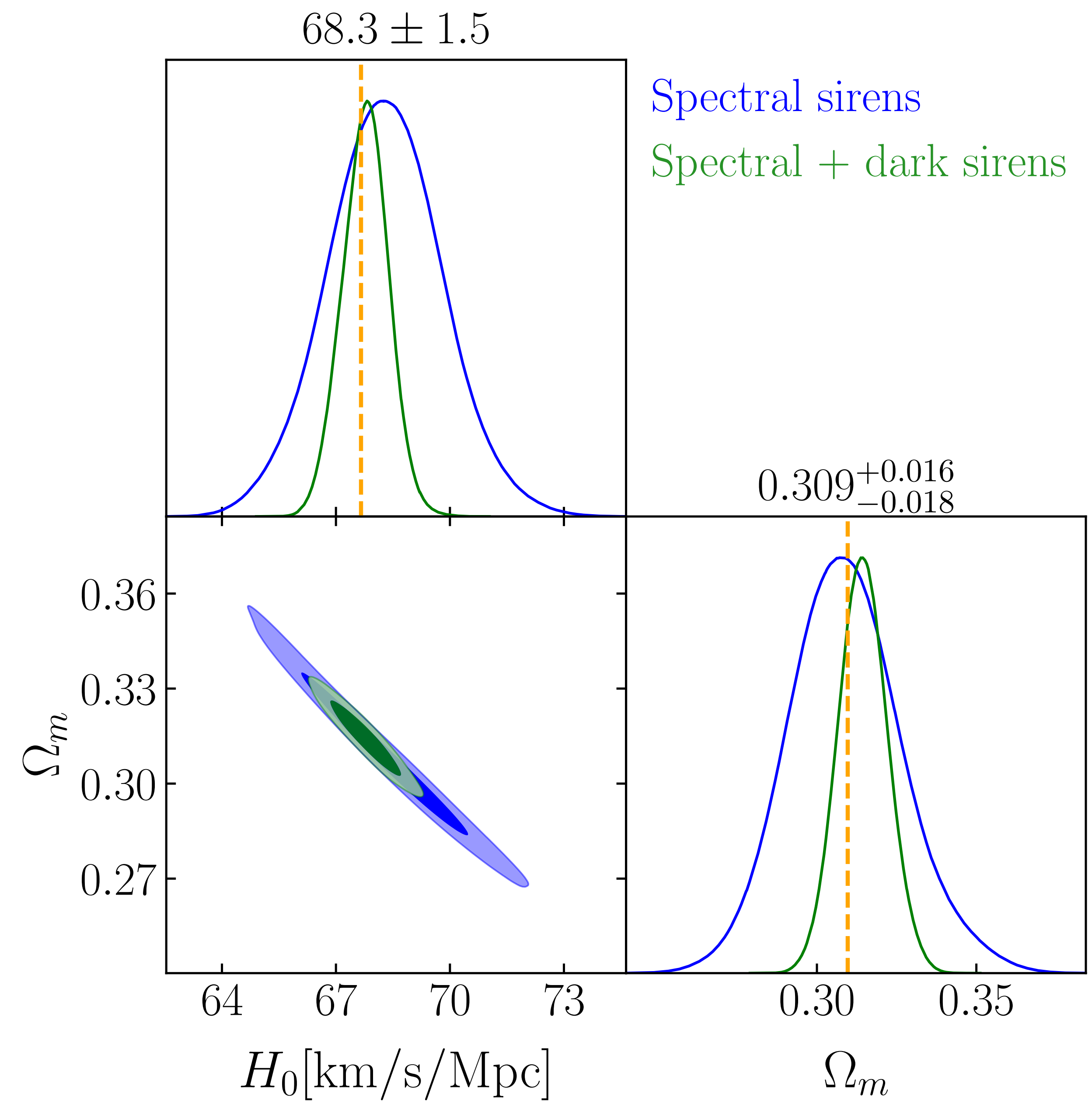
Expansion rate at high redshift *H(z)*

Combining sirens **sub-percent** precision across cosmic history!

Spectral sirens are competitive with cosmic surveys

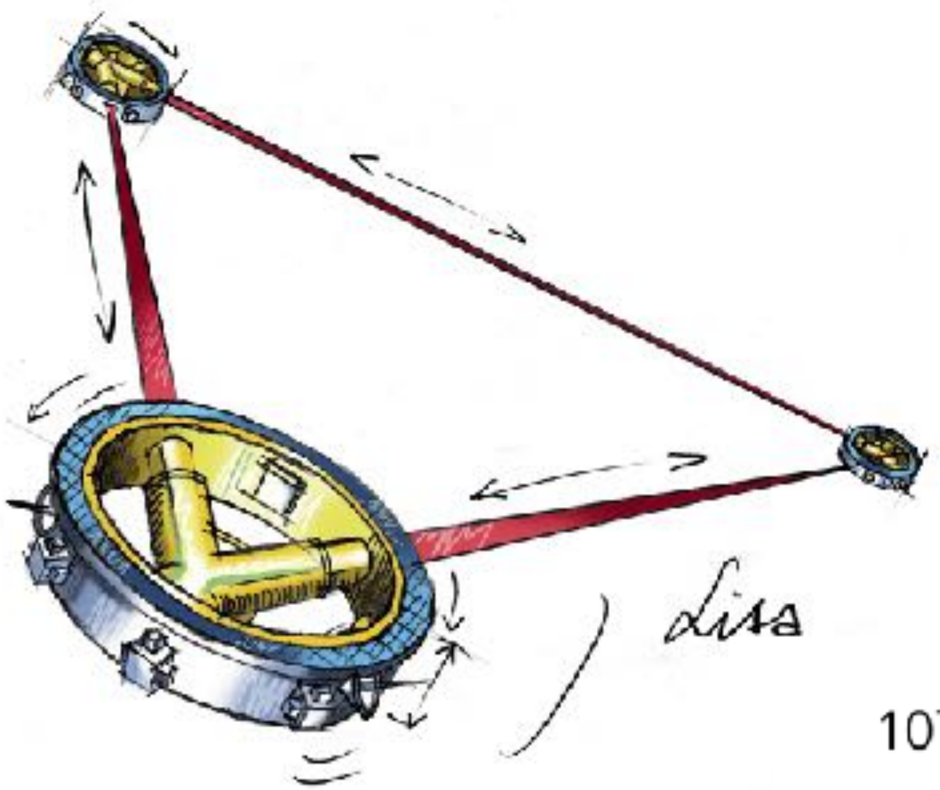


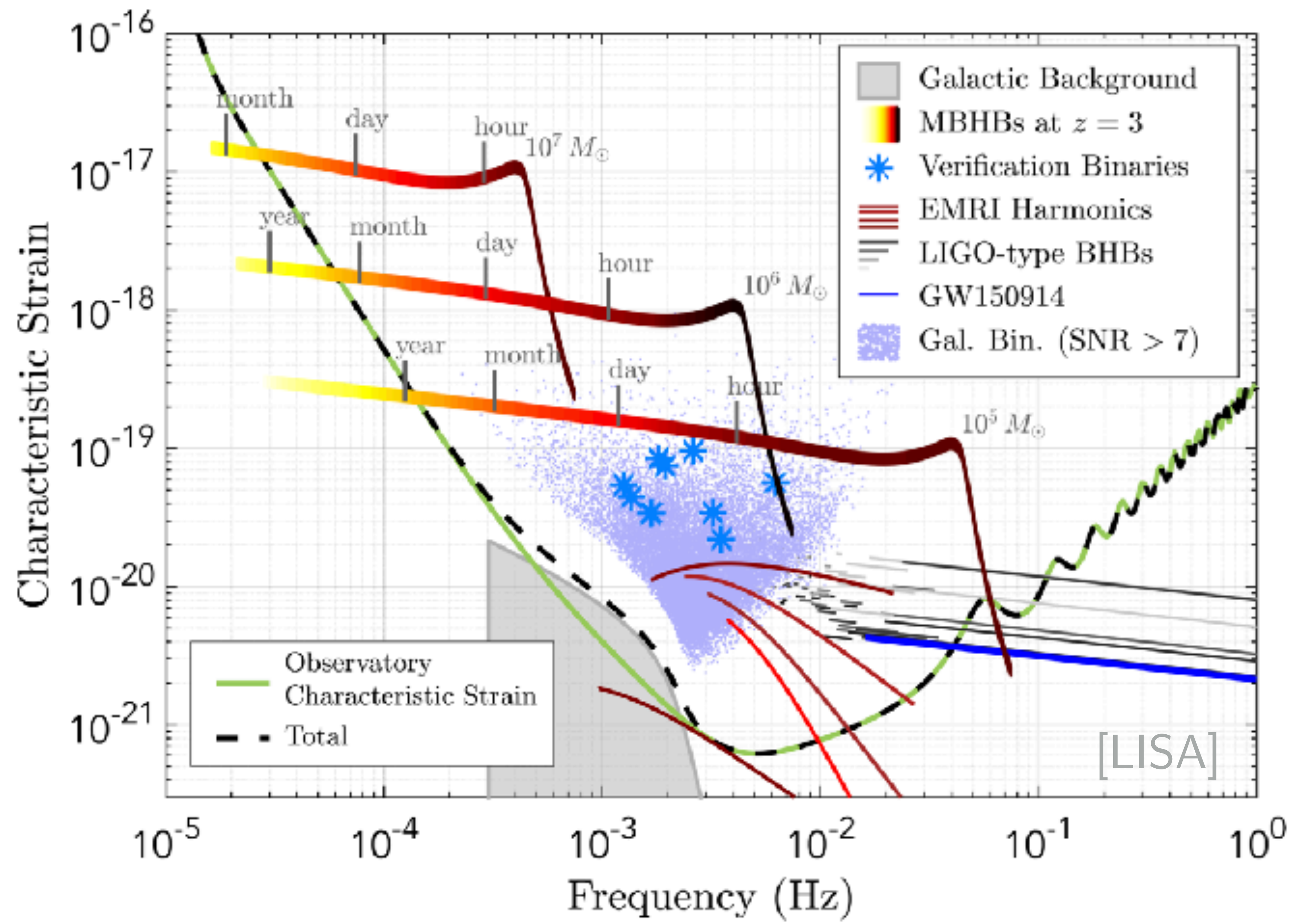
[**Ezquiaga** & Holz (PRL'22)]



[Chen, **Ezquiaga** & Gupta (CQG'24)]

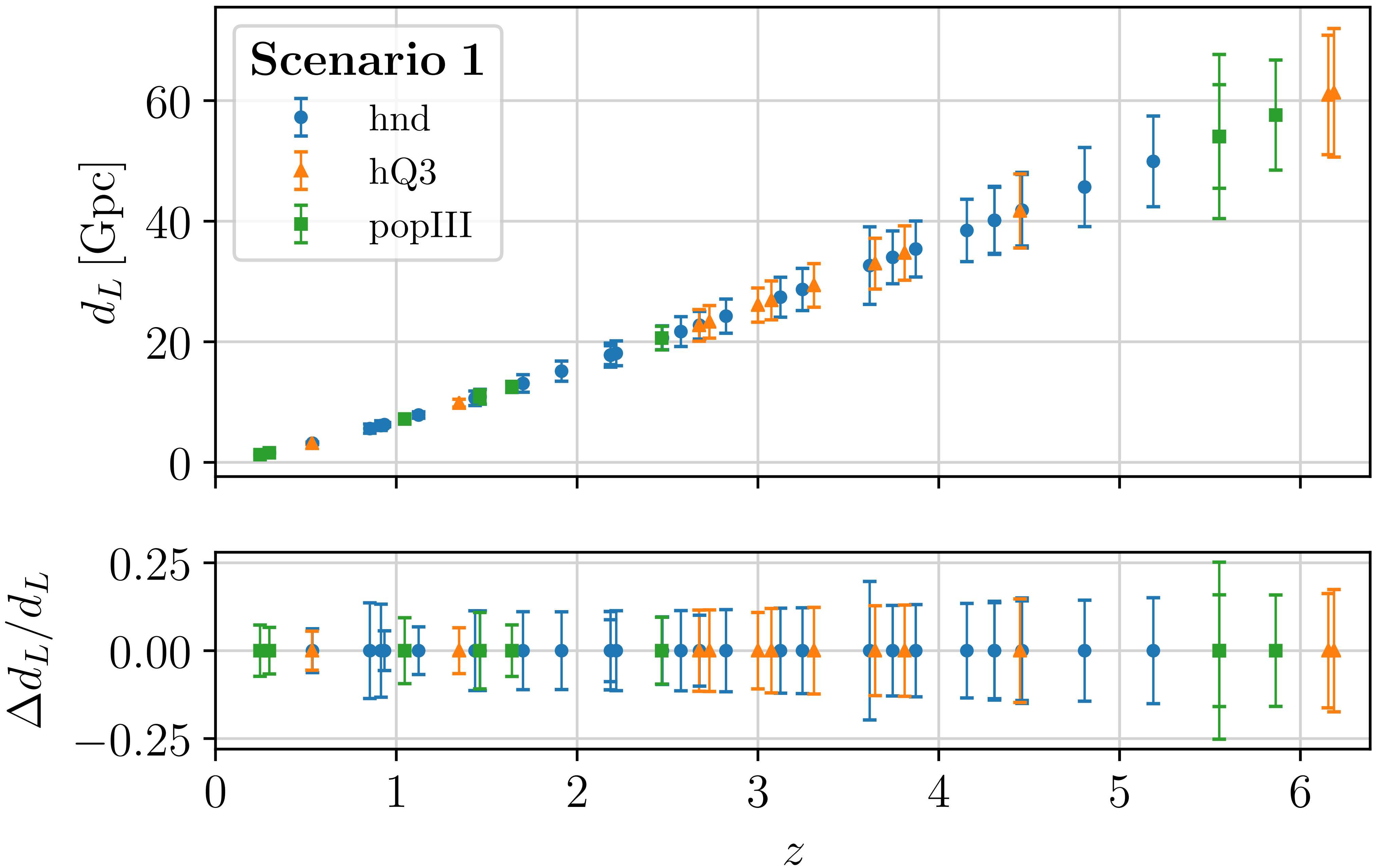
LISA's perspective





LISA forecasts: super massive BBHs

[approx. 10-30 **bright sirens** (4 yrs)]

