



# A premier on gravitational wave lensing

[O'Keefe, 1965]

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Niels Bohr Institute

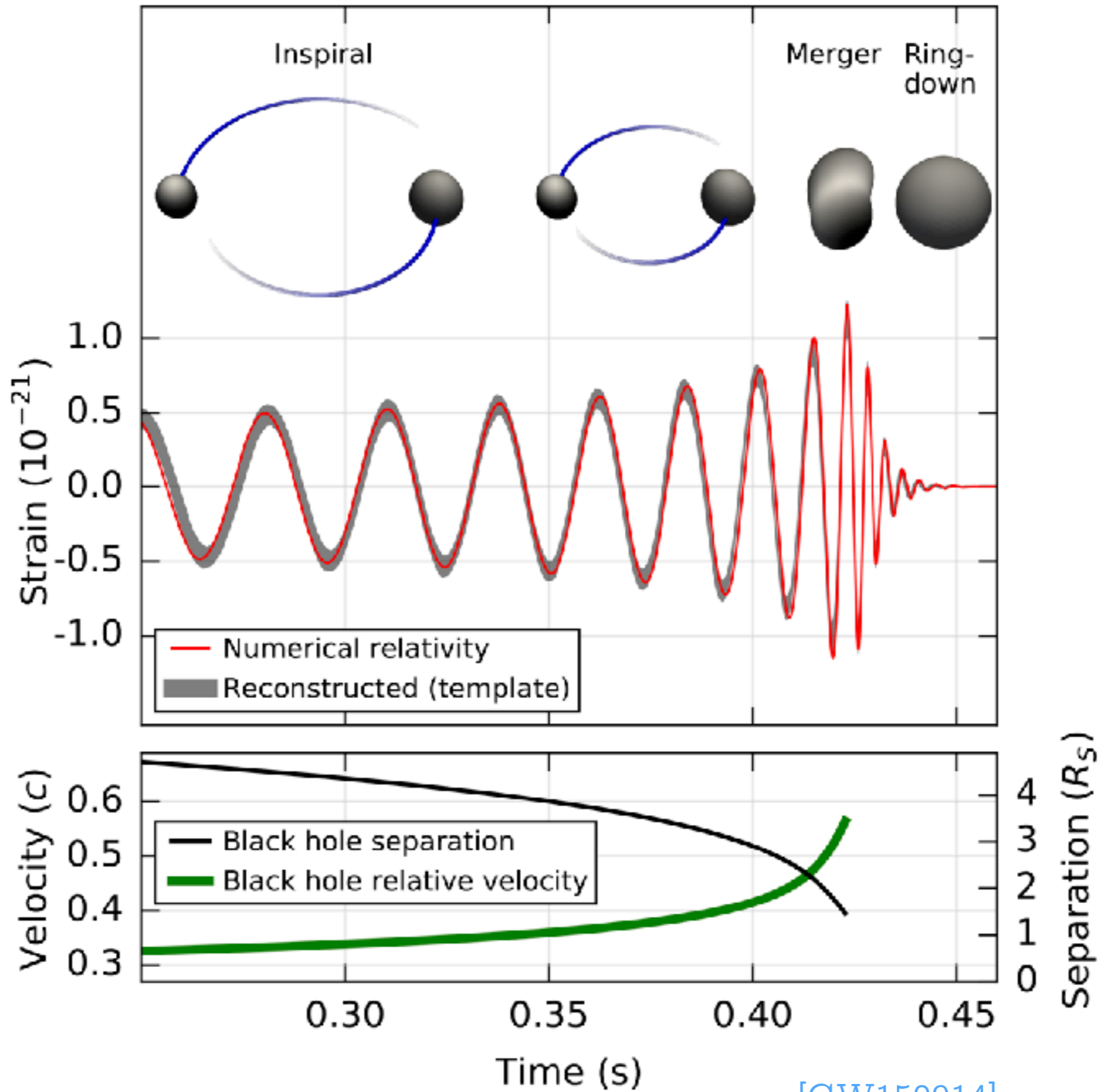
[jose.ezquiaga@nbi.ku.dk](mailto:jose.ezquiaga@nbi.ku.dk)

[ezquiaga.github.io](https://ezquiaga.github.io)

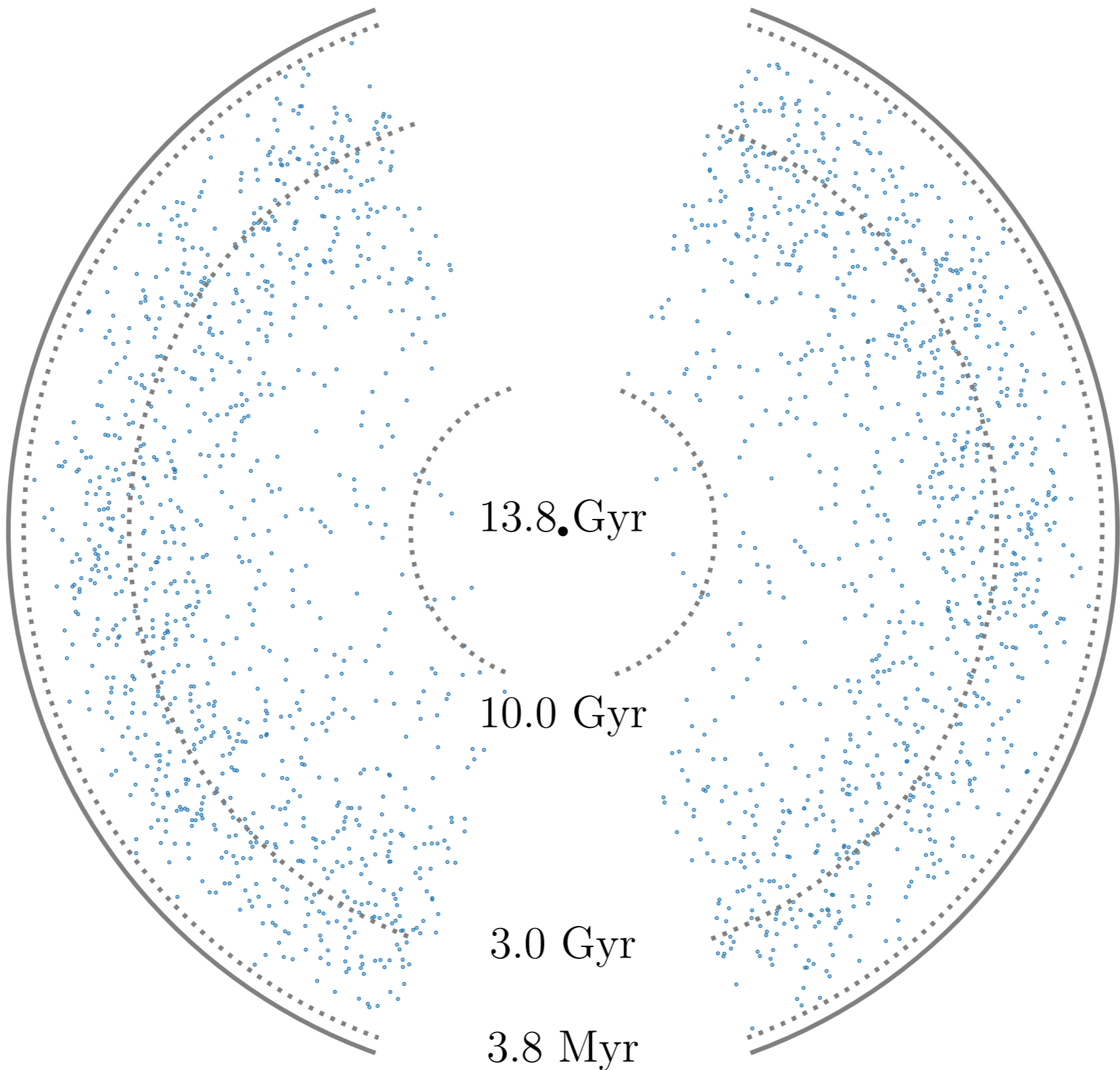
VILLUM FONDEN



KØBENHAVNS  
UNIVERSITET



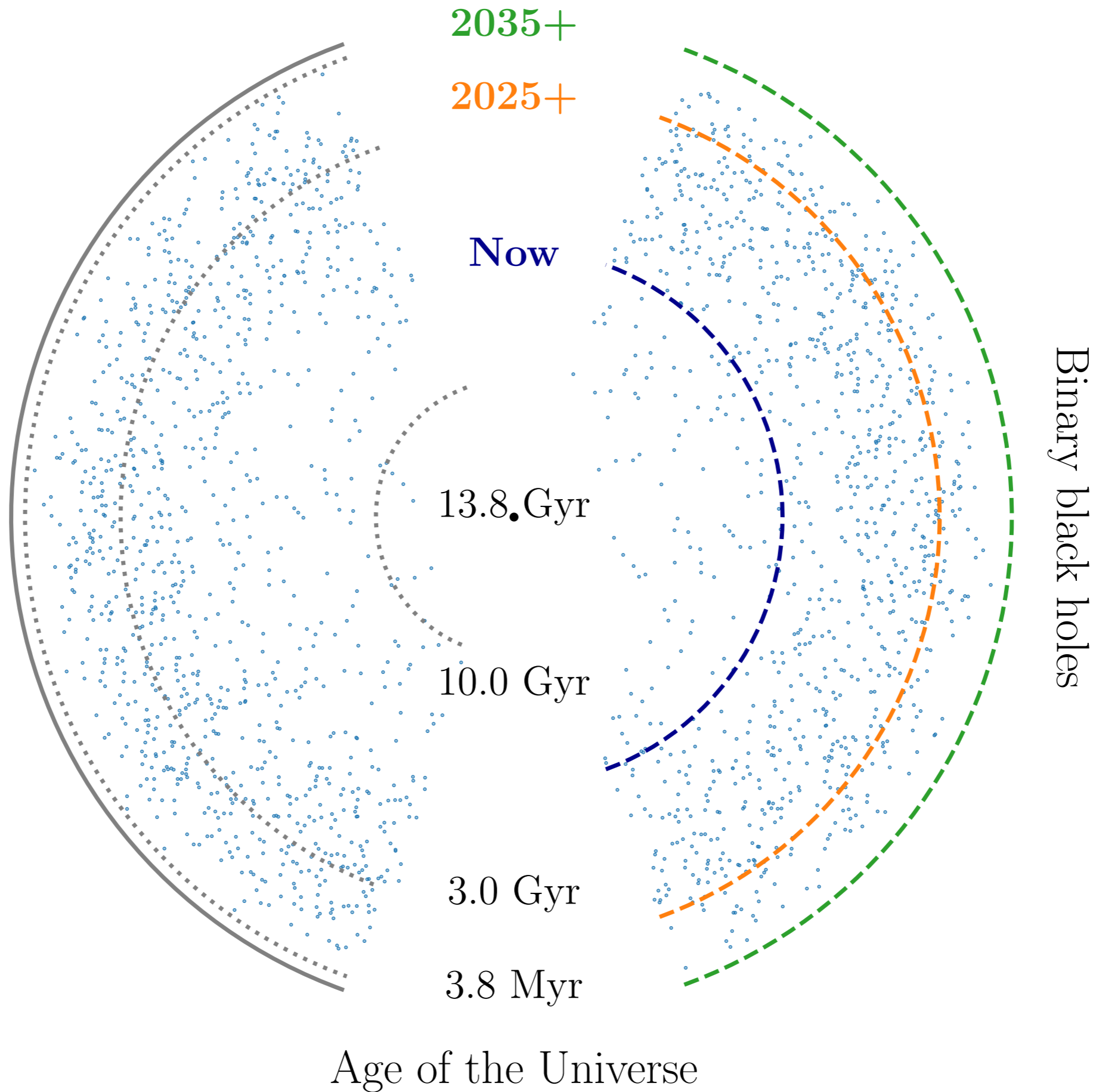
[GW150914]



Binary black holes

Age of the Universe

# Gravitational Wave horizons



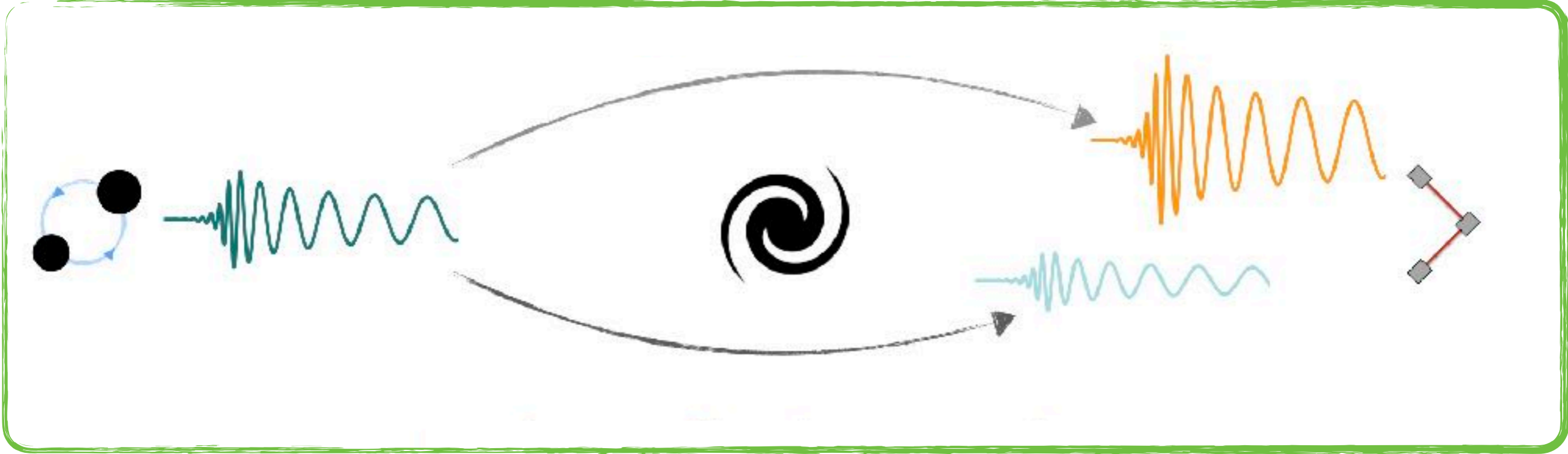


# Gravitational waves

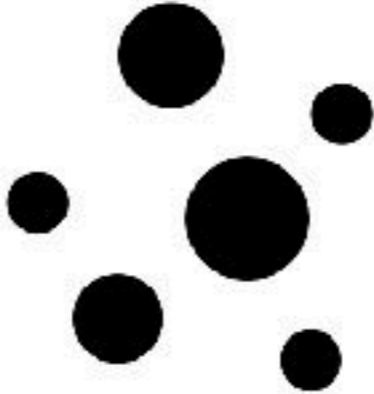
- **Signals from first principles:** *general relativity* predicts the waveform of a compact binary coalescence
- **Well understood selection effects:** signals can be injected in real noise
- **Powerful population studies:** all sky coverage up to cosmological distances, not affected by medium
- **Wavelengths of astrophysical scale:** frequency from the orbital motion



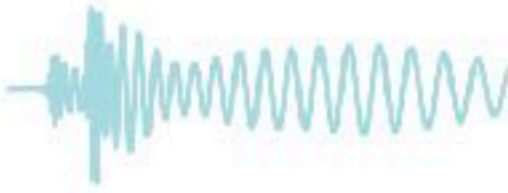
# What lensing?