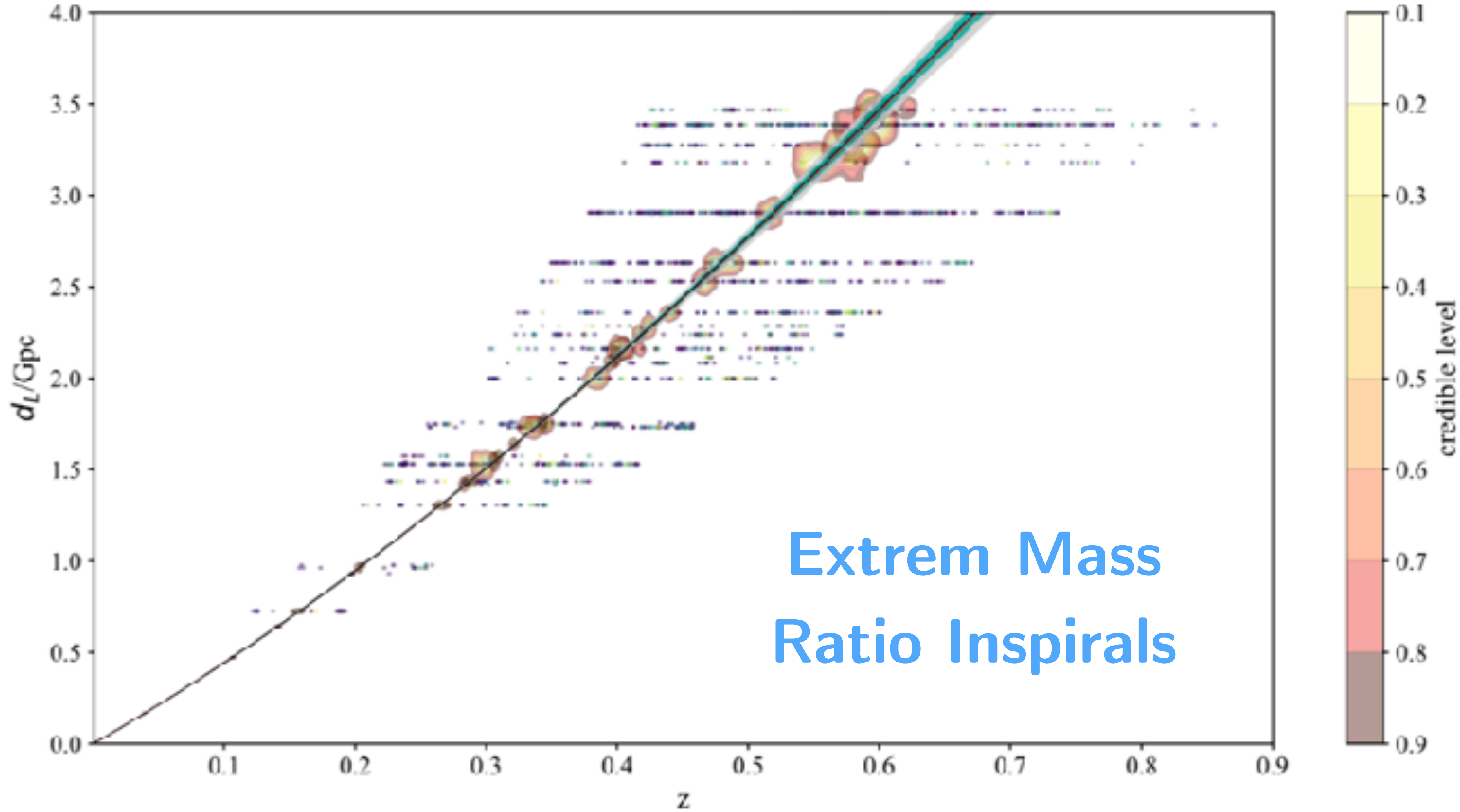


hnd: heavy seeds no delay

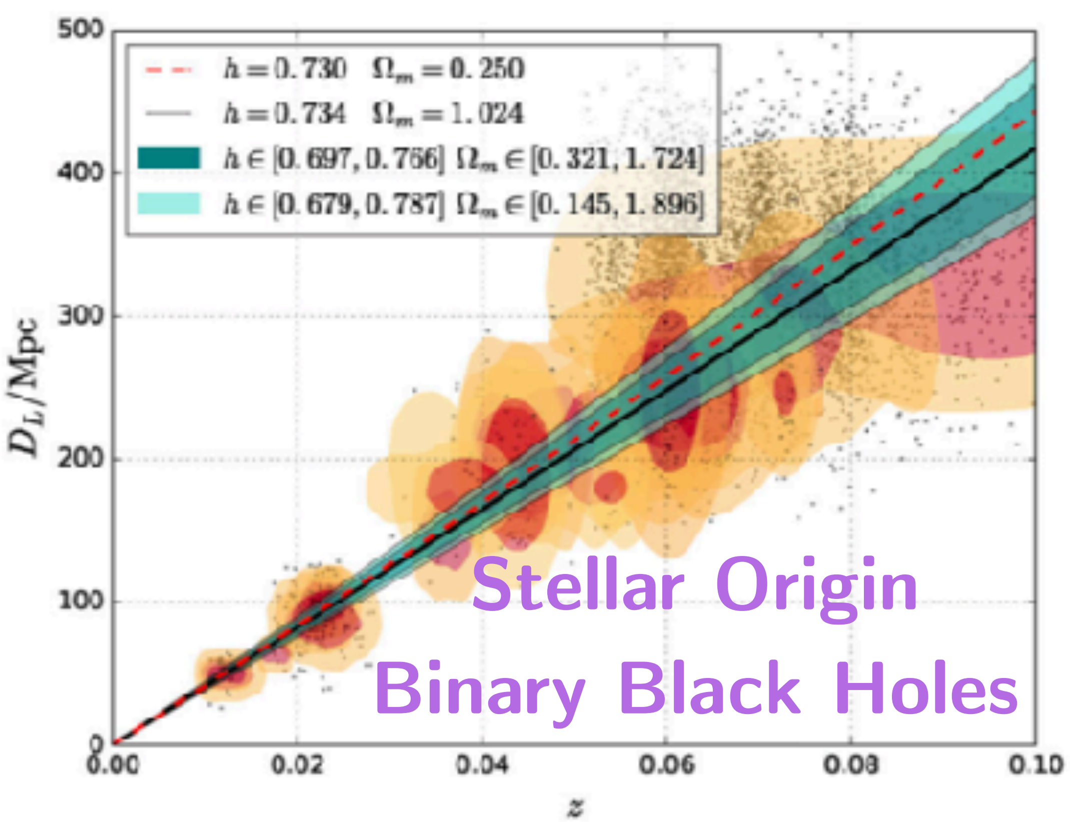
hQ3: heavy seeds with delay

popIII: light seeds

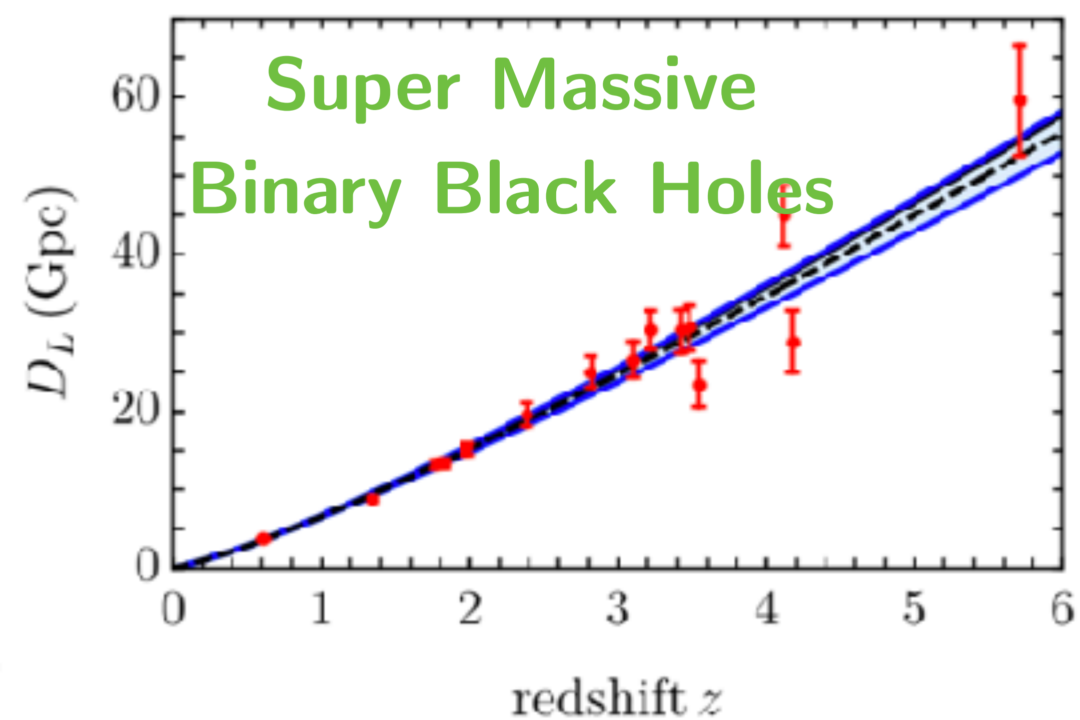
[Belgacem et al.; JCAP'19]



Extrem Mass
Ratio Inspirals
d_L/Gpc
z
credible level



Stellar Origin
Binary Black Holes
$h = 0.730$ $\Omega_m = 0.250$
$h = 0.734$ $\Omega_m = 1.024$
$h \in [0.697, 0.766]$ $\Omega_m \in [0.321, 1.724]$
$h \in [0.679, 0.787]$ $\Omega_m \in [0.145, 1.896]$
D_L/Mpc
z



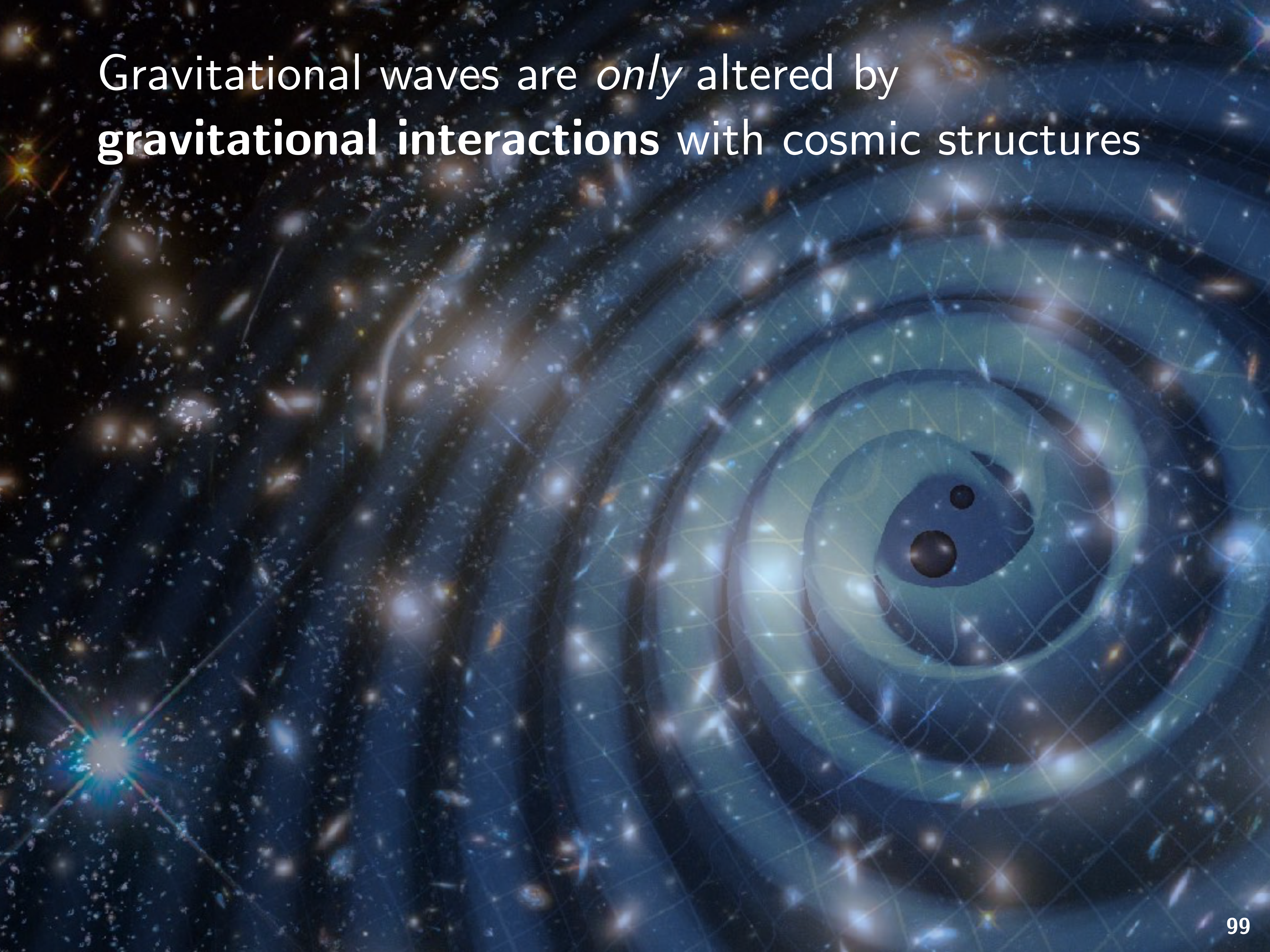
Super Massive
Binary Black Holes
D_L (Gpc)
redshift z

3. Key takeaways

- Gravitational waves carry information about their *luminosity distance* and *redshifted masses*
- With a direct additional information on redshift we have a *bright siren*. Using a galaxy catalog we have a *dark siren*
- Cosmology and astrophysics can be inferred simultaneously using the *spectral siren* method
- Current constraints dominated by *GW170817*. Spectral siren allow to look further.
- In the future, constrain *H(z)* at high redshift!