Source

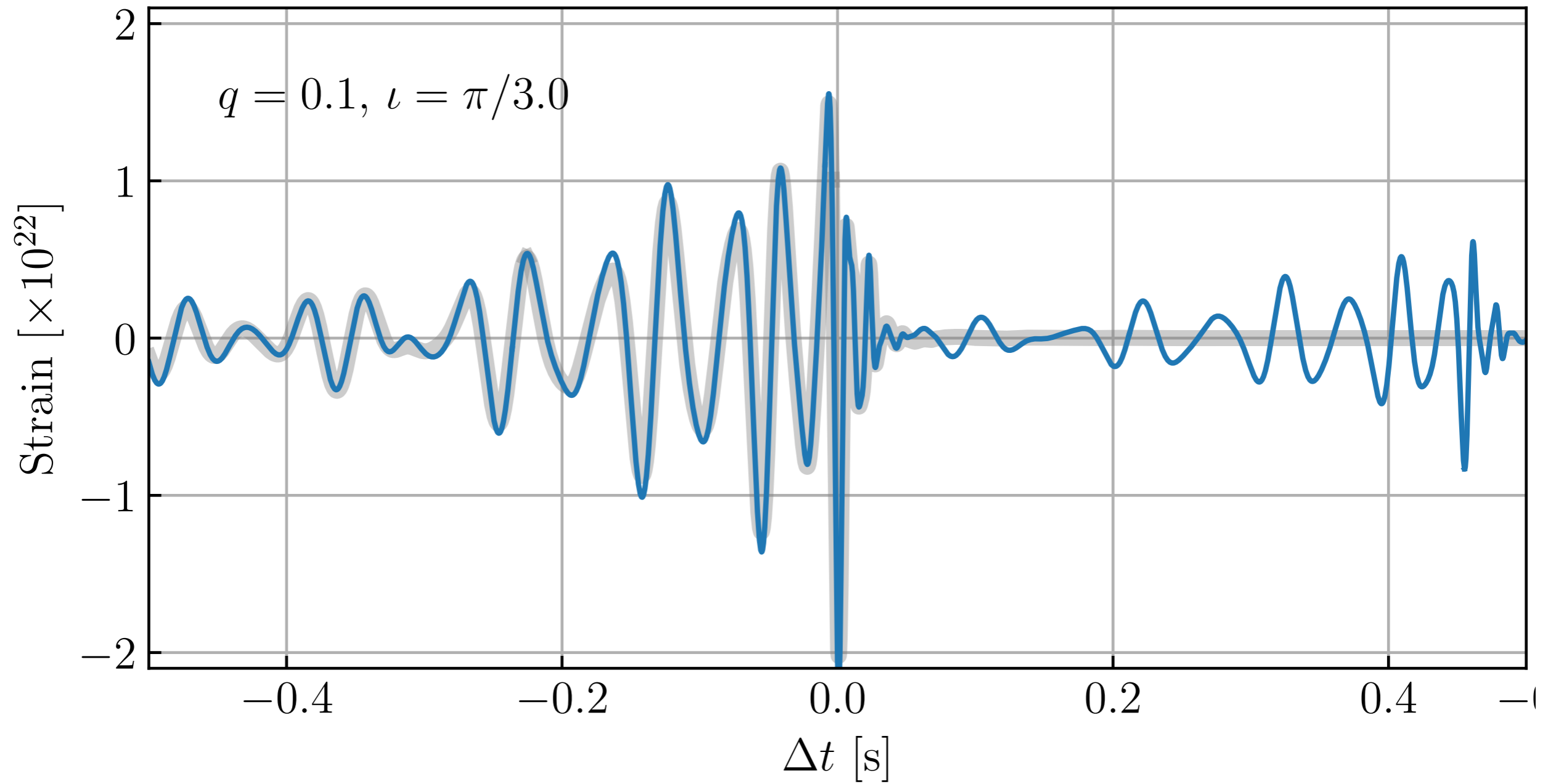


Lens

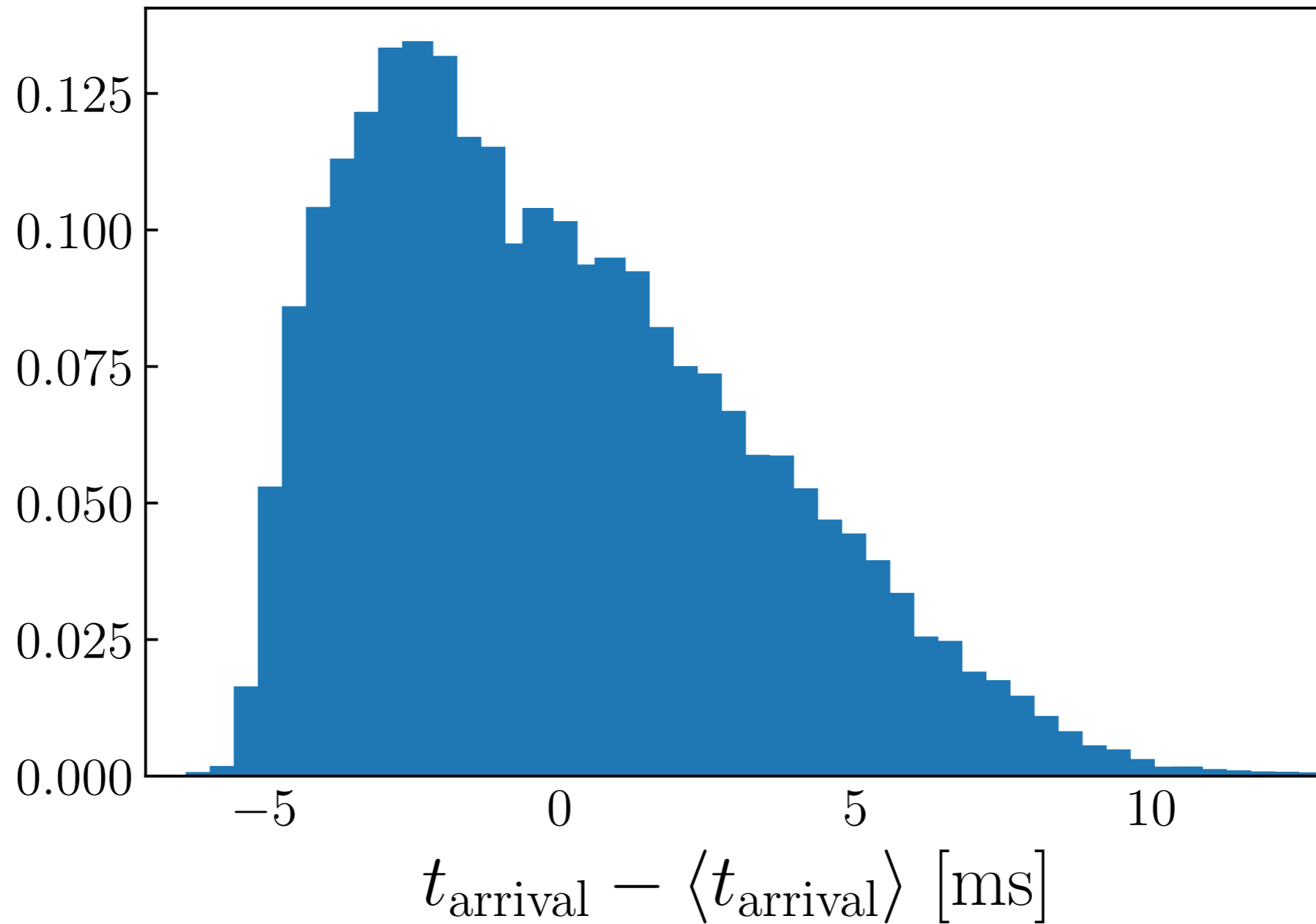


Detector

# Repeated signals



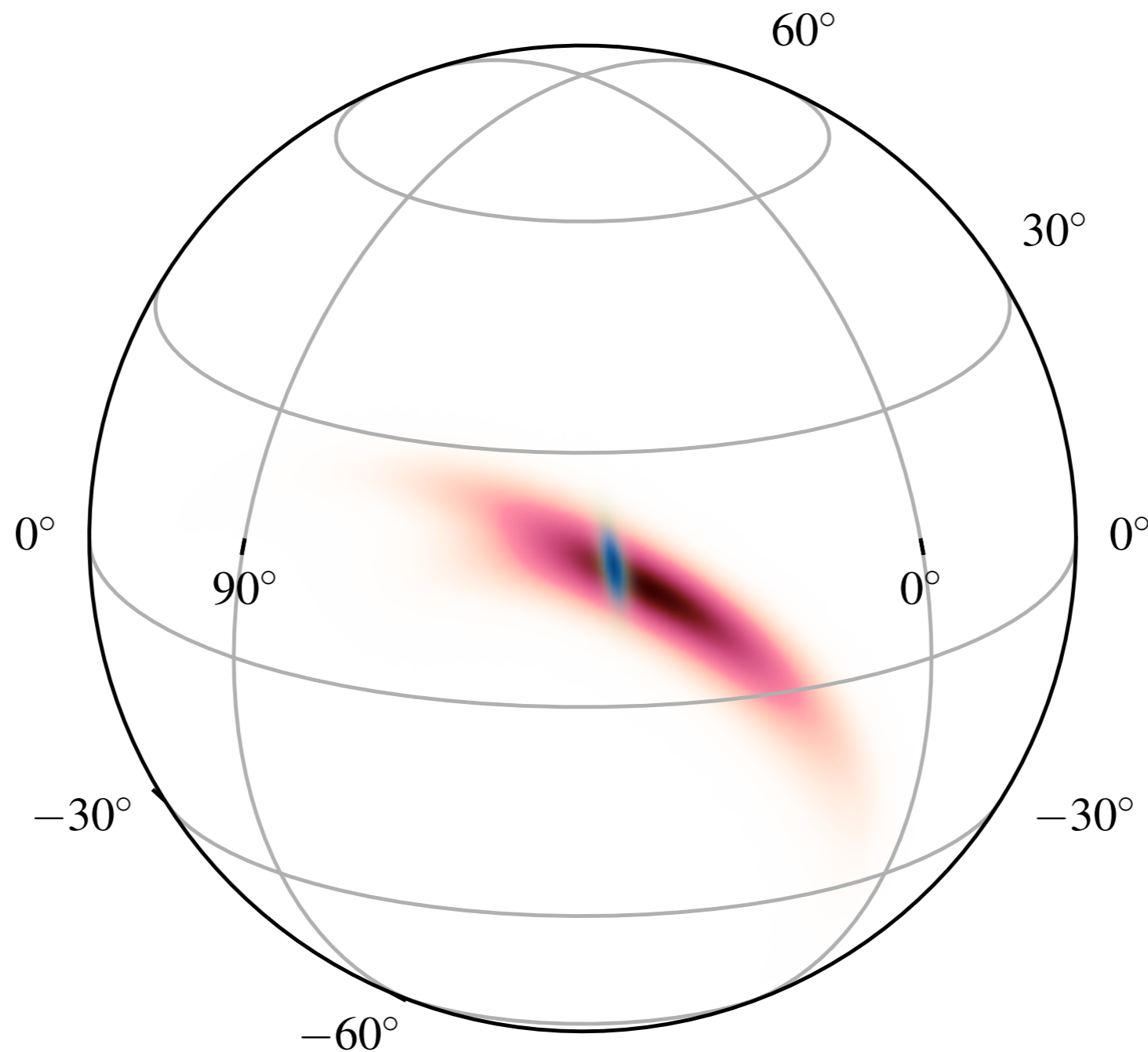
# Precise timing





# Poor sky localization

$$\theta_E \sim 1'' \sqrt{\frac{M}{10^{12} M_\odot}} \sqrt{\frac{1 \text{ Gpc}}{D}}$$



# Gravitational lensing of gravitational waves

- Repeated signals
- Precise timing
- Poor sky localization
- Coherent detection of waveform

# Strong lensing

$$\Delta t_d \cdot \omega \gg 1$$

$$h_L(\omega) = F(\omega, \theta_S) \cdot h(\omega)$$

$$F \approx \sum_j |\mu_j|^{1/2} \exp(i\omega t_j - i\pi n_j)$$

Magnification  
Time delay  
Phase shift

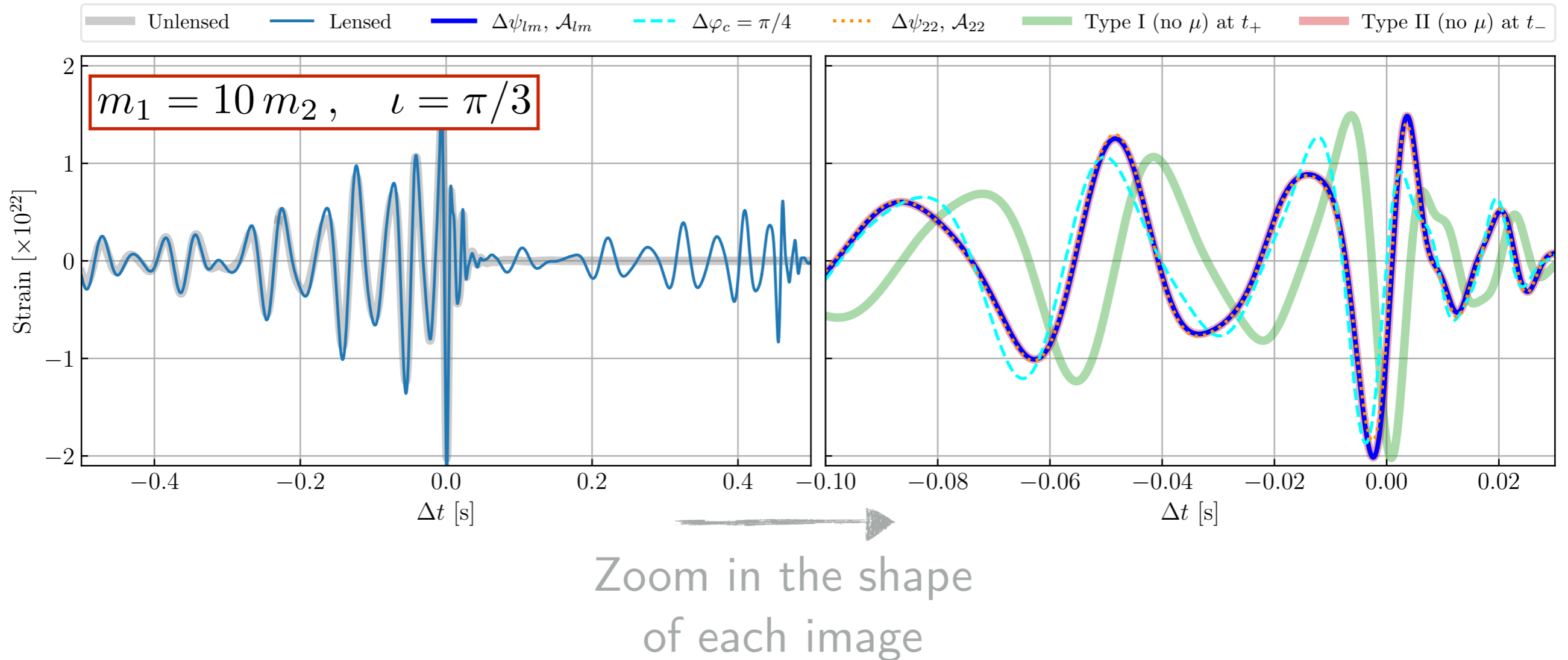
- Each image type (I, II and III) acquire a different phase shift

$$n_j = 0, 1/2, 1$$



# Waveform distortions in **type II** images

- Lensing imprints *small* but *characteristic* modifications in the signals that cannot be mapped to other astrophysical parameters





# Strong lensing

$$\Delta t_d \cdot \omega \gg 1$$

$$h_L(\omega) = F(\omega, \theta_S) \cdot h(\omega)$$

$$F \approx \sum_j |\mu_j|^{1/2} \exp(i\omega t_j - i\pi n_j)$$

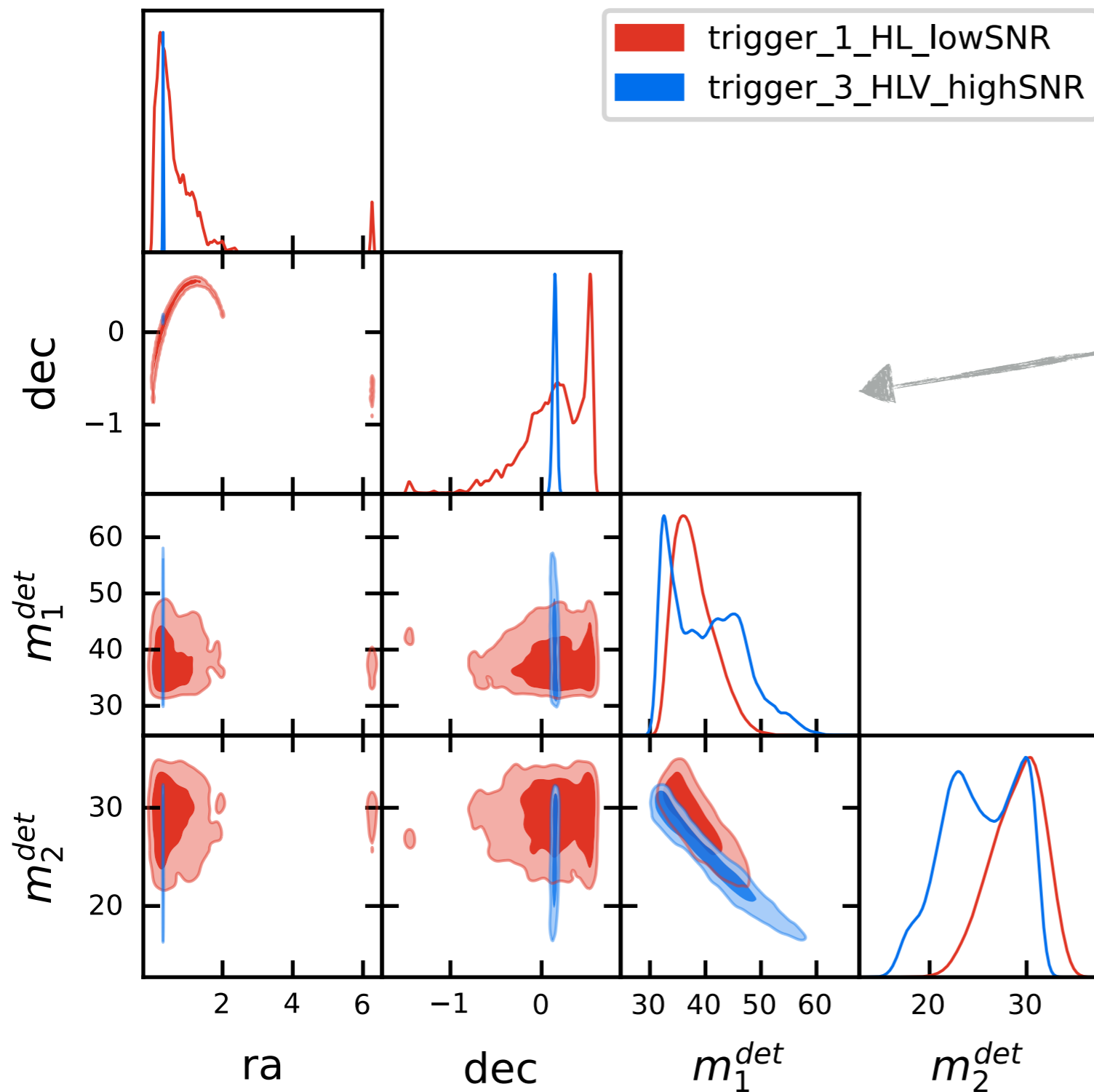
Magnification  
Time delay  
Phase shift

- Each image type (I, II and III) acquire a different phase shift

$$n_j = 0, 1/2, 1$$

- Lensed GWs *can differ* from (unlensed) GR wave-forms
- *Identify* strong lensing with *single image*

# Searching for strongly lensed GWs



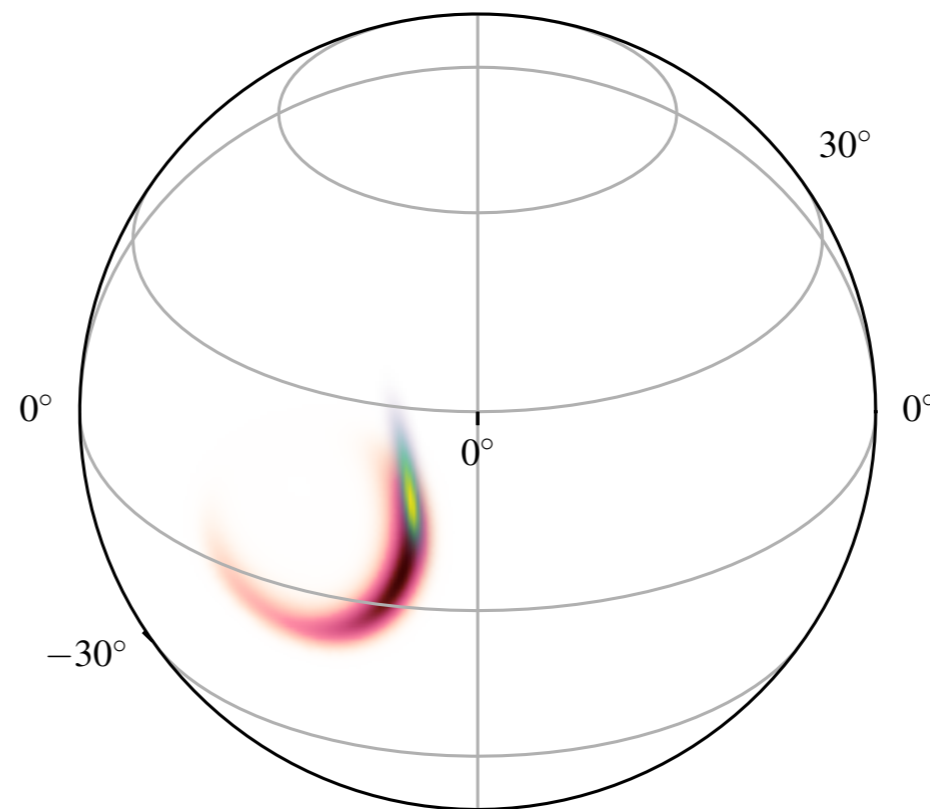
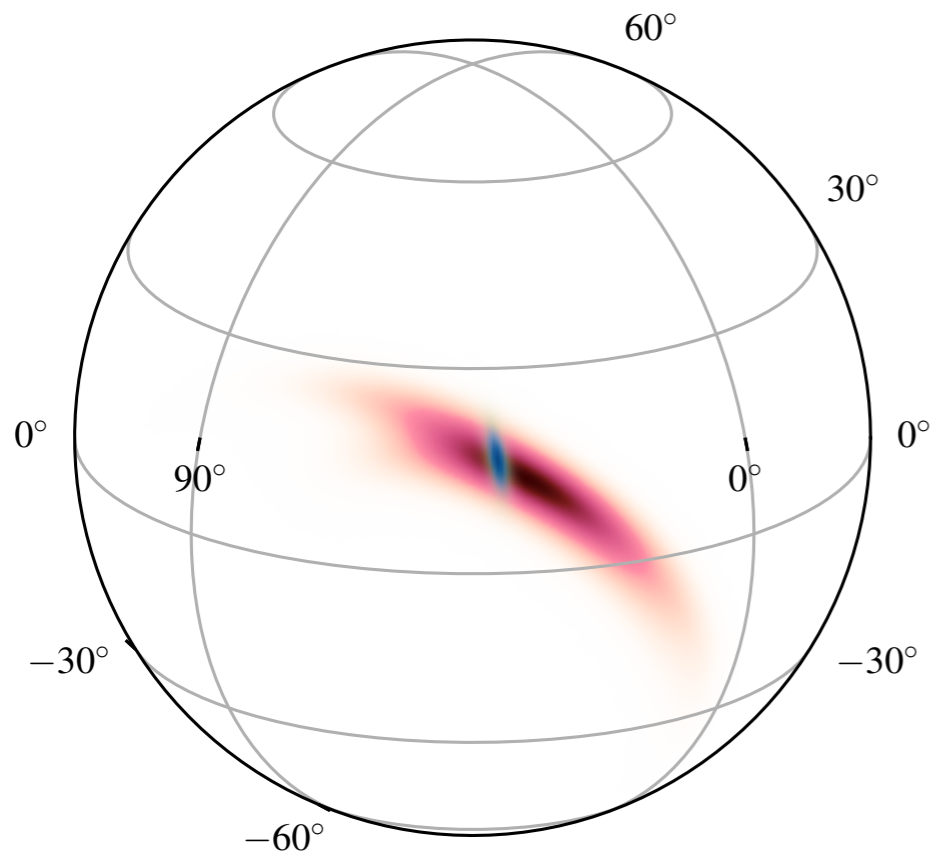
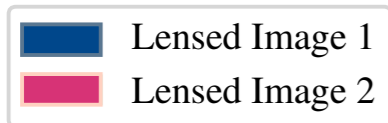
Look for events with similar properties: masses, sky positions, spins...  
+  
Joint parameter estimation