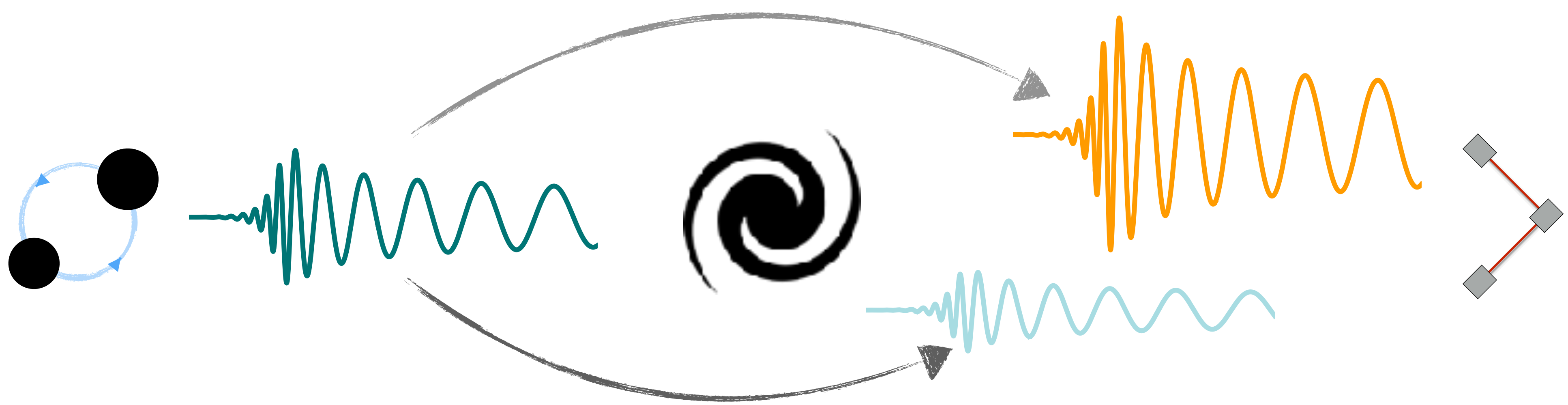
4. Gravitational wave lensing

Gravitational waves are *only* altered by **gravitational interactions** with cosmic structures

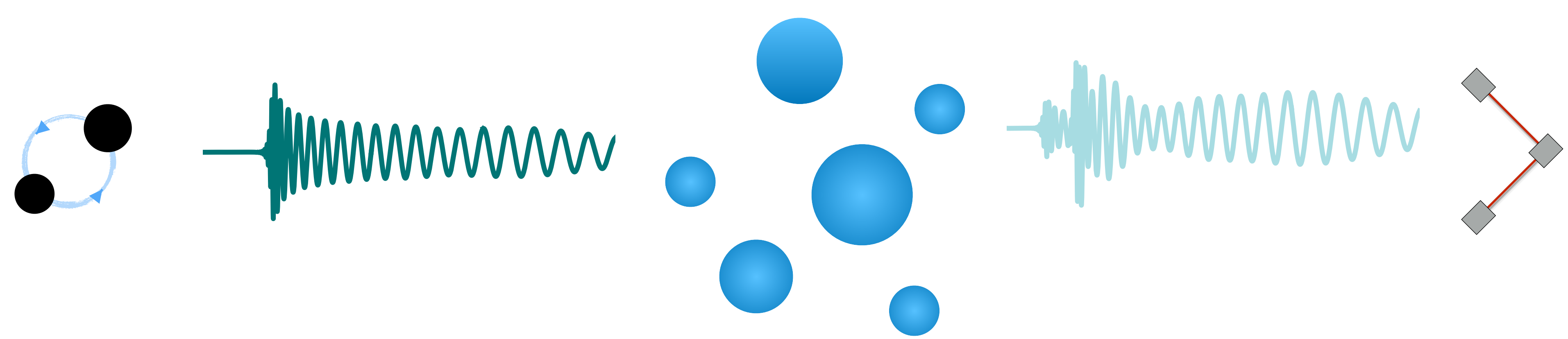


Gravitational lensing - gravitational wave spectrum

Repeated chirps due to strong lensing

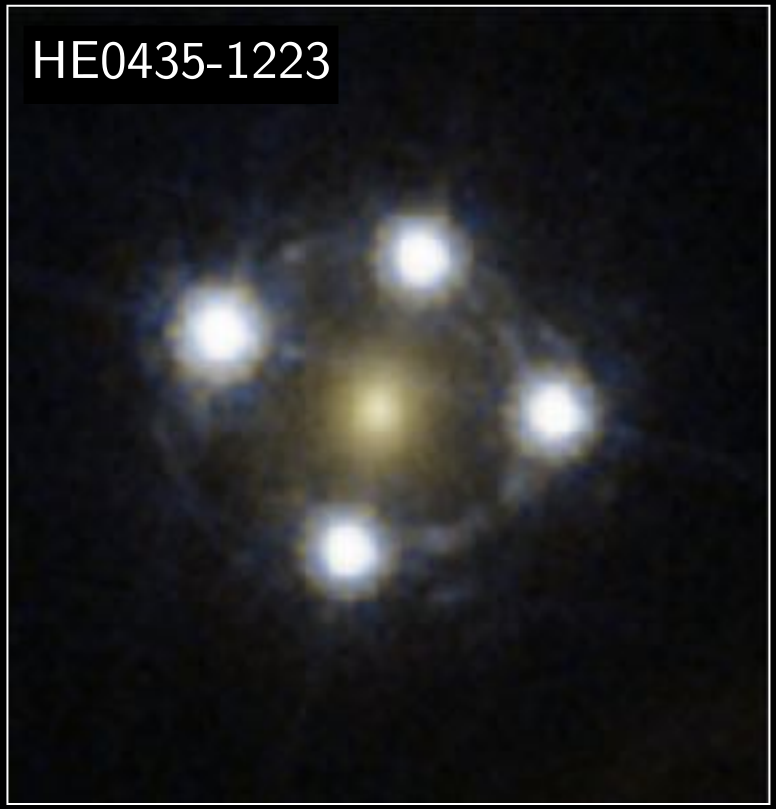


Waveform *distortions* by substructures



Source Lens Detector

Gravitational lensing - electromagnetic spectrum



[multiple images]

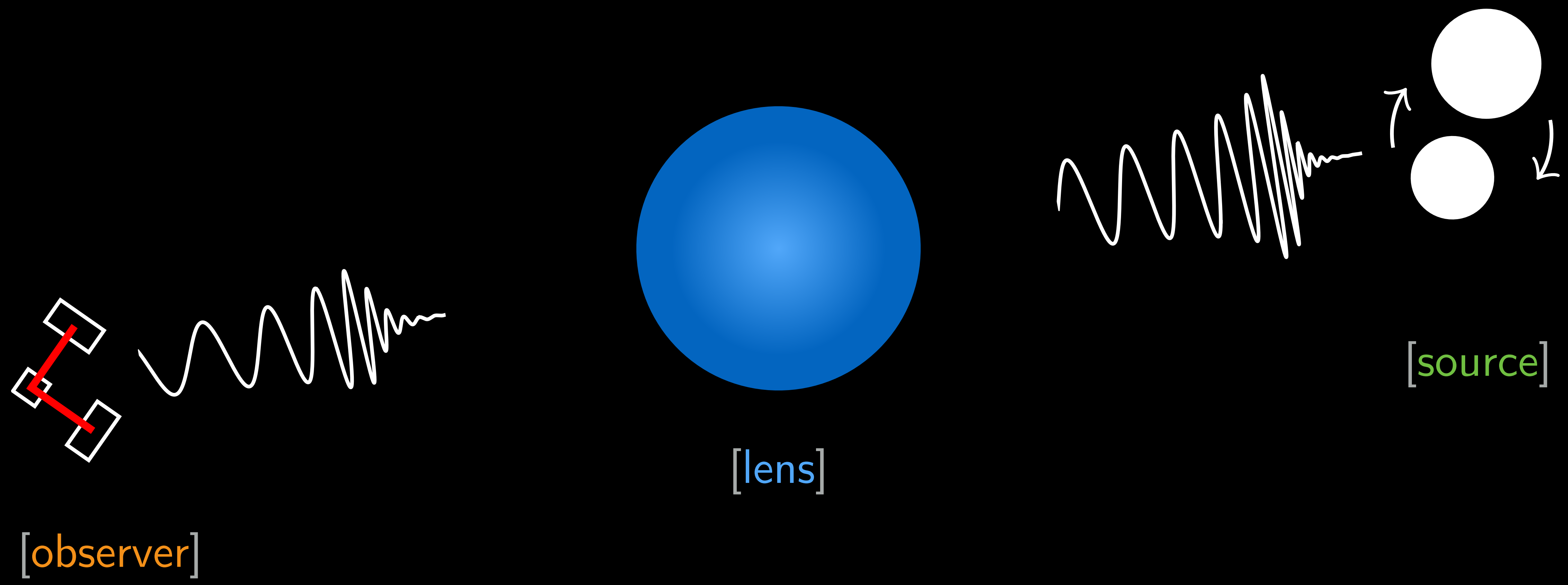
[arcs and rings]

Gravitational lensing

- Solve GW propagation on a curved background

$$\Box \bar{h}_{\mu\nu} + 2\bar{R}_{\alpha\mu\beta\nu}\bar{h}^{\alpha\beta} = 0$$

- We want to make a mapping between the source and the observer through the lens



Gravitational lensing

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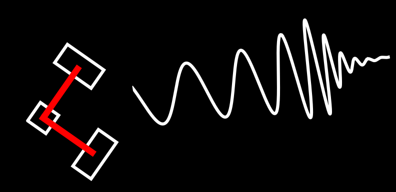
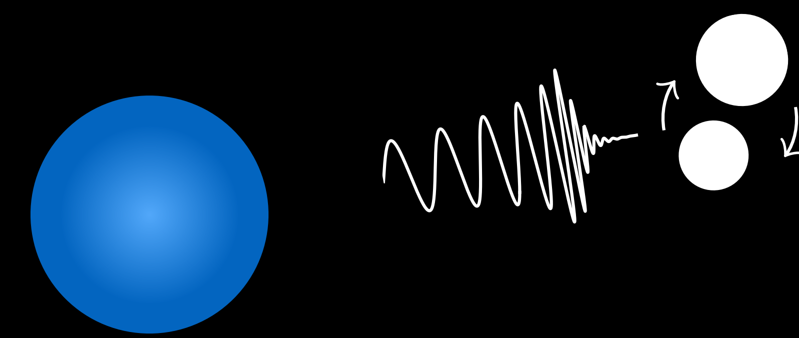




- Within *weak-gravity* & *thin lens* approximations, in *Fourier* space:

$$h_L(\omega) = F(\omega, \theta_S) \cdot h(\omega)$$

$$F(w, \vec{y}) = \frac{w}{2\pi i} \int \mathrm{d}^2 x \;\; \exp[i w T_d(\vec{x}, \vec{y})]$$

[Dimensionless variables] $\vec{x} \equiv \vec{\theta}/\theta_*\,, \quad \vec{y} \equiv \vec{\theta}_S/\theta_*\,, \quad w \equiv \tau_D \theta_*^2 \omega$

$$T_d \equiv t_d/\tau_D\theta_*^2 \qquad \tau_D \equiv (1+z_L)D_L D_S/cD_{LS}$$

Stationary Phase Approximation

- Solve integral in the limit of highly oscillatory integrand

$$F(w,\vec{y}) = \frac{w}{2\pi i}\int \mathrm{d}^2x \;\; \exp[iwT_d(\vec{x},\vec{y})]$$

- Stationary points define the images:

$$\left.\frac{\partial t_d}{\partial \theta_a}\right|_{\vec{\theta}=\vec{\theta}_j} = 0$$

$$T_d(\vec{\theta}) \approx T_d(\vec{\theta}_j) + \frac{1}{2}\sum_{(a,b)=1}^{2} \delta\theta_a \delta\theta_b \frac{\partial^2 T_d(\vec{\theta}_j)}{\partial\theta_a\partial\theta_b} + \cdots$$

- Hessian matrix determines magnifications

$$\mu(\theta_j) = 1/\mathrm{det}(T_{ab}(\theta_j))$$

$$T_{ab} \equiv \tau_D^{-1}\partial^2 t_d/\partial\theta_a\partial\theta_b$$

Strong lensing

$\Delta t_d \cdot \omega \gg 1$

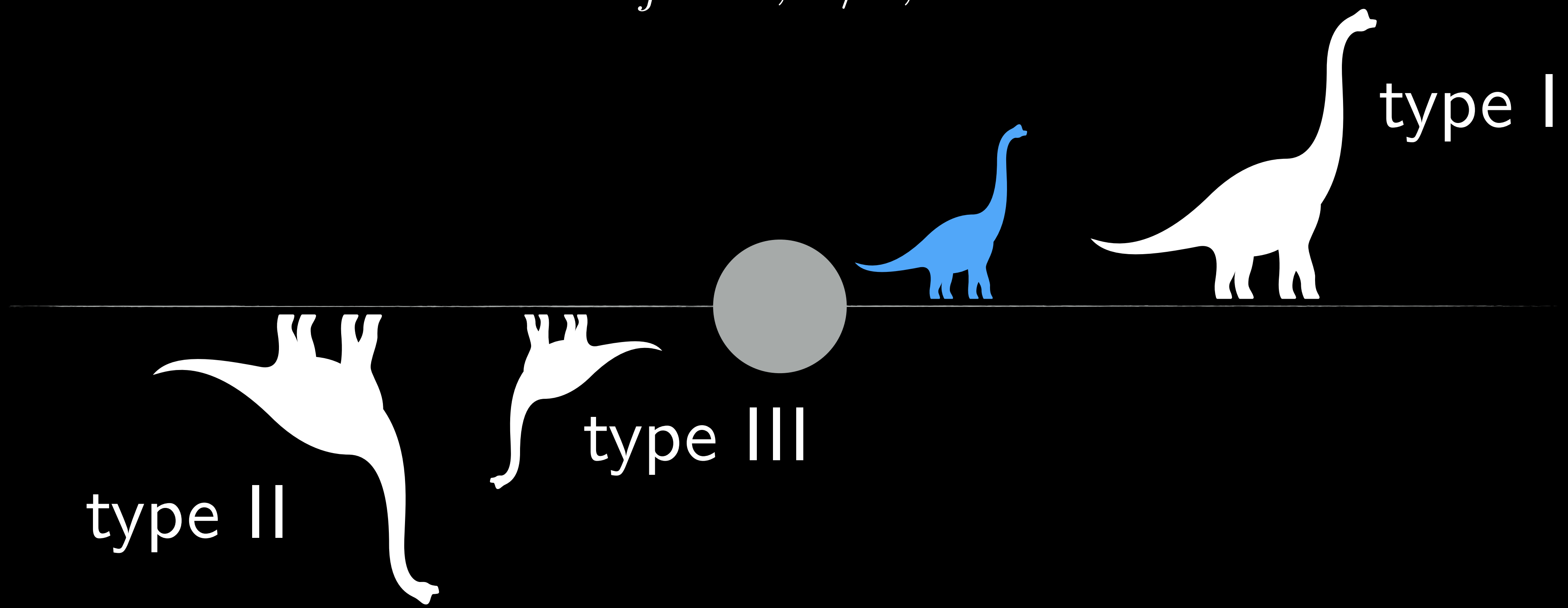
$$h_L(\omega) = F(\omega, \theta_S) \cdot h(\omega)$$