# Lensing or luck?

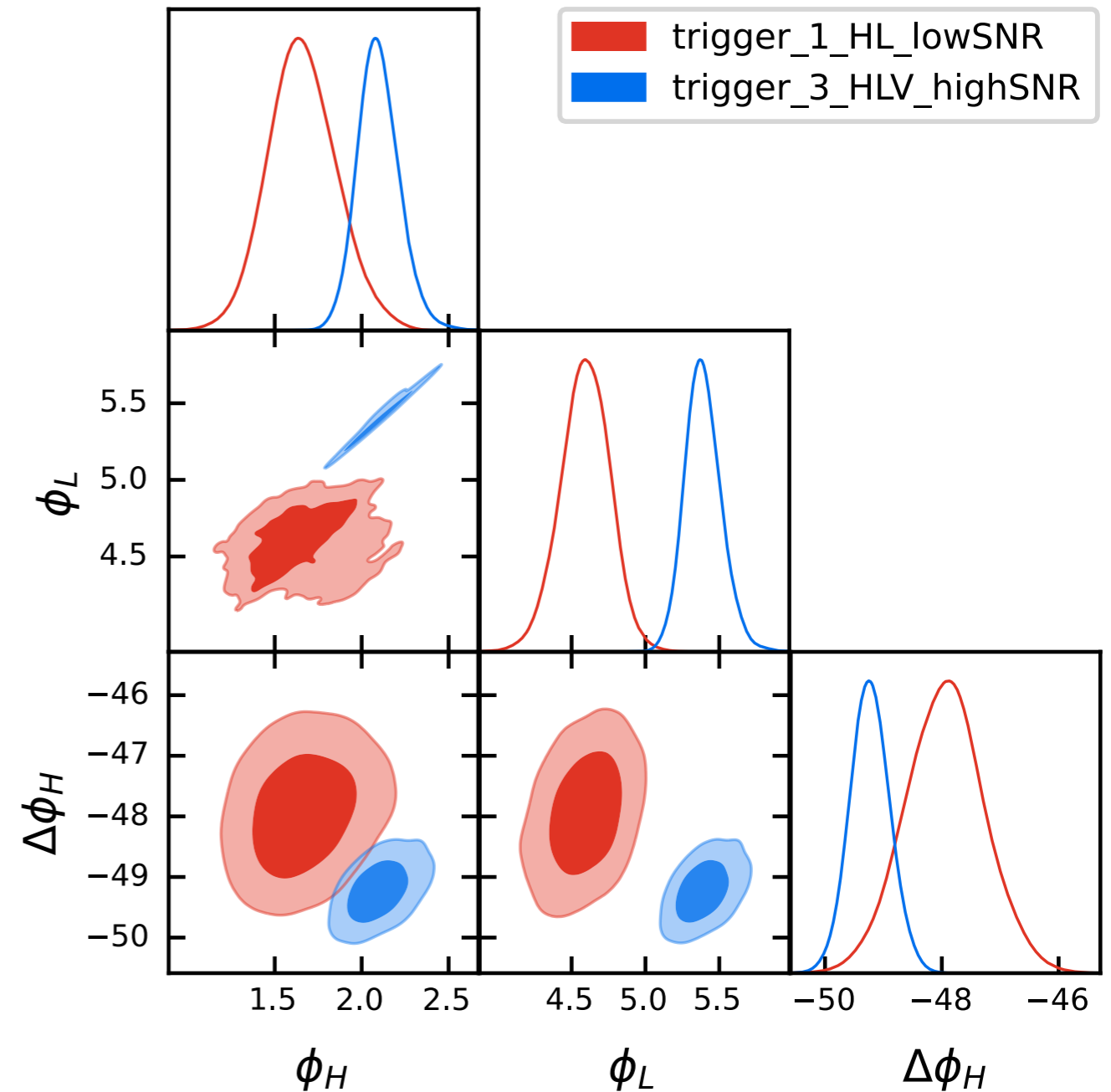
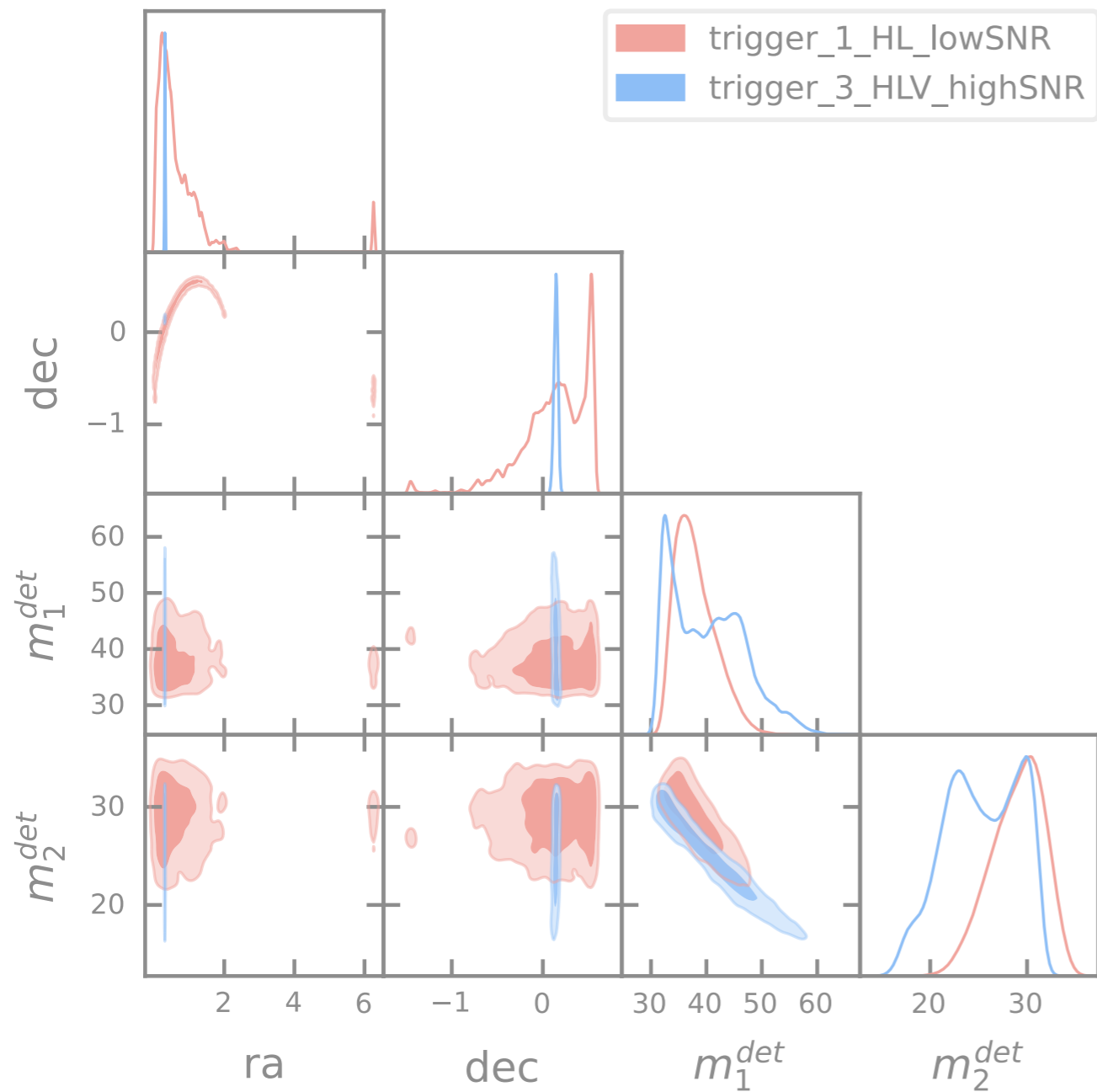
$$N_{\text{false alarm}} \sim N^2$$



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(JHU)

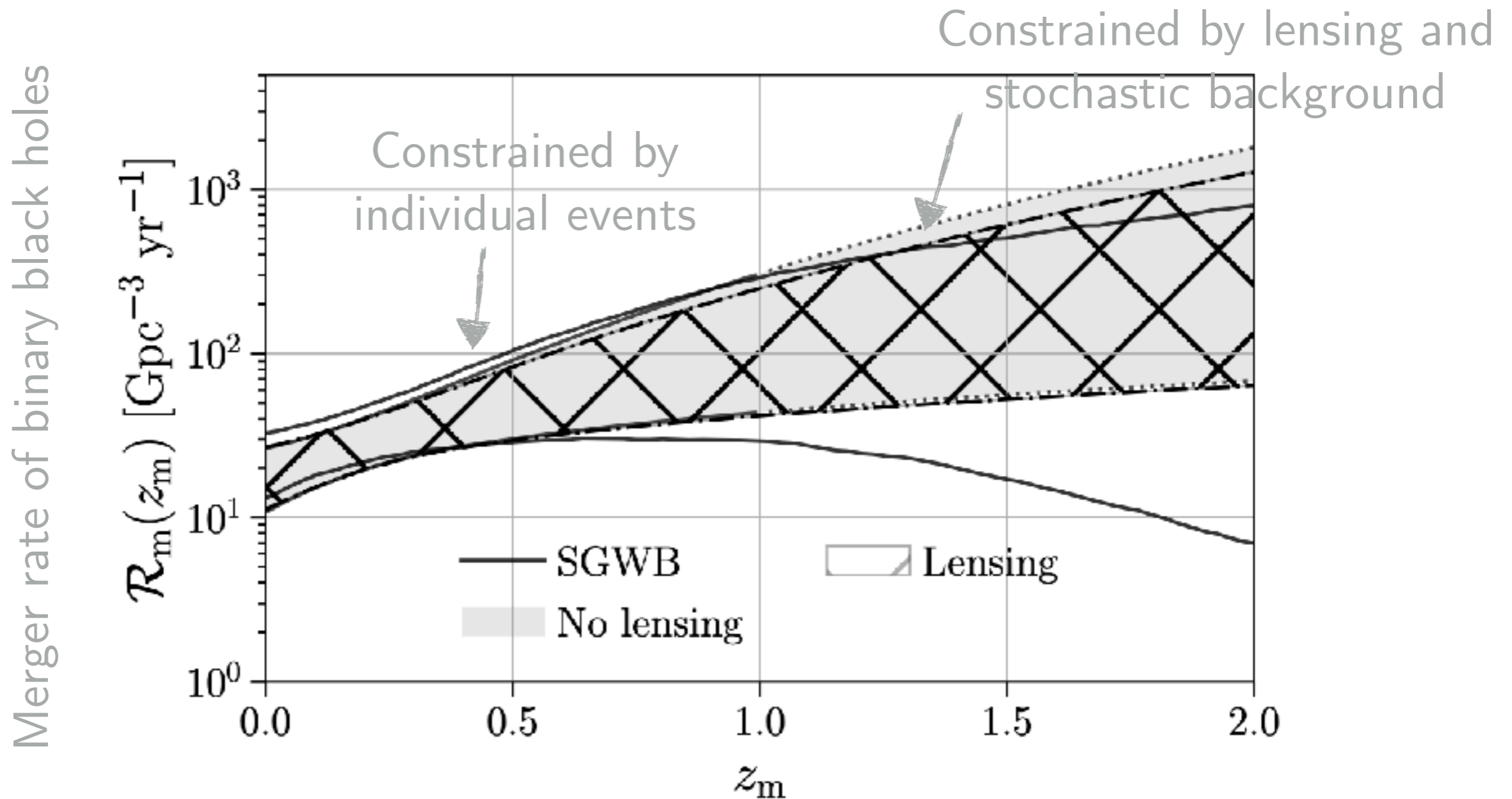


# Fight false alarms: **phase consistency**





# LVK: no evidence of strong lensing so far...

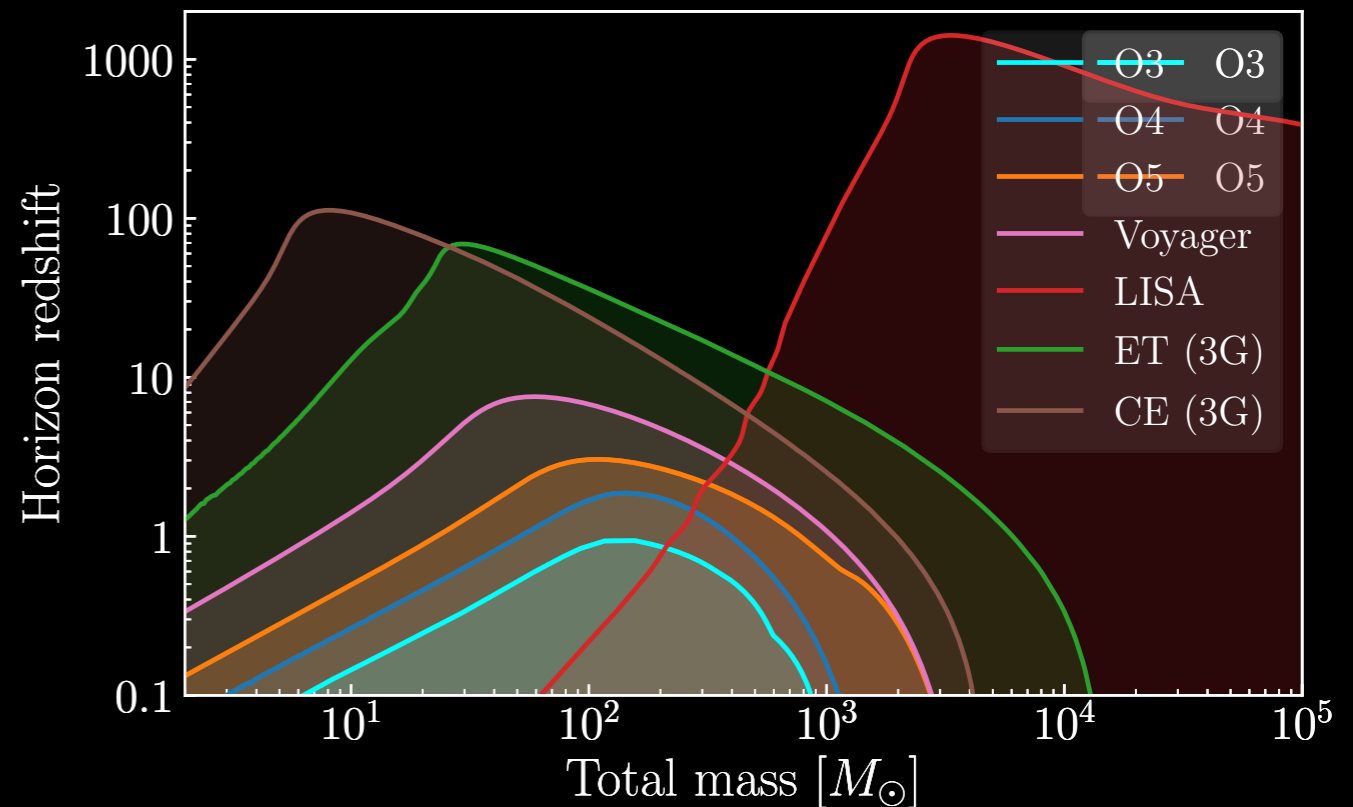
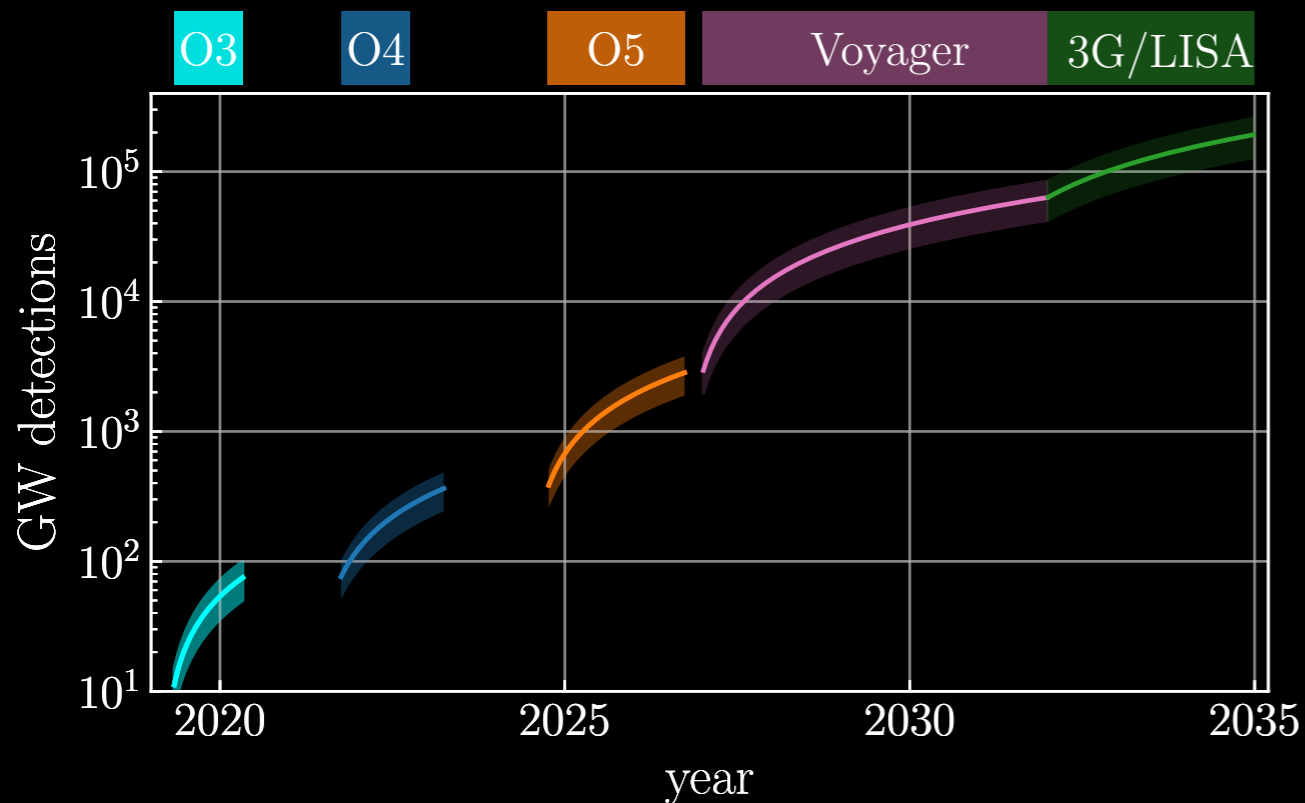


LVC (incl. **Ezquiaga**); *Search GW lensing O3a* (ApJ, [arXiv 2105.06384](#), [science summary](#))

LVK (incl. **Ezquiaga**); *Search GW lensing full O3* ([arXiv 2304.08393](#), [science summary](#))

# Looking ahead

*Now* **2G**: current generation ground-based GW detectors  
**3G**: next generation ground-based GW detectors



*Approx. 100*  
 events typically at  
 $z < 0.6$



*1000s / year*  
 with some  $z > 1$



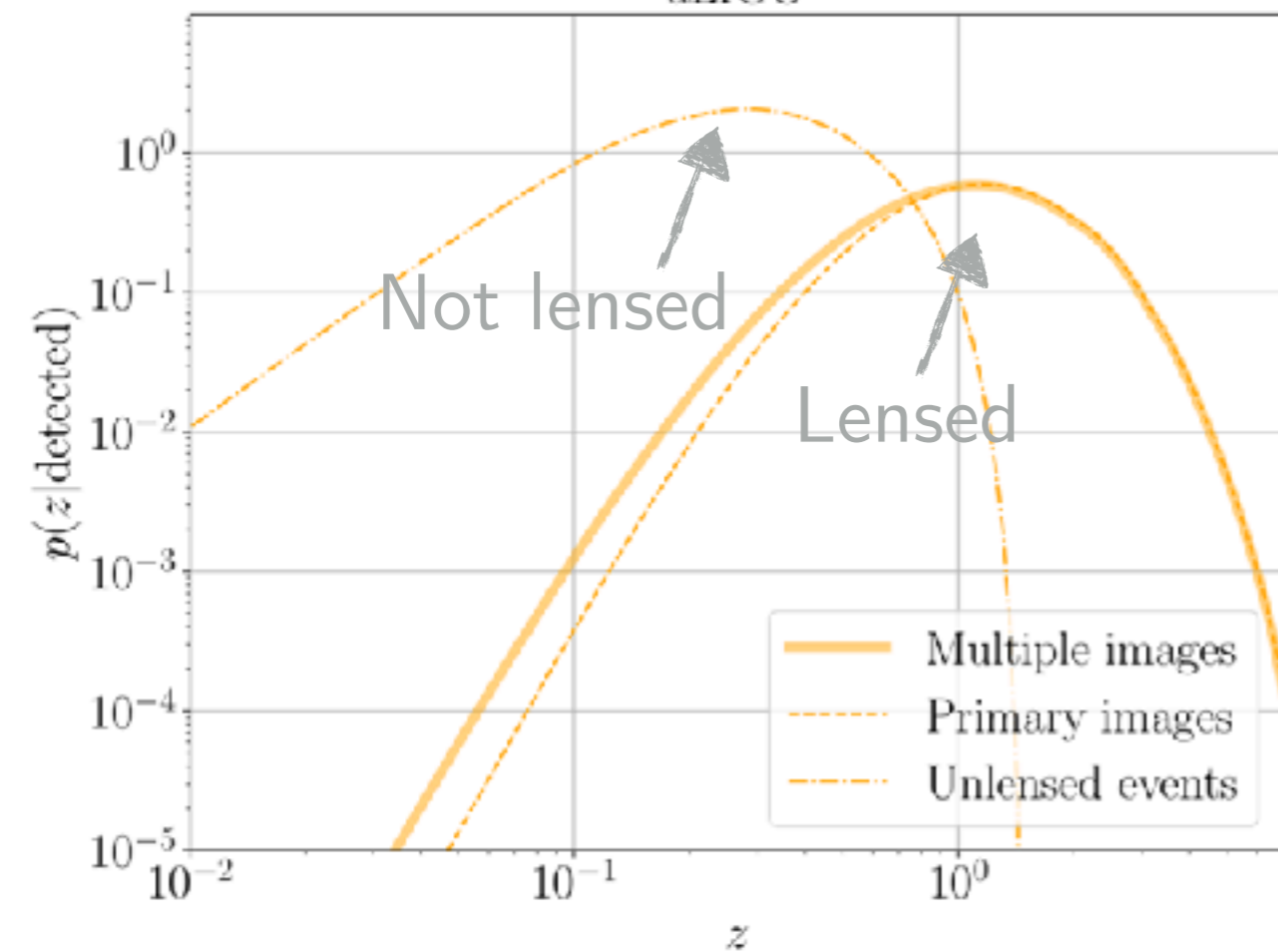
*100,000s / year*  
 with most  $z > 1$

# Probing source and lens populations

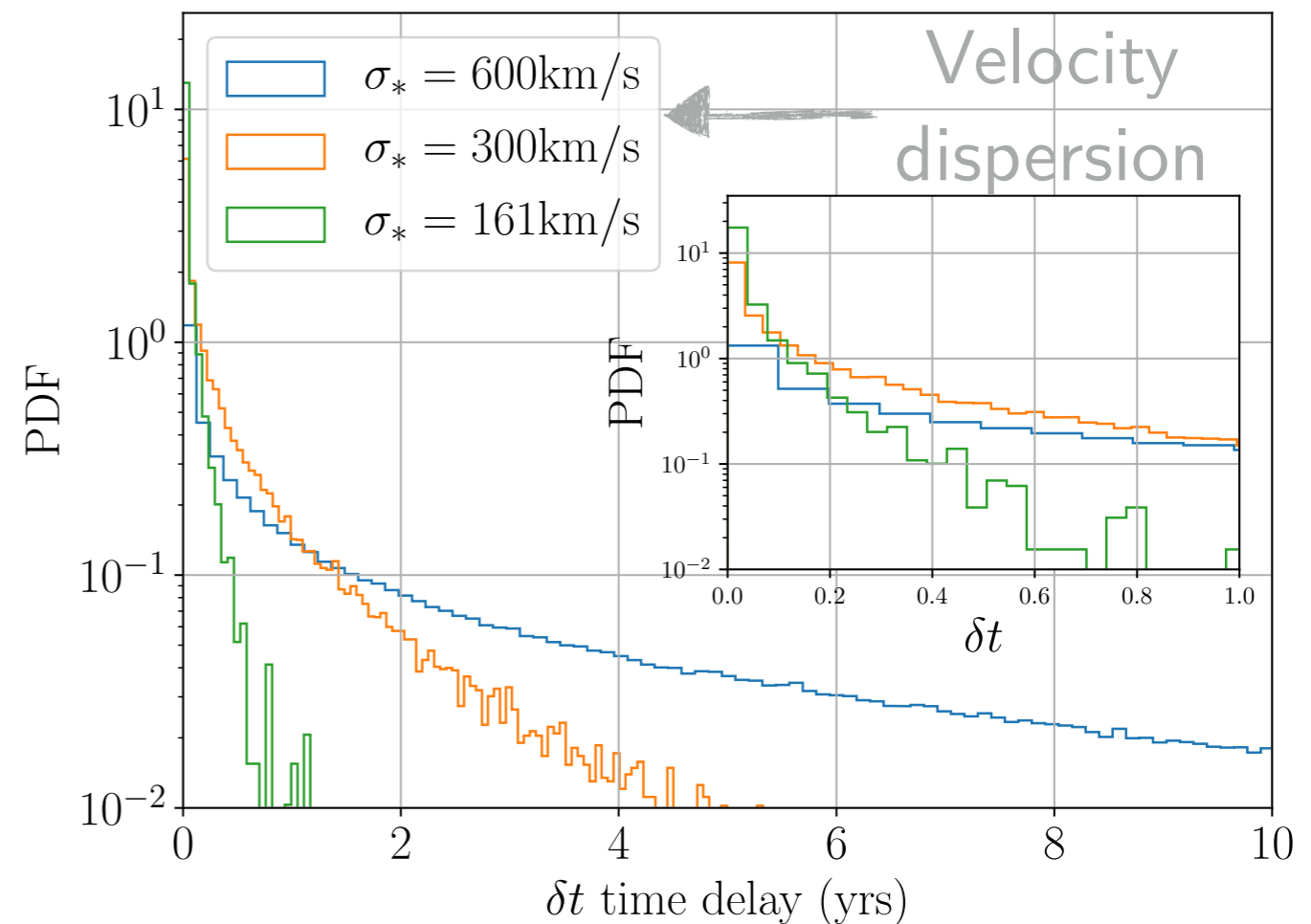


Fei Xu  
(UChicago)