$$F \approx \sum_j |\mu_j|^{1/2} \exp\left(i\omega t_j - i\pi n_j\right)$$

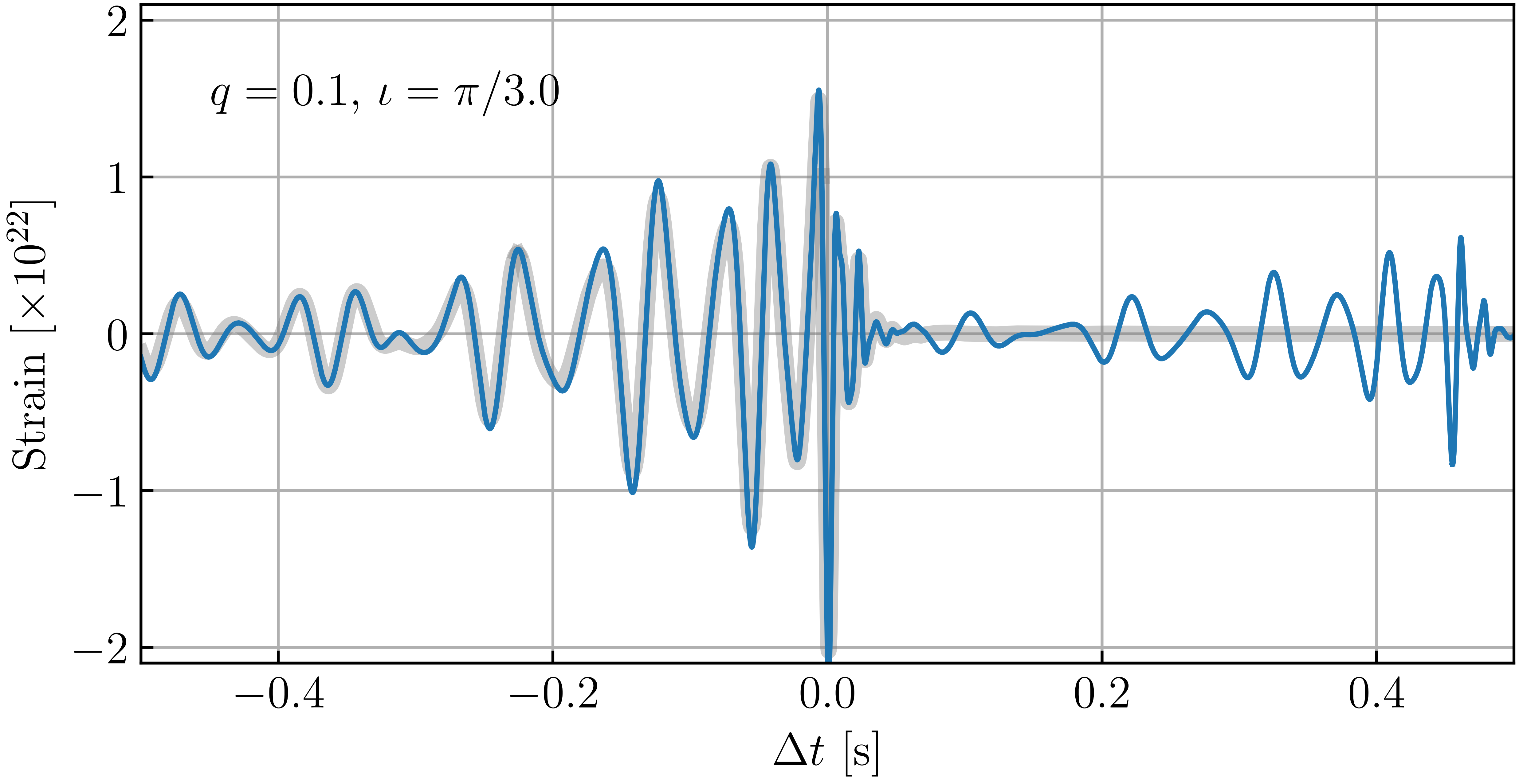
Magnification
Time delay
Phase shift

- Each image type (I, II and III) acquire a different phase shift

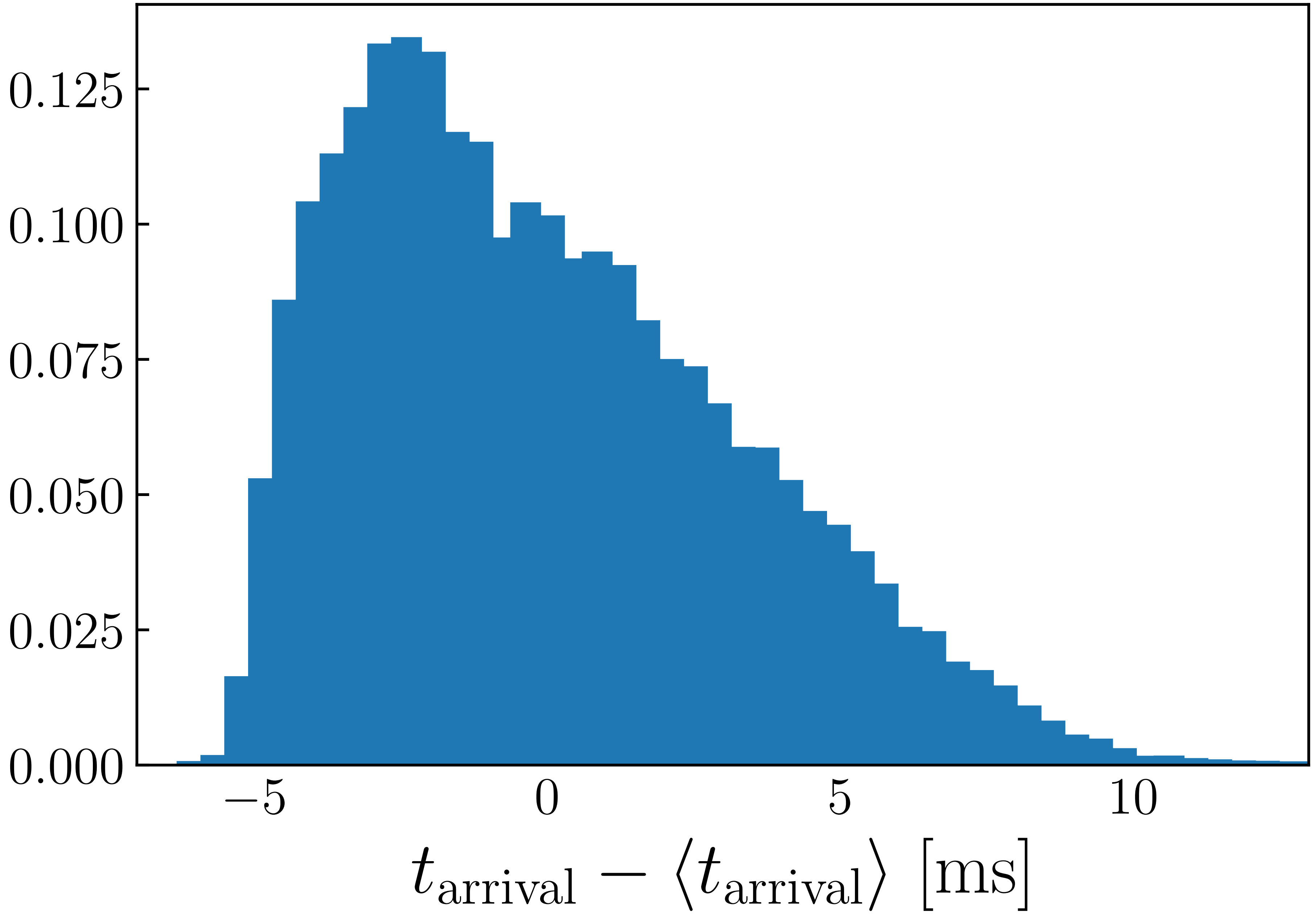
$$n_j = 0,\ 1/2,\ 1$$



Repeated, coherent signals

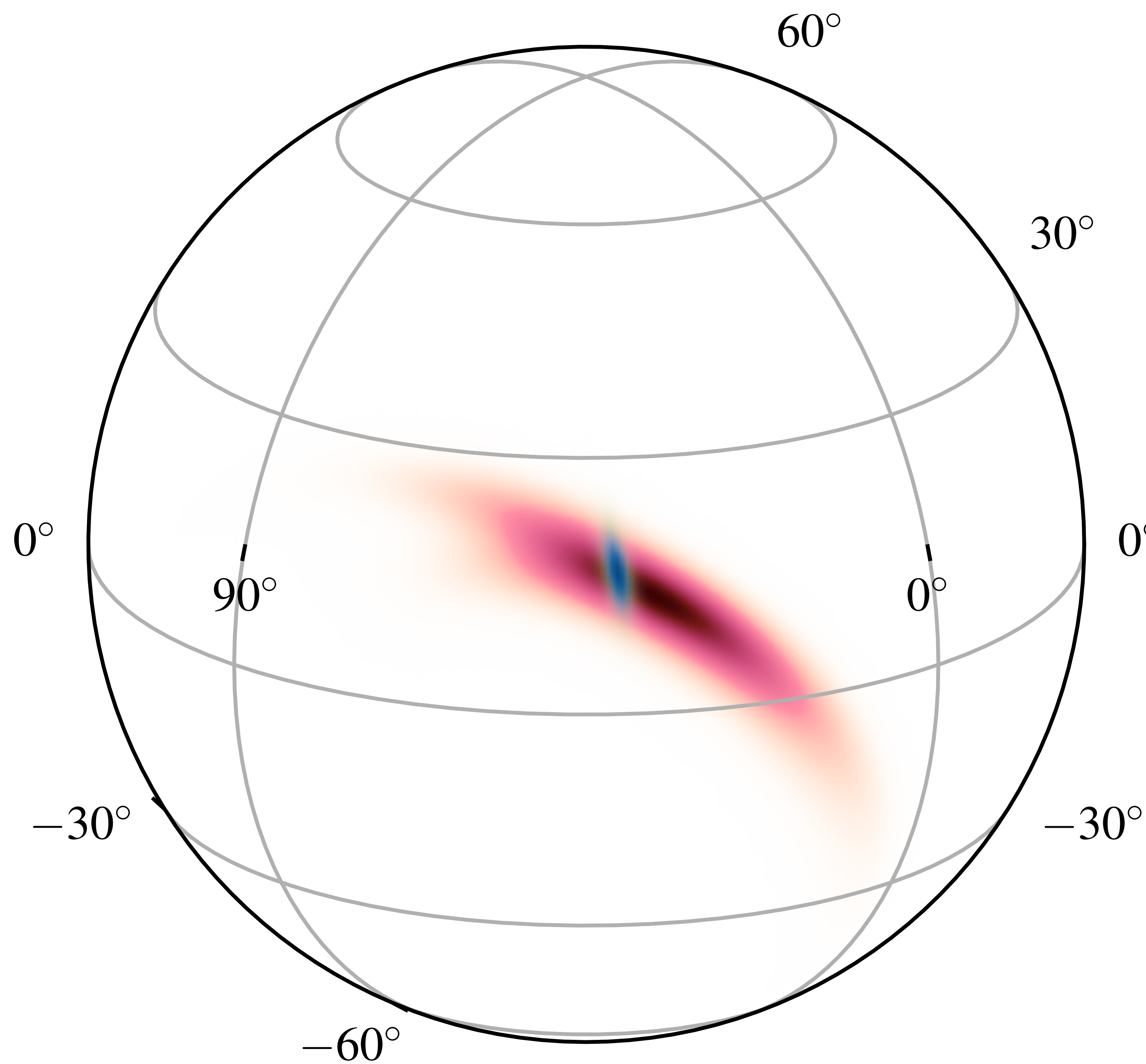


Precise timing



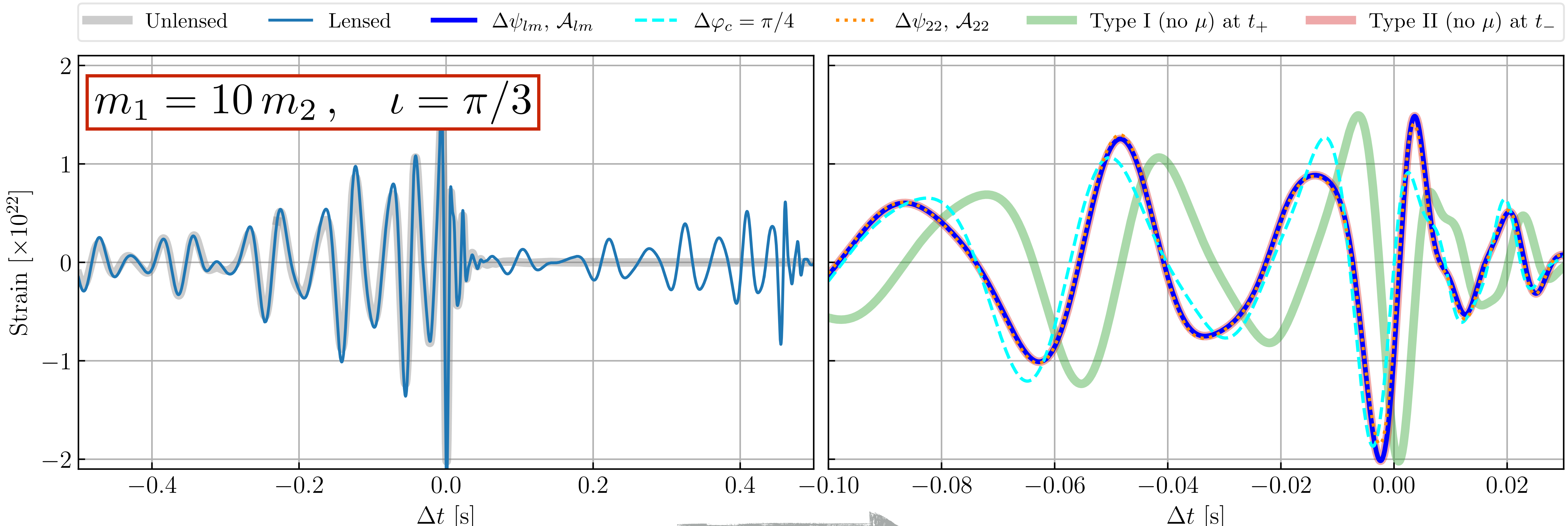
Poor sky localization

$$\theta_E \sim 1'' \sqrt{\frac{M}{10^{12} M_\odot}} \sqrt{\frac{1\mathrm{Gpc}}{D}}$$



Waveform distortions in **type II** images

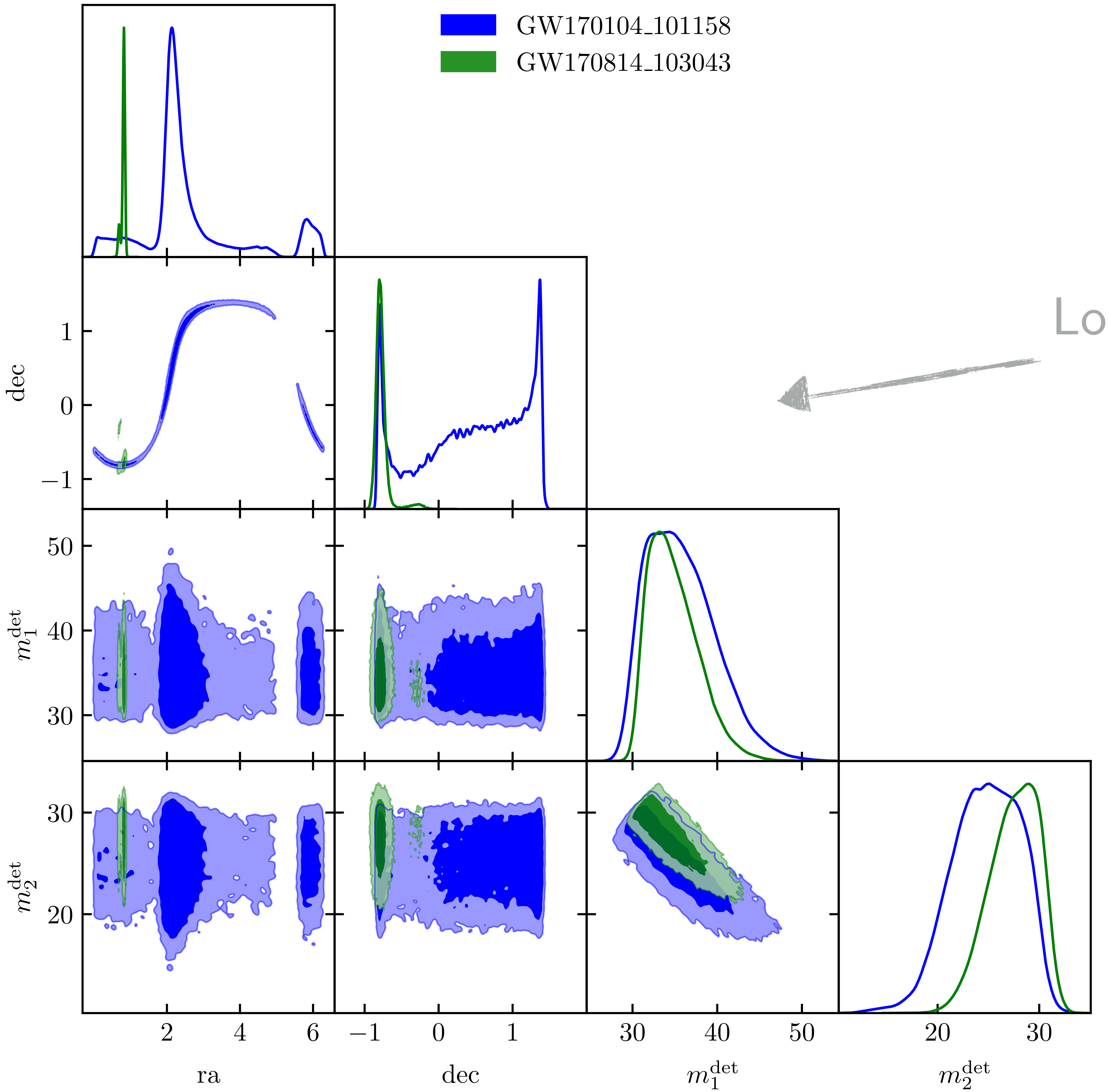
- Lensing imprints *small* but *characteristic* modifications in the signals that cannot be mapped to other astrophysical parameters



Zoom in the shape of each image

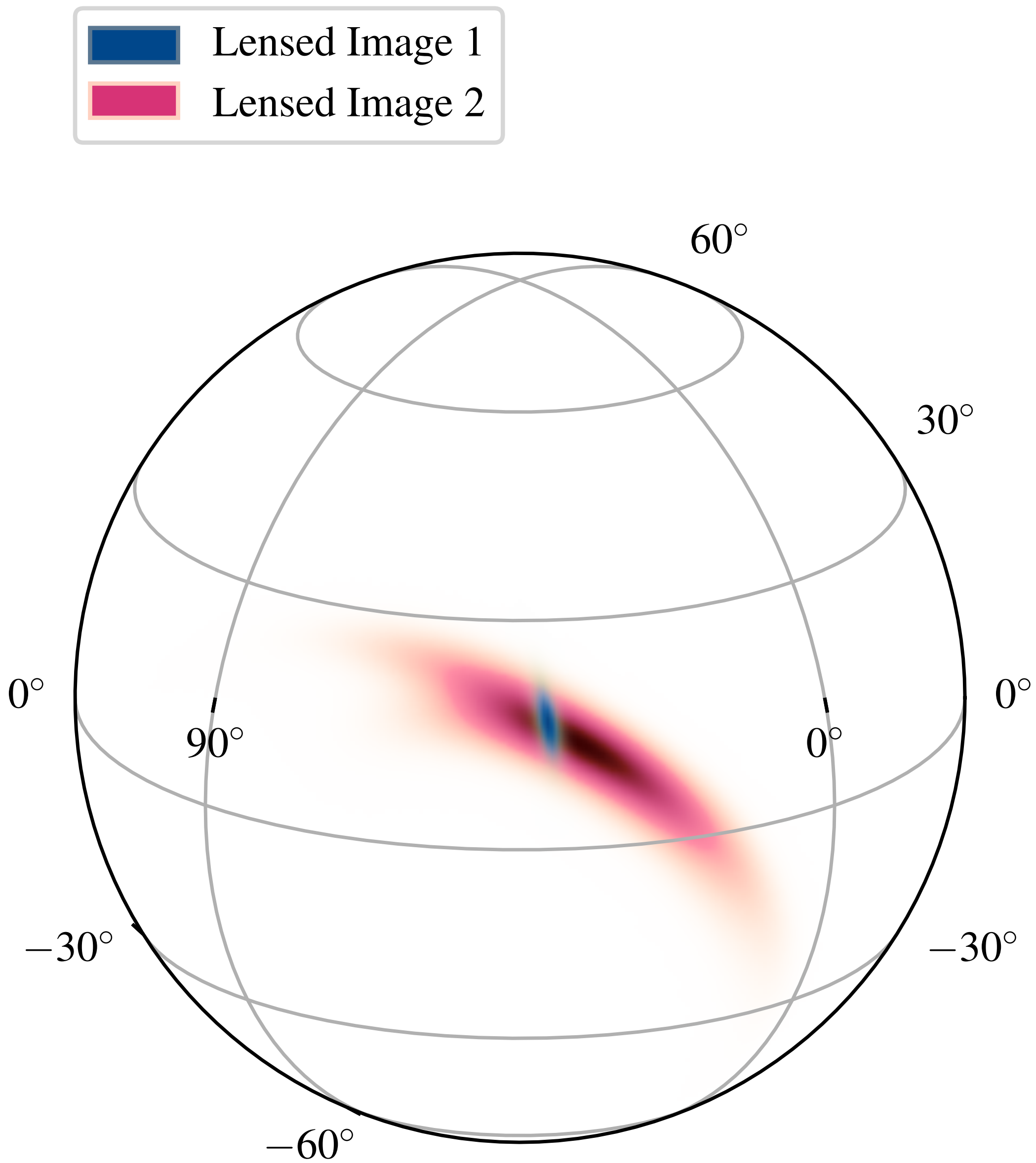
[**Ezquiaga** *et al.*; PRD'20]

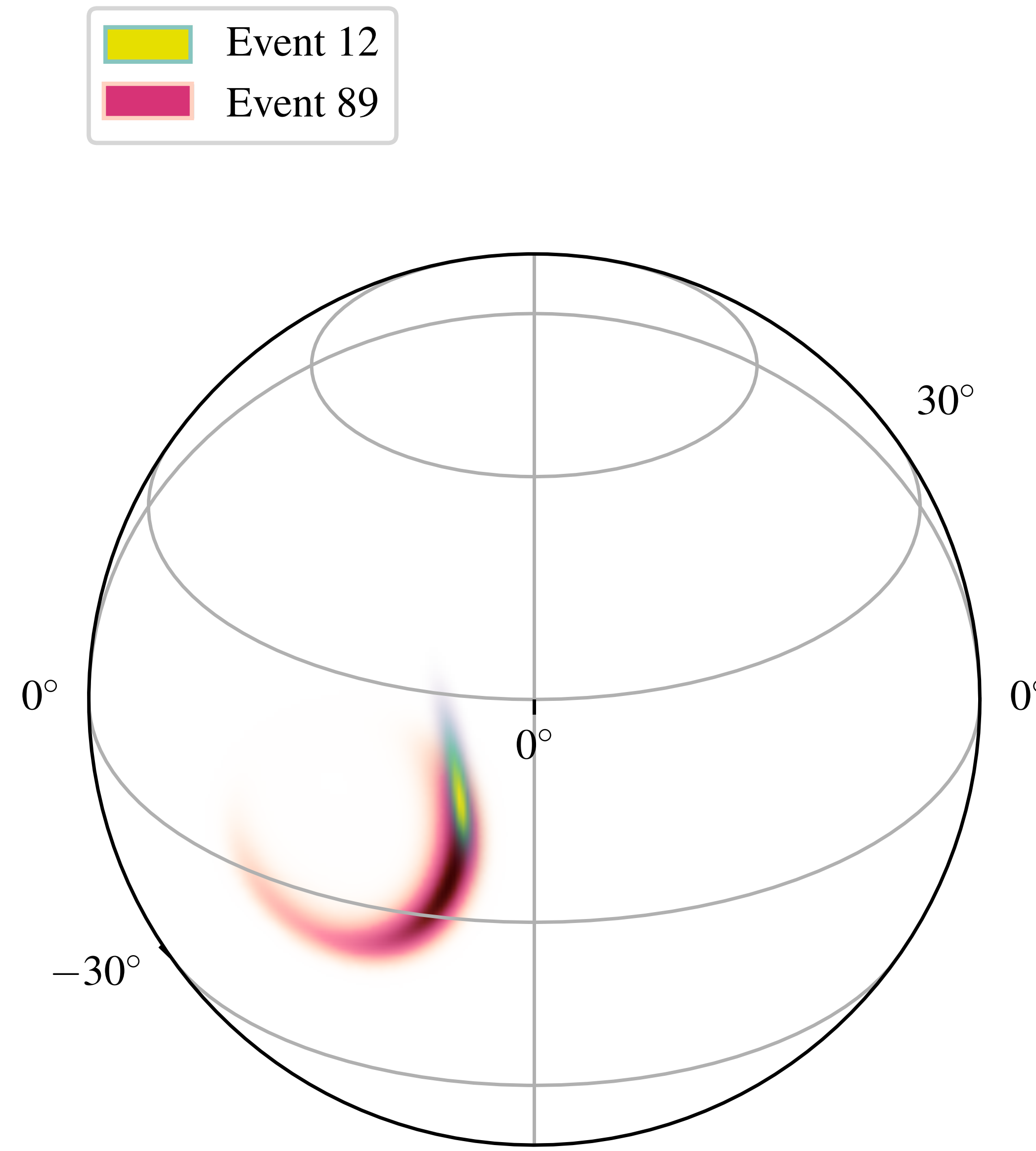
Searching for repeated chirps



Look for events with similar properties: masses, sky positions, spins...