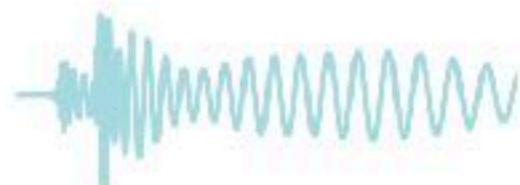
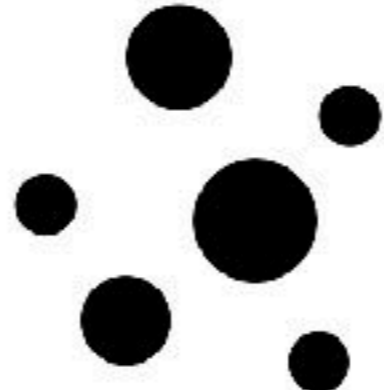
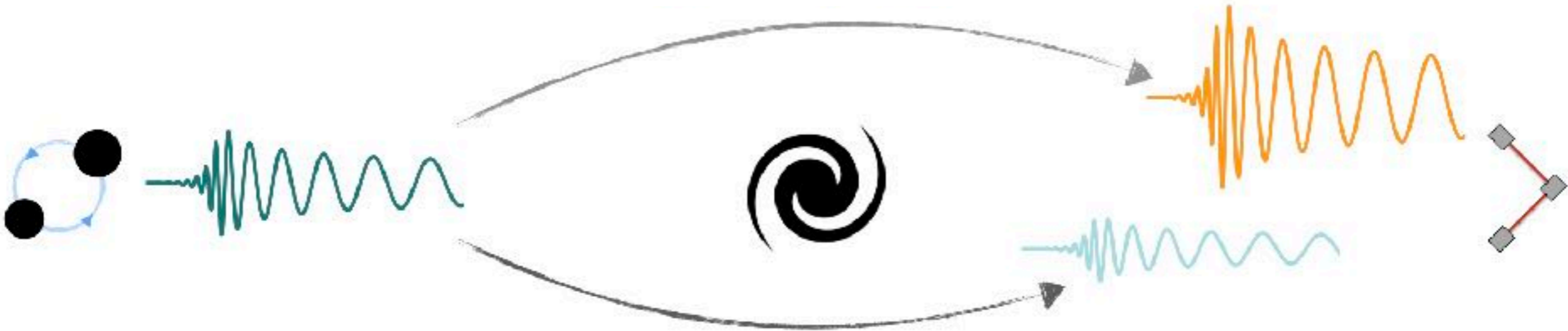
Detected populations  
aLIGO



Time delay distributions



# What lensing?



Source

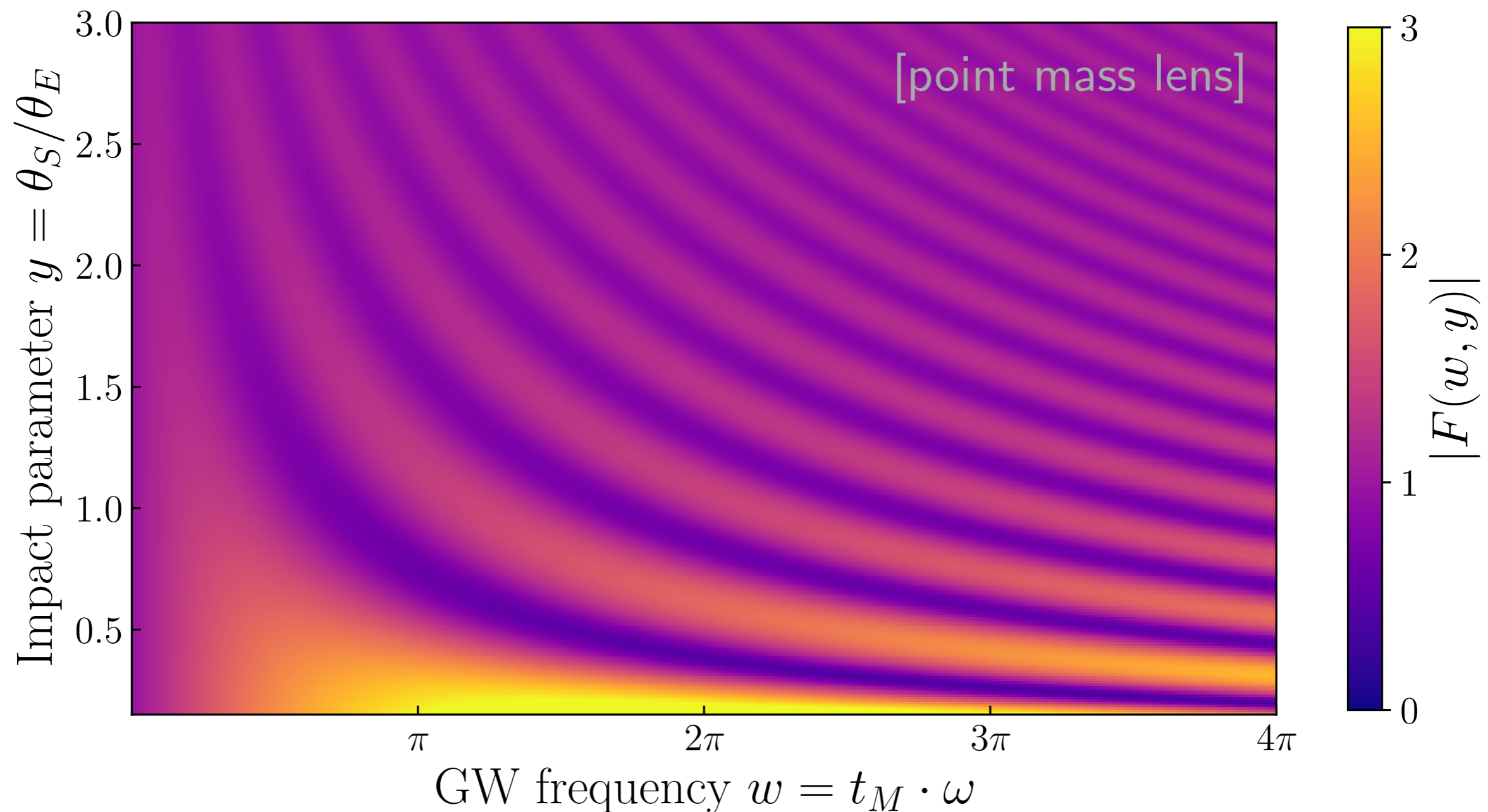
Lens

Detector



# Wave effects:

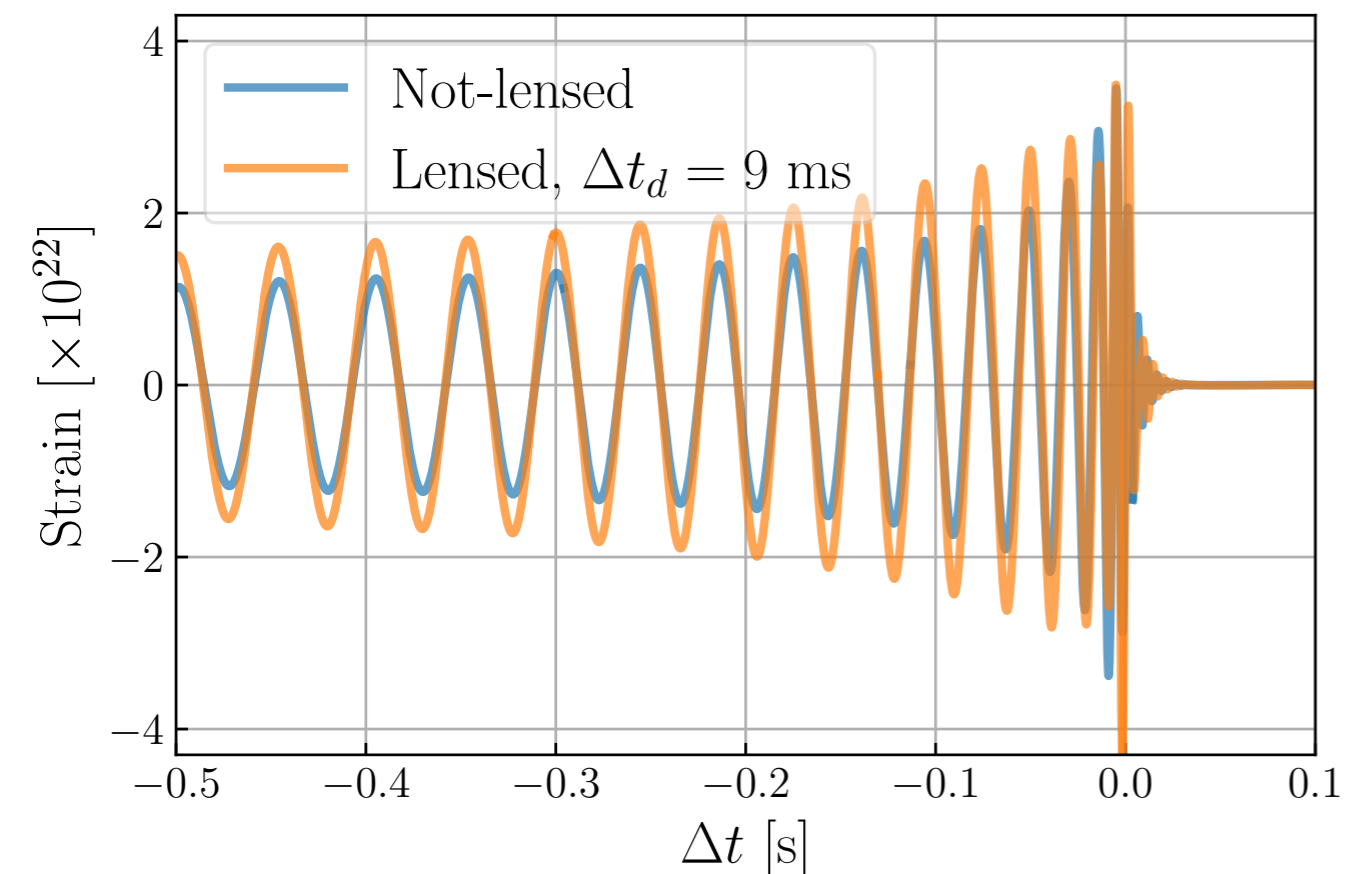
$$F(\omega, \vec{y}) = \frac{\omega}{2\pi i} \int d^2x \exp [i\omega T(\vec{x}, \vec{y})]$$