Searching for repeated chirps

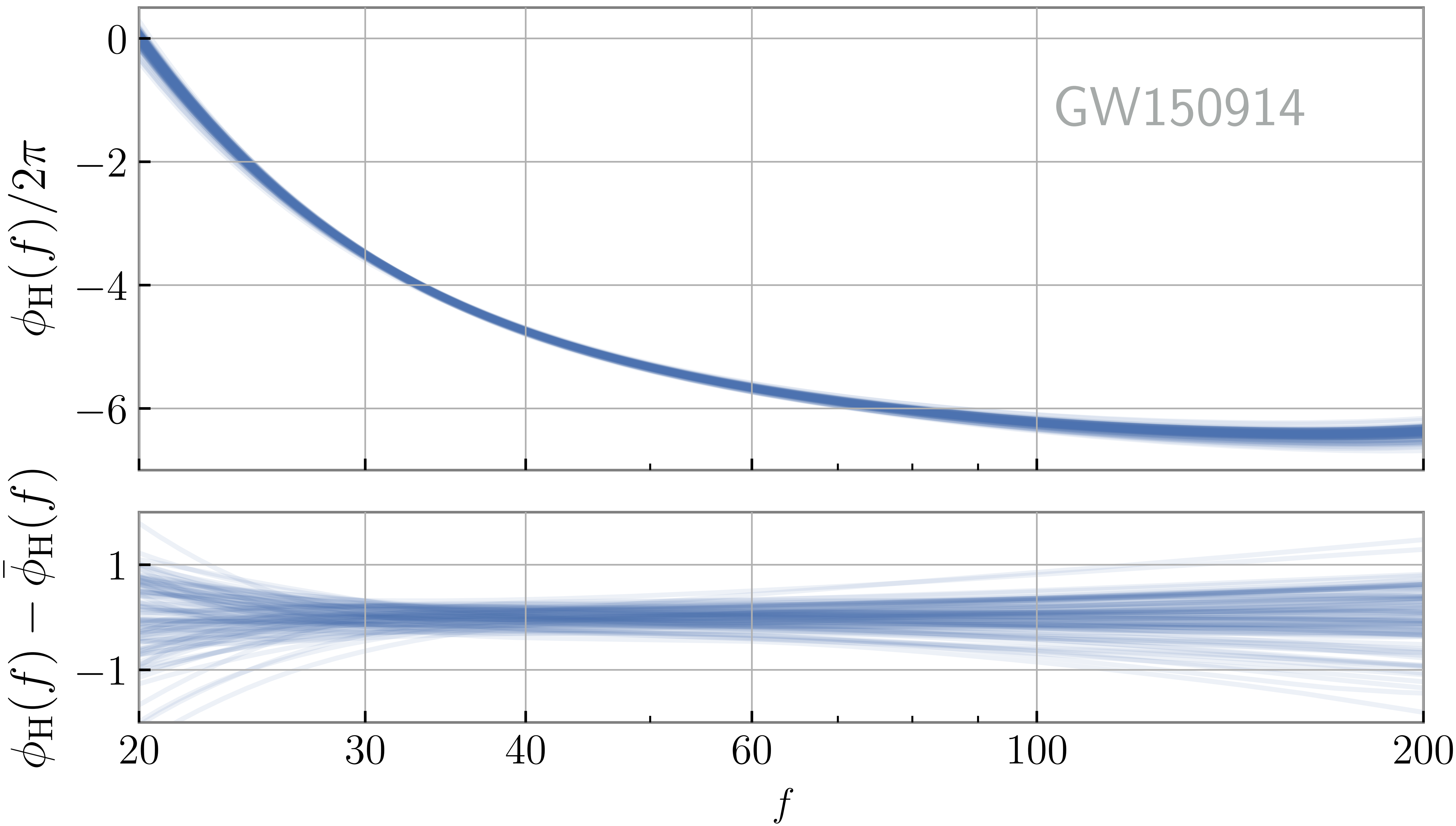




$$N_{\text{false alarm}} \sim N^2$$

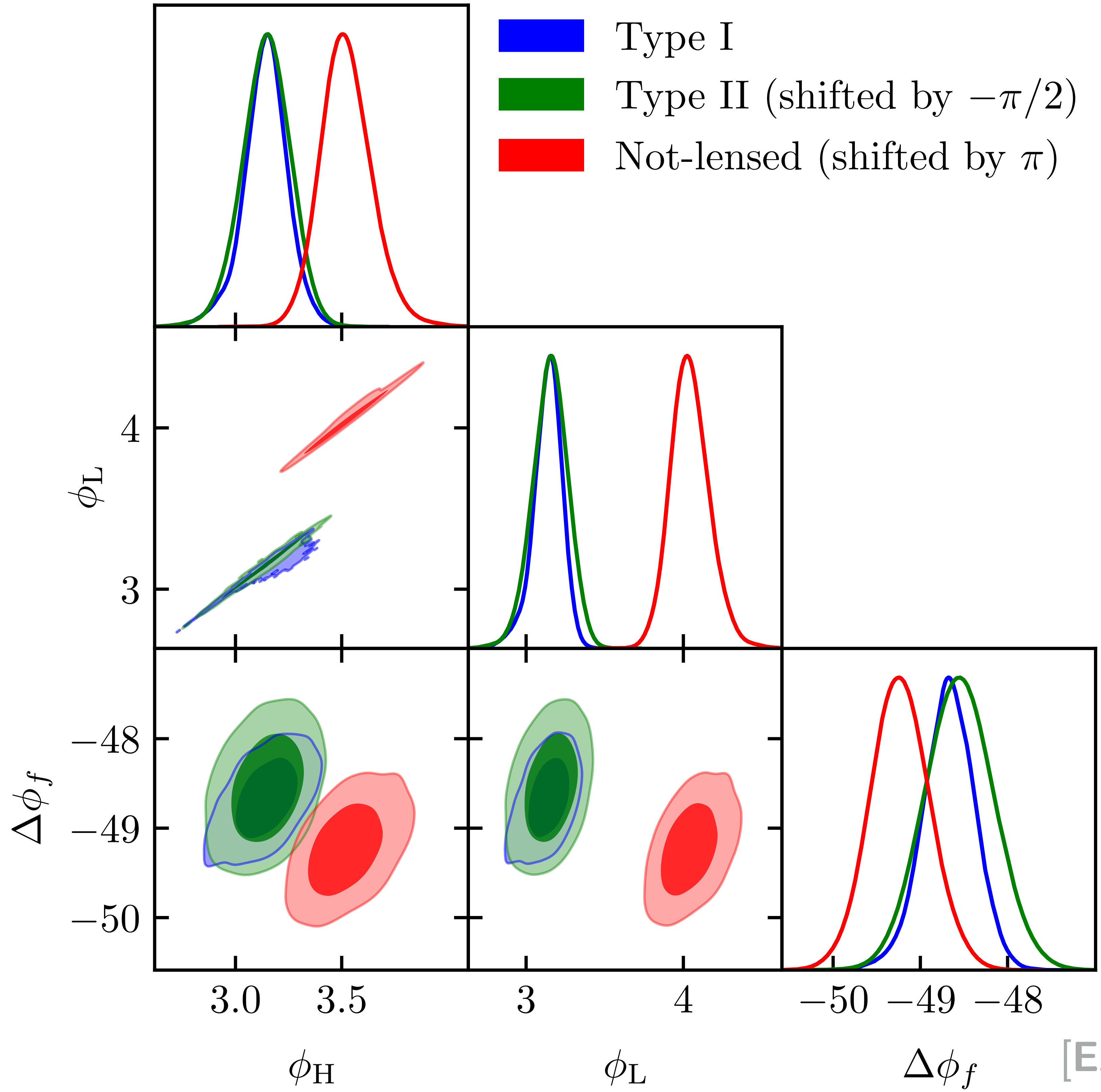
[*Çalışkan*, **Ezquiaga**, Hannuksela and Holz; PRD'22]

Fight false alarms: **phase consistency**



[**Ezquiaga**, Hu, Lo; PRD'23]

Fight false alarms: **phase consistency**



[**Ezquiaga**, Hu, Lo; PRD'23]

Wave optics

$\Delta t_d \cdot \omega$

- Time delay scales with the lens mass

$$\Delta t_d(y=1) \simeq 4 \left(\frac{(1+z_L)M_L}{100 M_\odot} \right) \mathrm{ms}$$

[point mass lens]

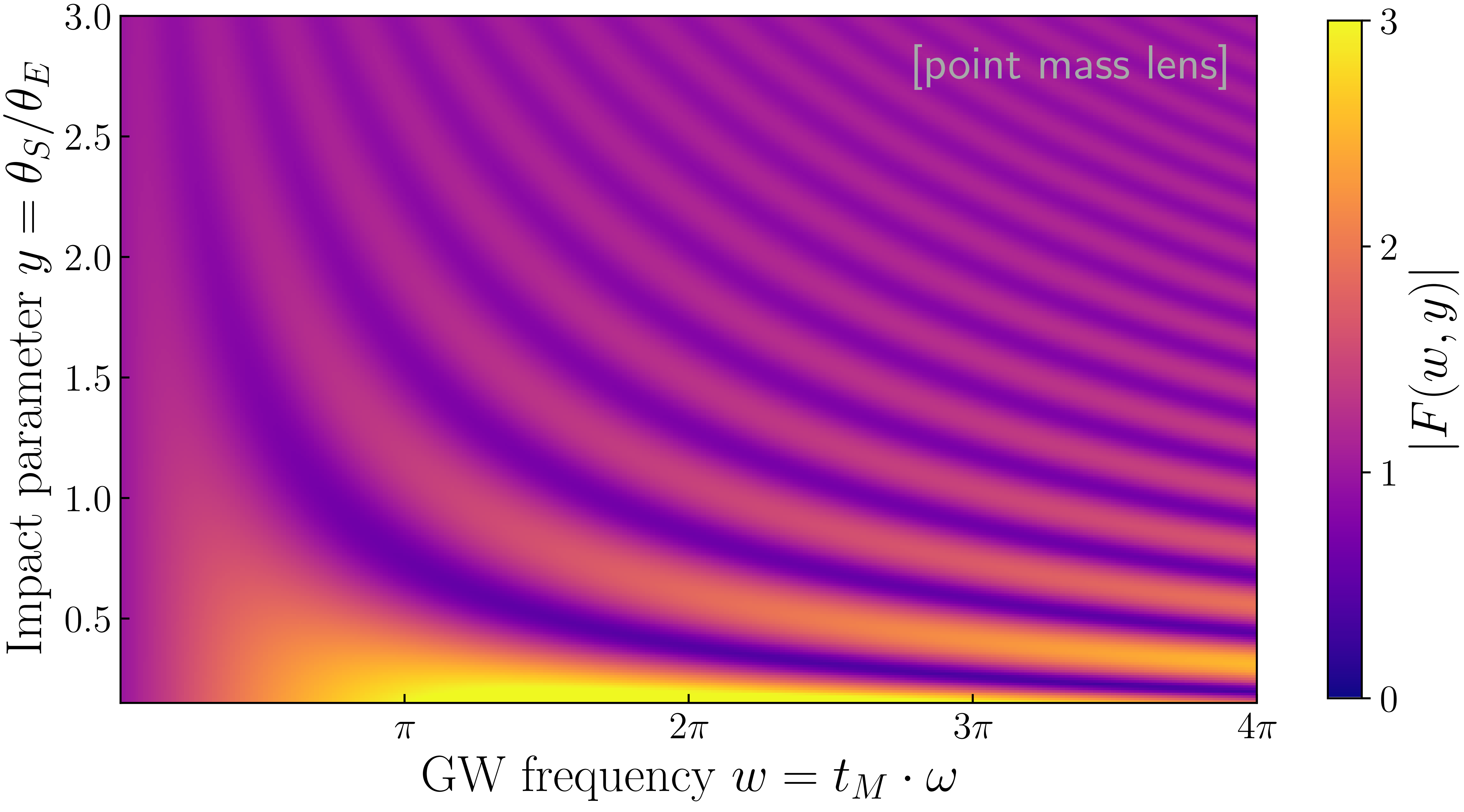
- GW frequency scales with binary mass *(has astrophysical size!)*

$$f \sim \frac{1}{2\pi} \frac{1}{2 t_{\mathrm{Sch}}} \sim 800\mathrm{Hz} \left(\frac{10 M_\odot}{M} \right)$$

- Wave optics regime: $\Delta t_d \cdot \omega \sim 1$

- Low-frequency limit has small lensing $\omega \to 0 \quad \Rightarrow \quad F \to 1$

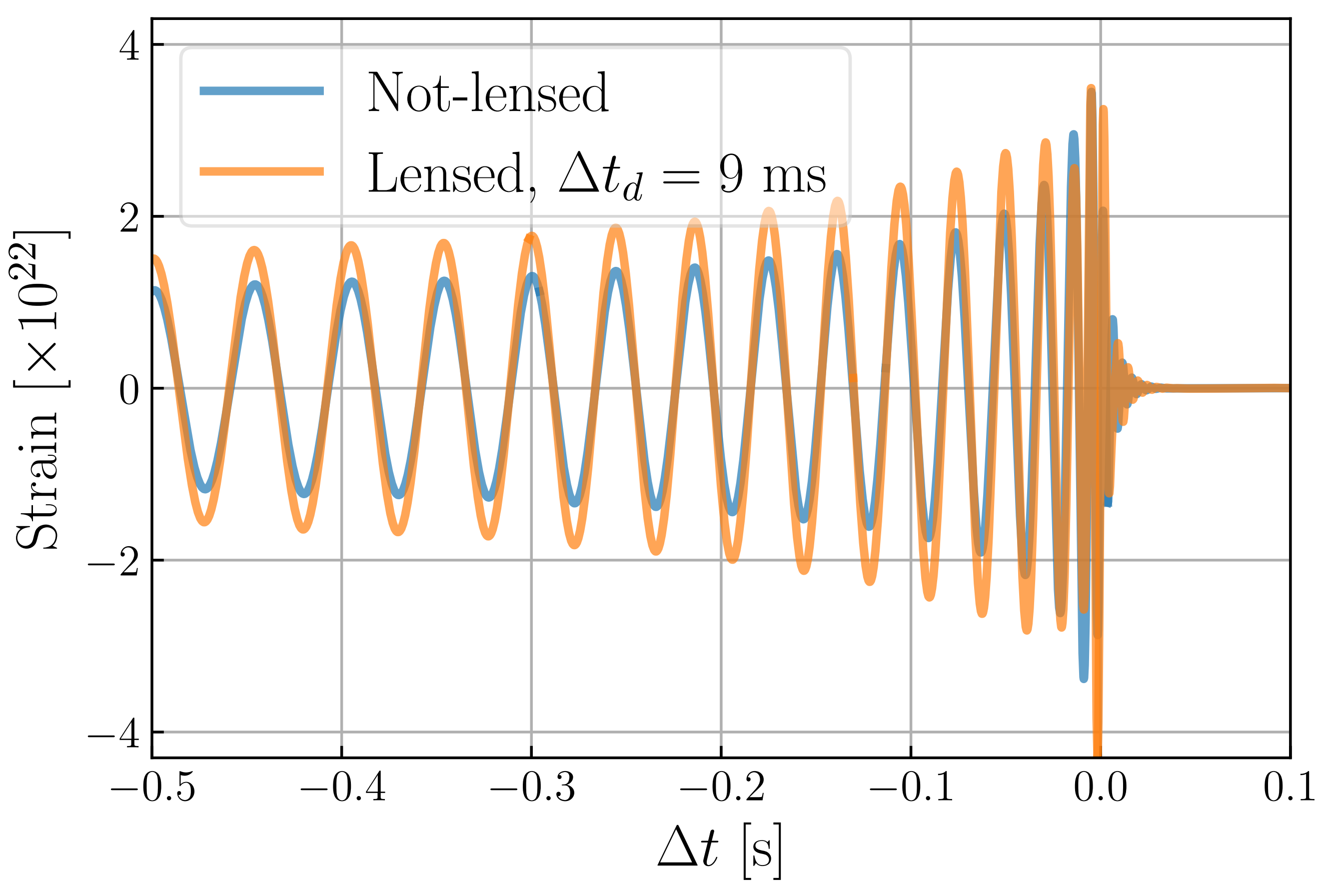
Wave optics: **diffraction**



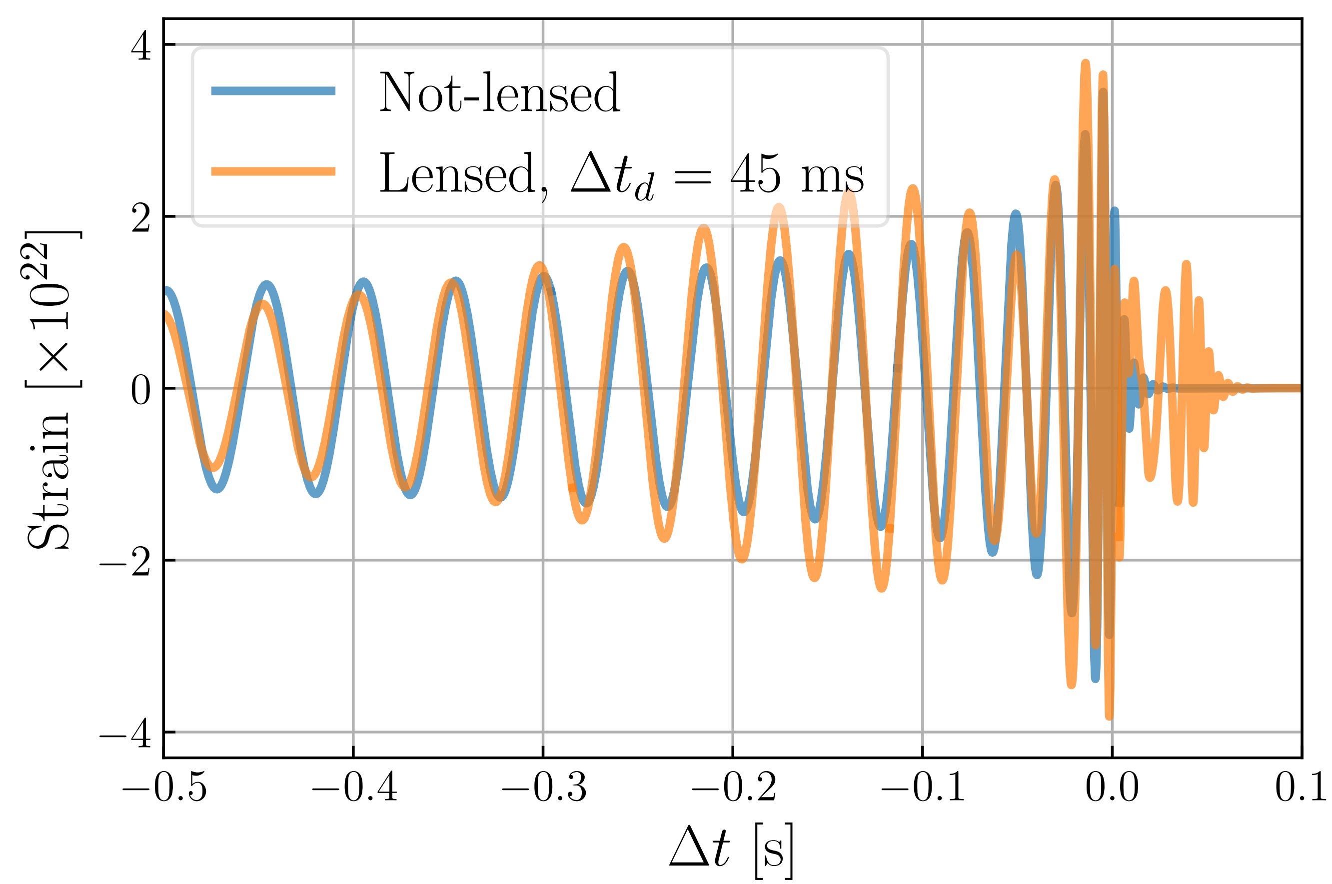
E.g. compact (point) lenses

$$\Delta t_d(y=1) \simeq 4 \left(\frac{(1+z_L)M_L}{100 M_\odot} \right) \text{ms}$$

Diffraction

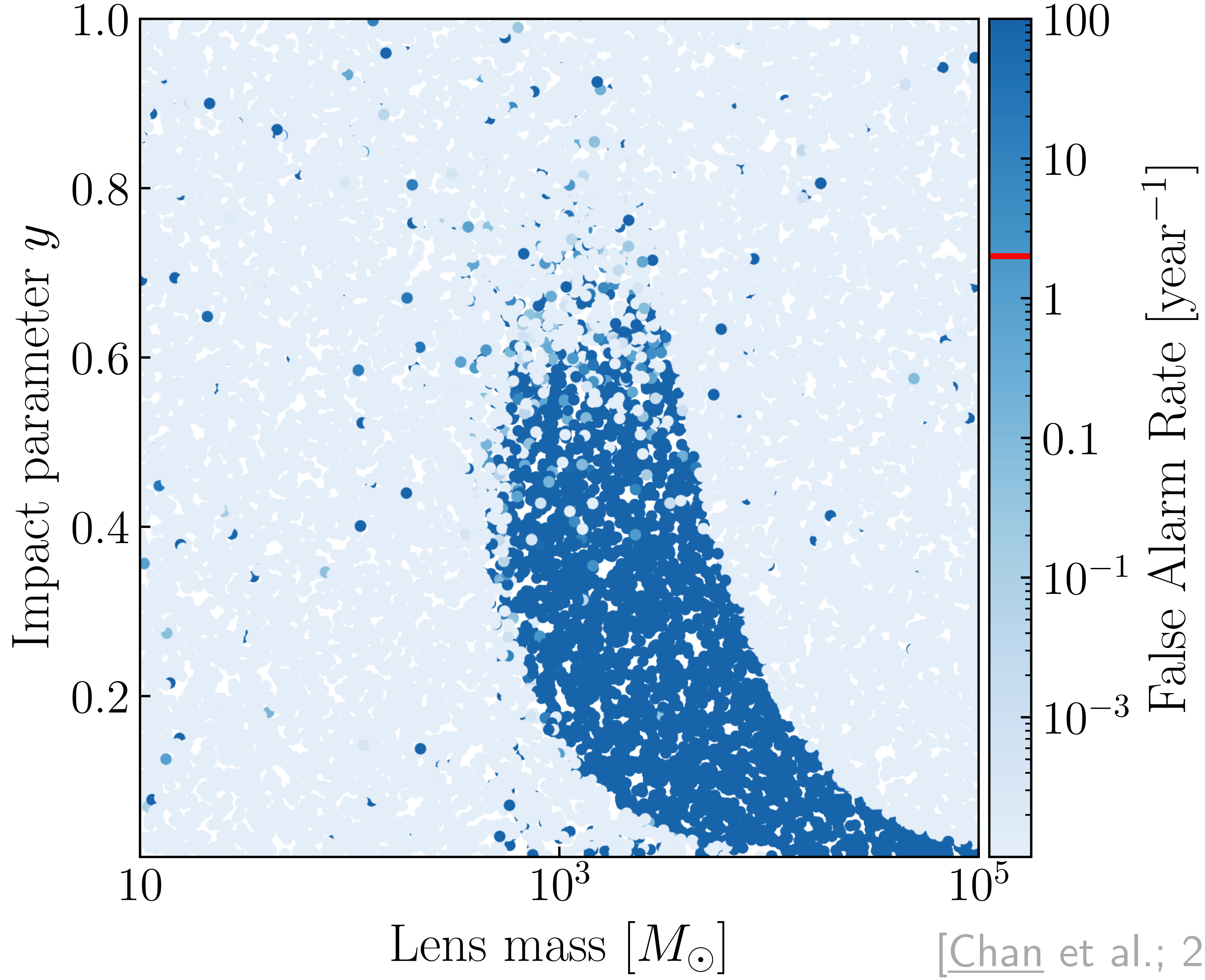


Interference



Searching for distorted lensed GWs

- Highly distorted waveforms could be missed by current searches

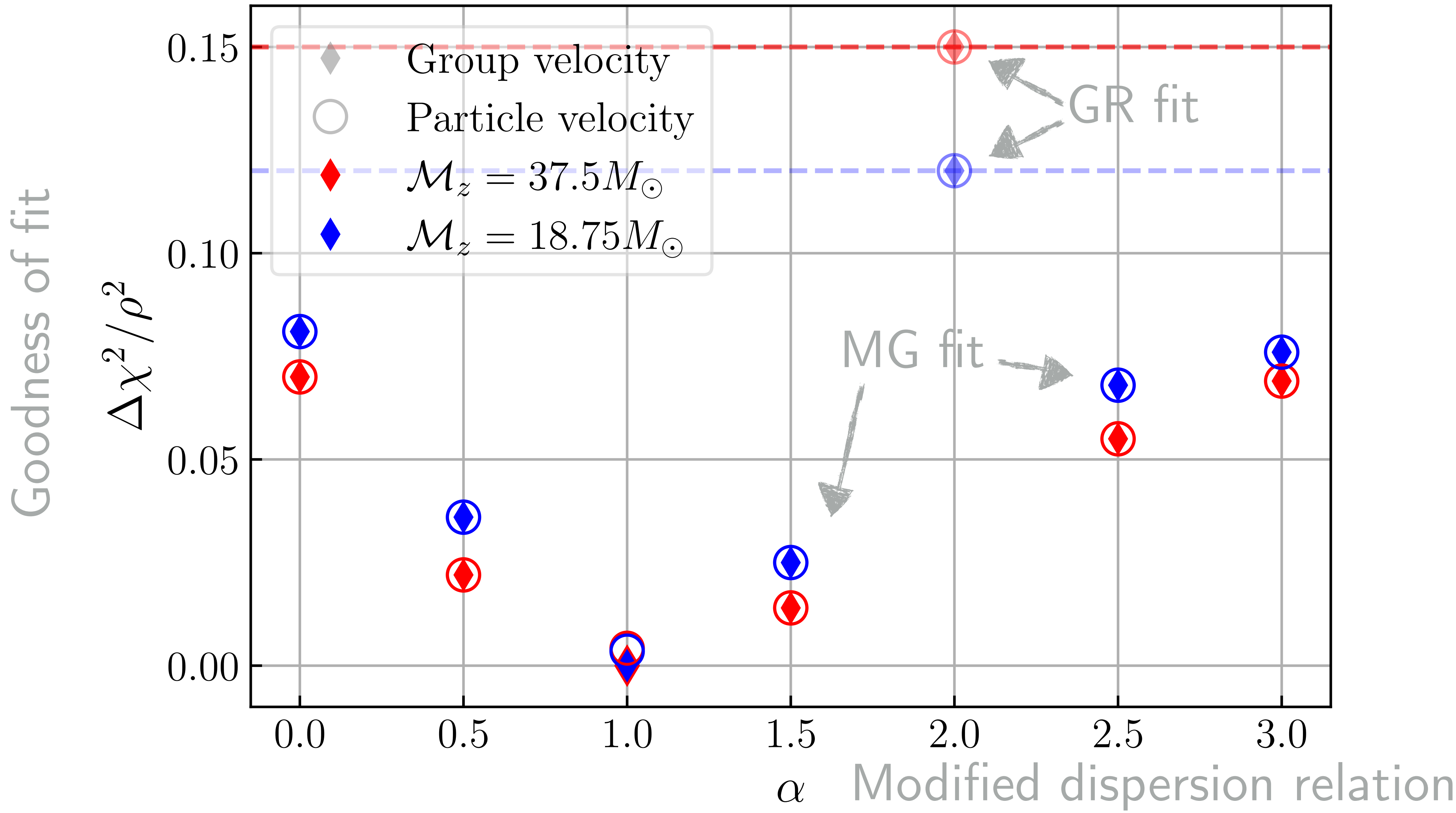


[Chan et al.; 2024]

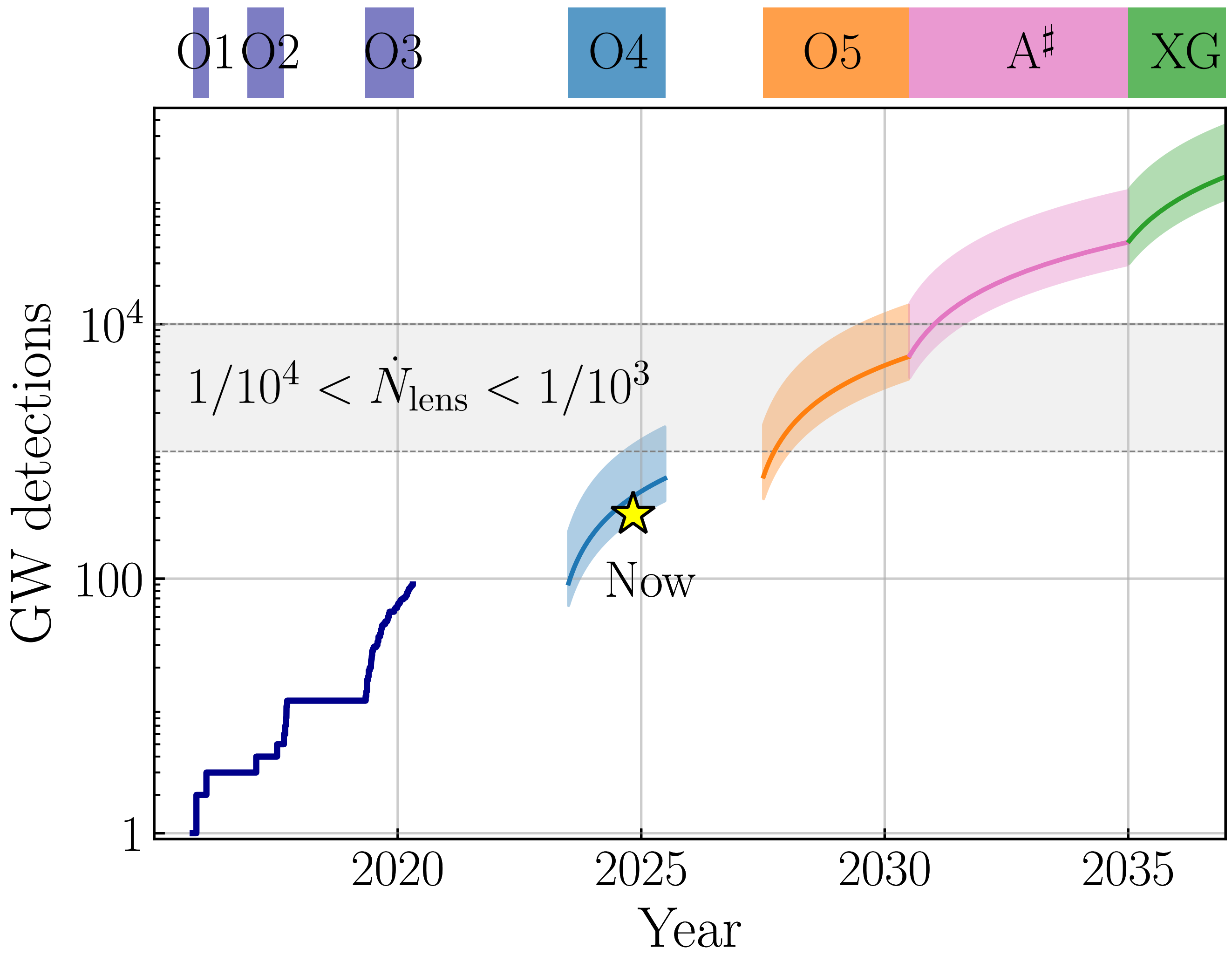
False violations of general relativity

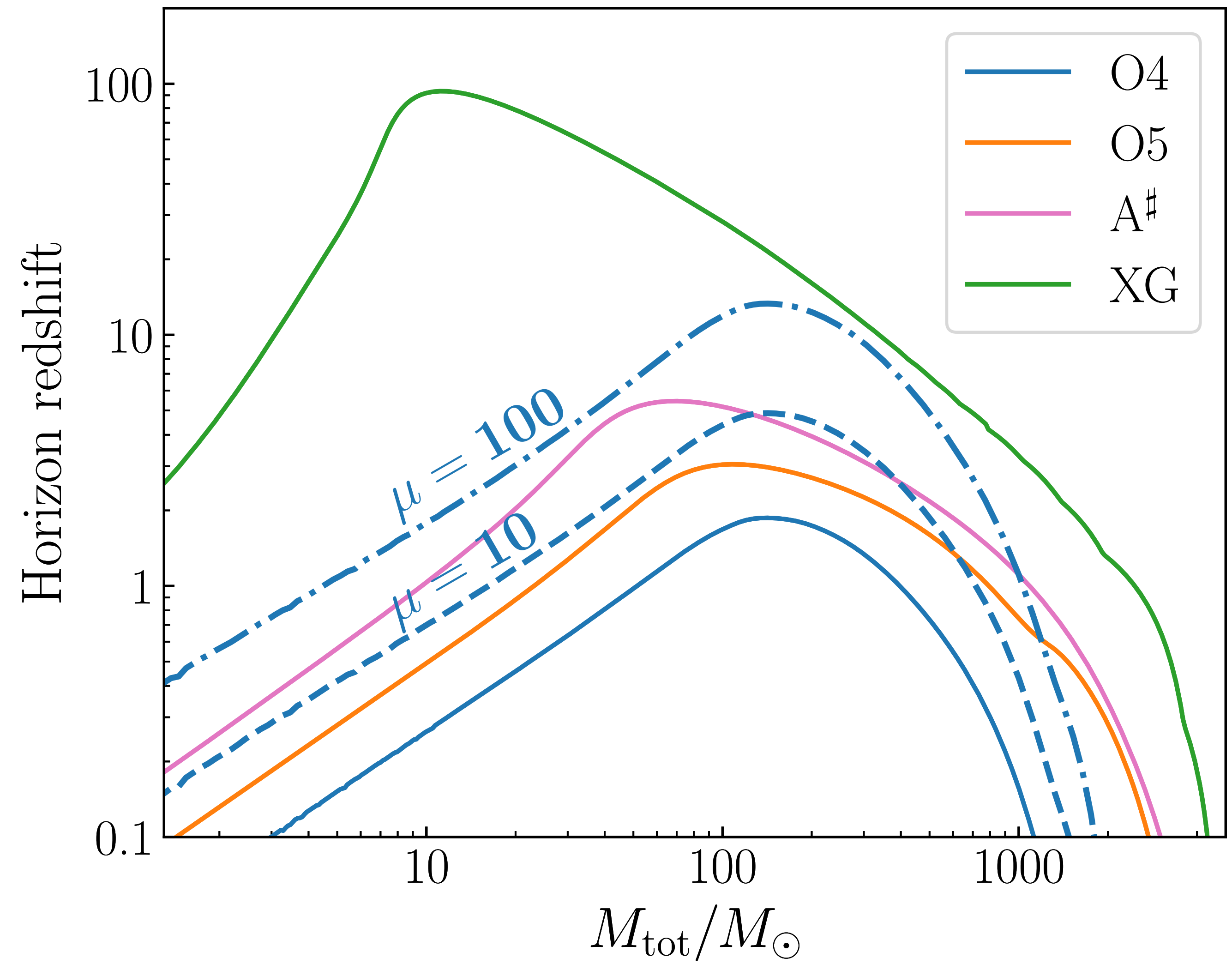
- Lensed waveforms can be different from (unlensed) general relativity waveforms
- E.g. type II images

[**Ezquiaga**, *et al.*; JCAP'22]



Gravitational wave lensing:
expanding *horizons*

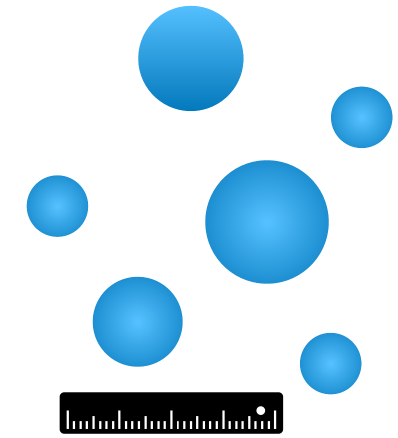


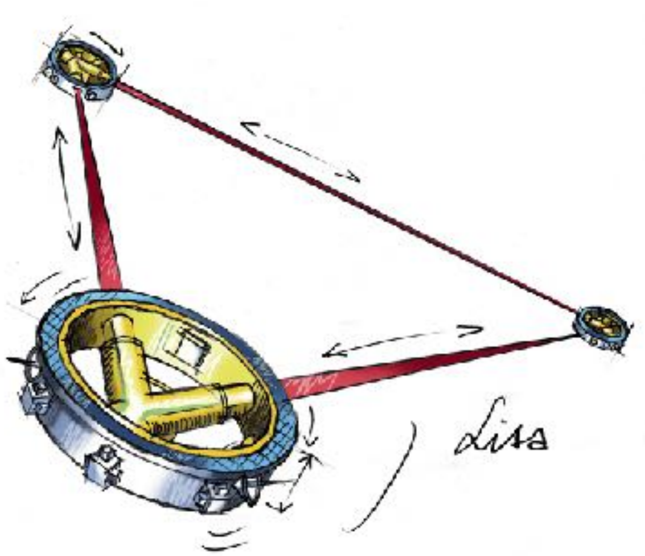


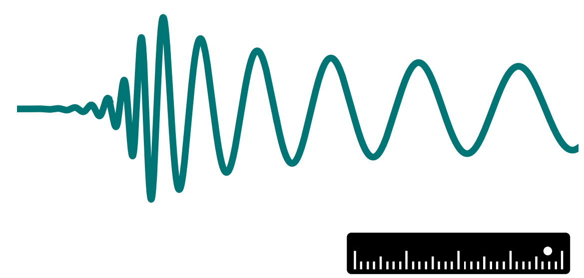
[Xu, **Ezquiaga**, Holz; ApJ'21]

[Lo, Vujeva, **Ezquiaga**, Chan; 2024]

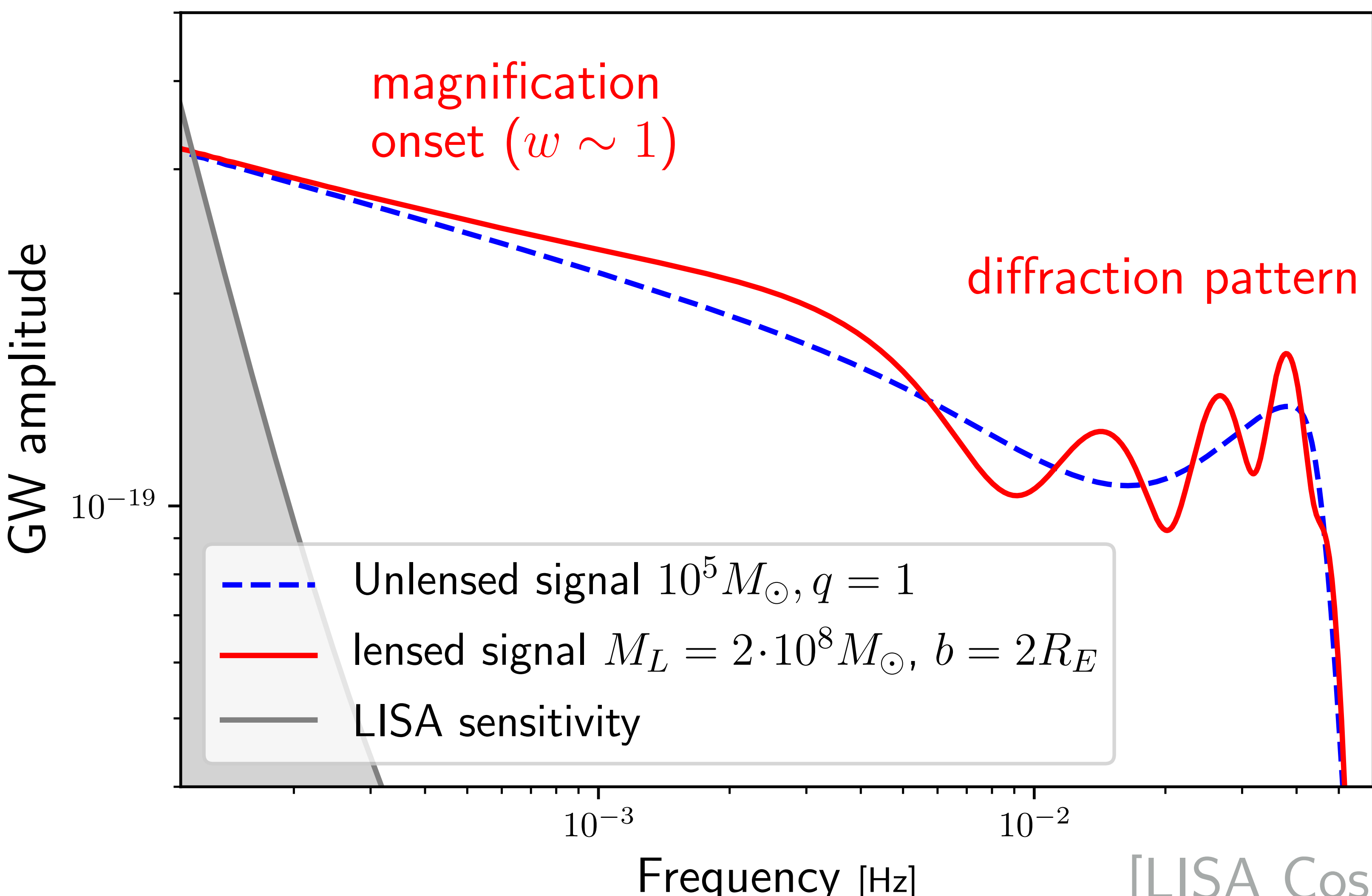
Probing *dark matter* structures





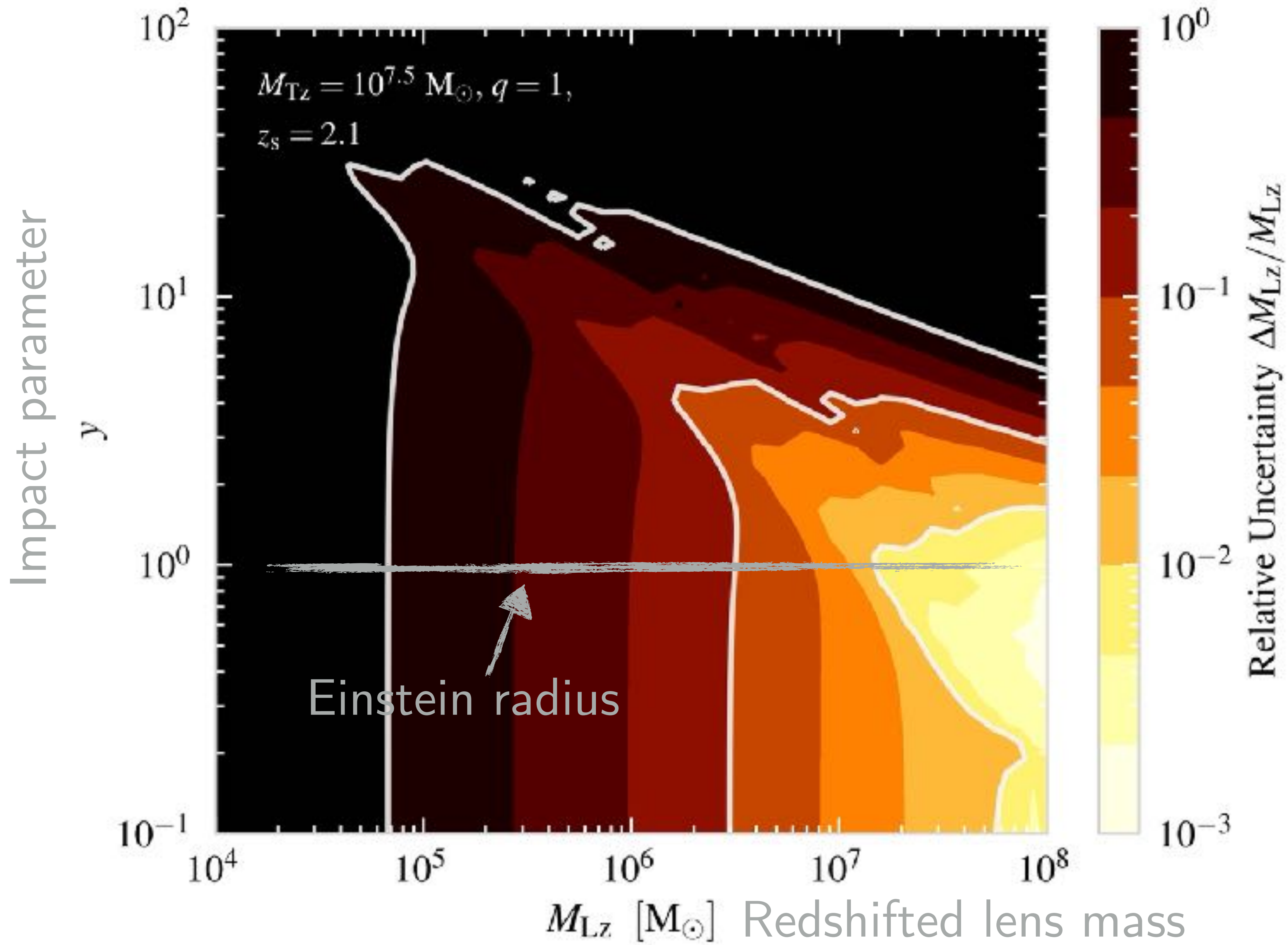


$$\lambda_{\rm gw} \sim 10^3 {\rm km} \left(\frac{M_{\rm bbh}}{10 M_\odot} \right)$$



[LISA Cosmo white paper]

Increased optical depth in wave optics



[Çalışkan *et al.*; PRD'23]

4. Key takeaways

- Gravitational waves are *only* altered by *gravitational interactions* with cosmic structures
- Strong lensing may produce *repeated chirps*. Searching for them is difficult, but first detections is around the corner
- Gravitational waves may be diffracted by cosmic structures producing *distorted waveforms*. This is unique!
- There are other unique observational signatures as phase shifts in *type II images*
- Lensed gravitational waves can probe small *compact lenses* and *dark matter subhalos*

Conclusions

Gravitational waves are precious cosmological probes:

- Well understood signals from general relativity
- Coherent detection of waveform

- Expansion rate at high redshift $H(z)$ with **binary black holes** mergers
- Probing origin of the observed black holes and dark matter substructures via **lensing**

- Future of gravitational wave astronomy is exciting. **Join us!**

There are MANY other things I did not have time to cover:

- cross correlations with other surveys
- stochastic backgrounds
- neutron star equation of state
- tests of gravity
- ....

Muchas gracias!

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Join us!

VILLUM FONDEN

ezquiaga.github.io/joinus