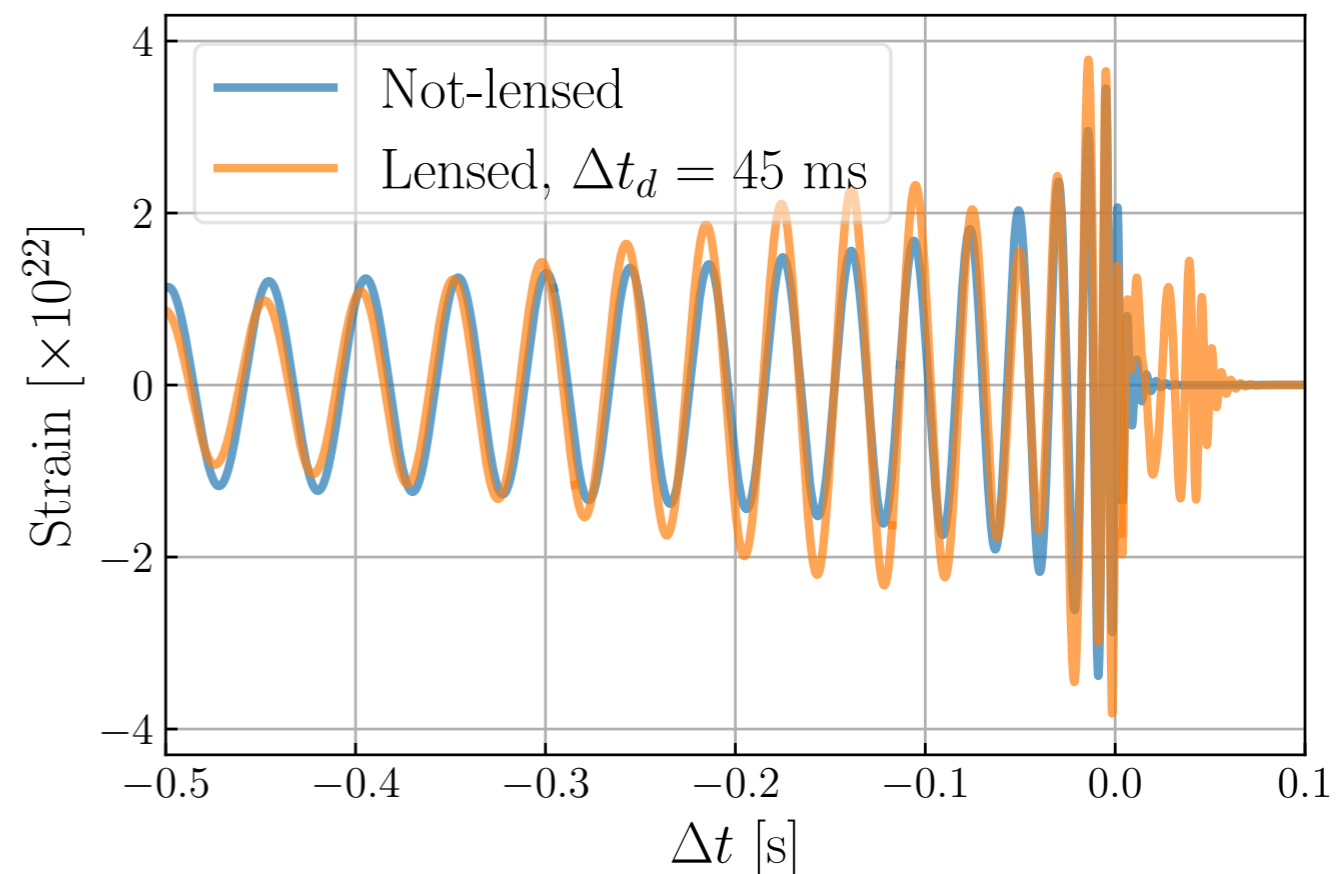
# Wave effects:

$$\Delta t_d(y = 1) \simeq 4 \left( \frac{(1 + z_L) M_L}{100 M_\odot} \right) \text{ ms}$$

## Diffraction

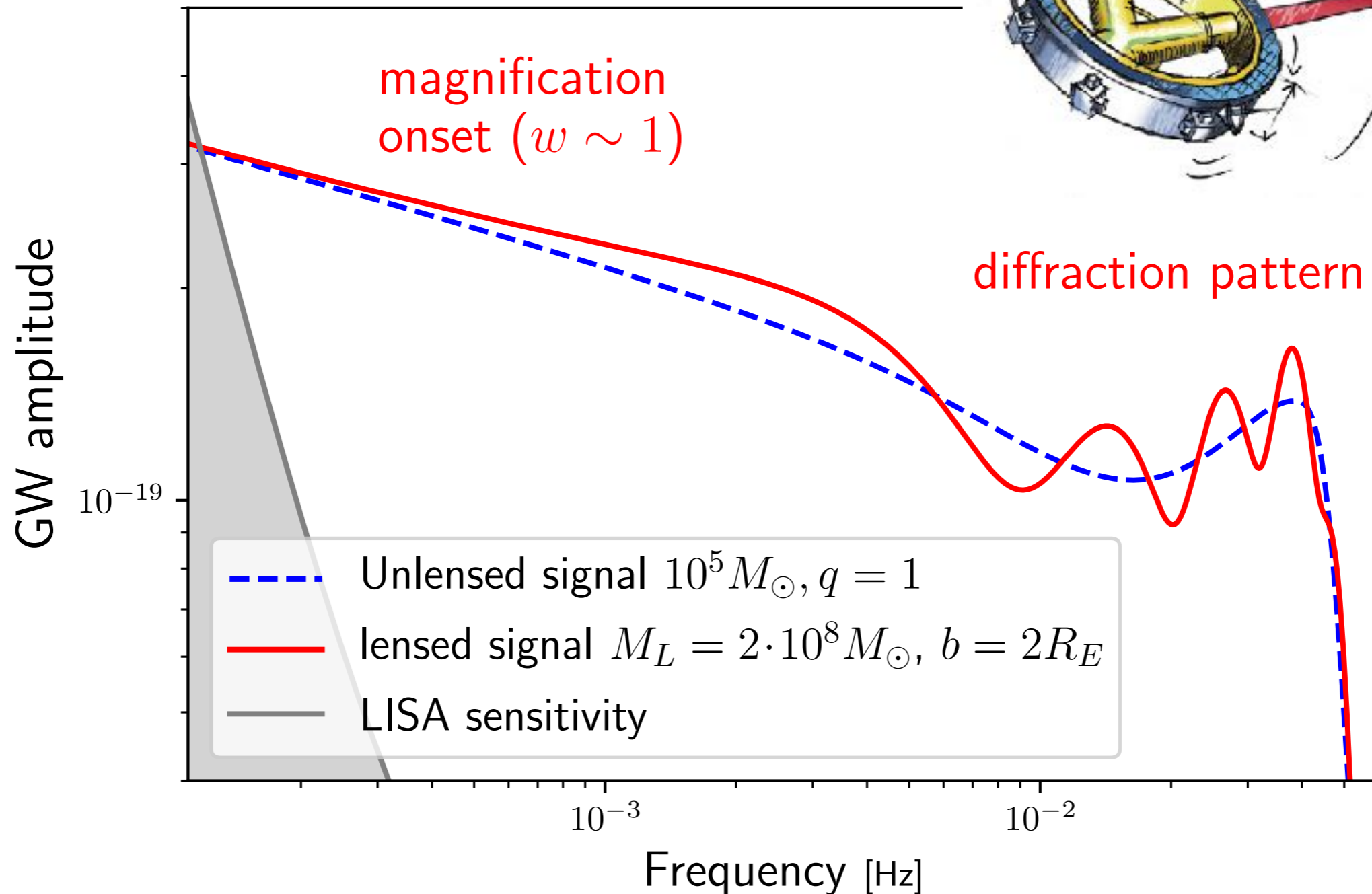
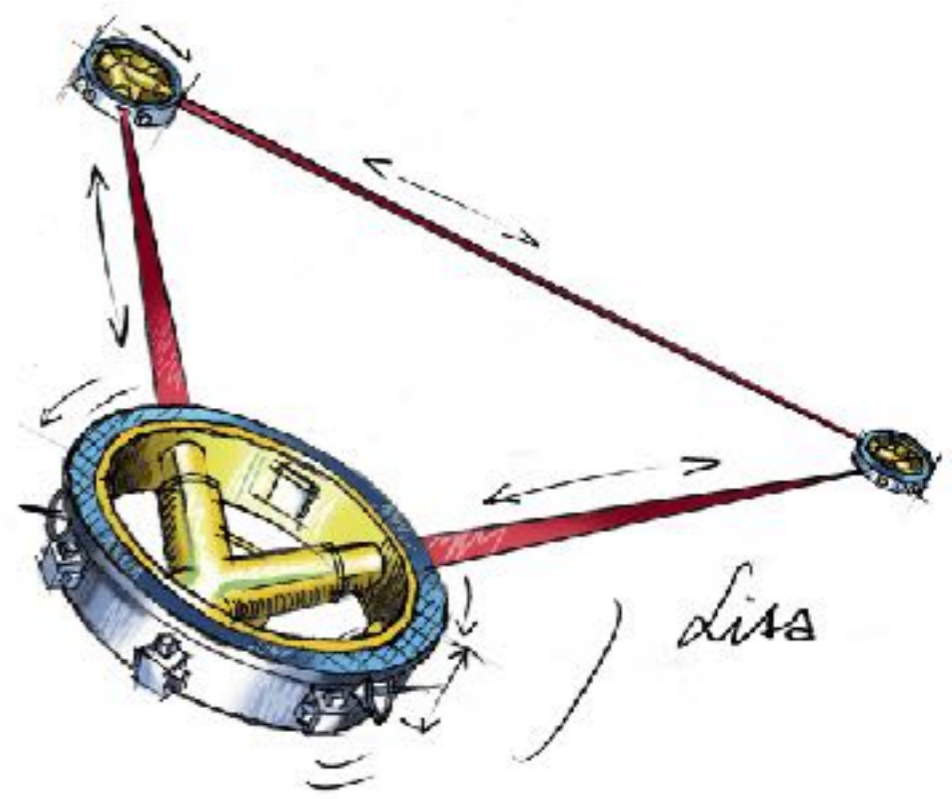


## Interference

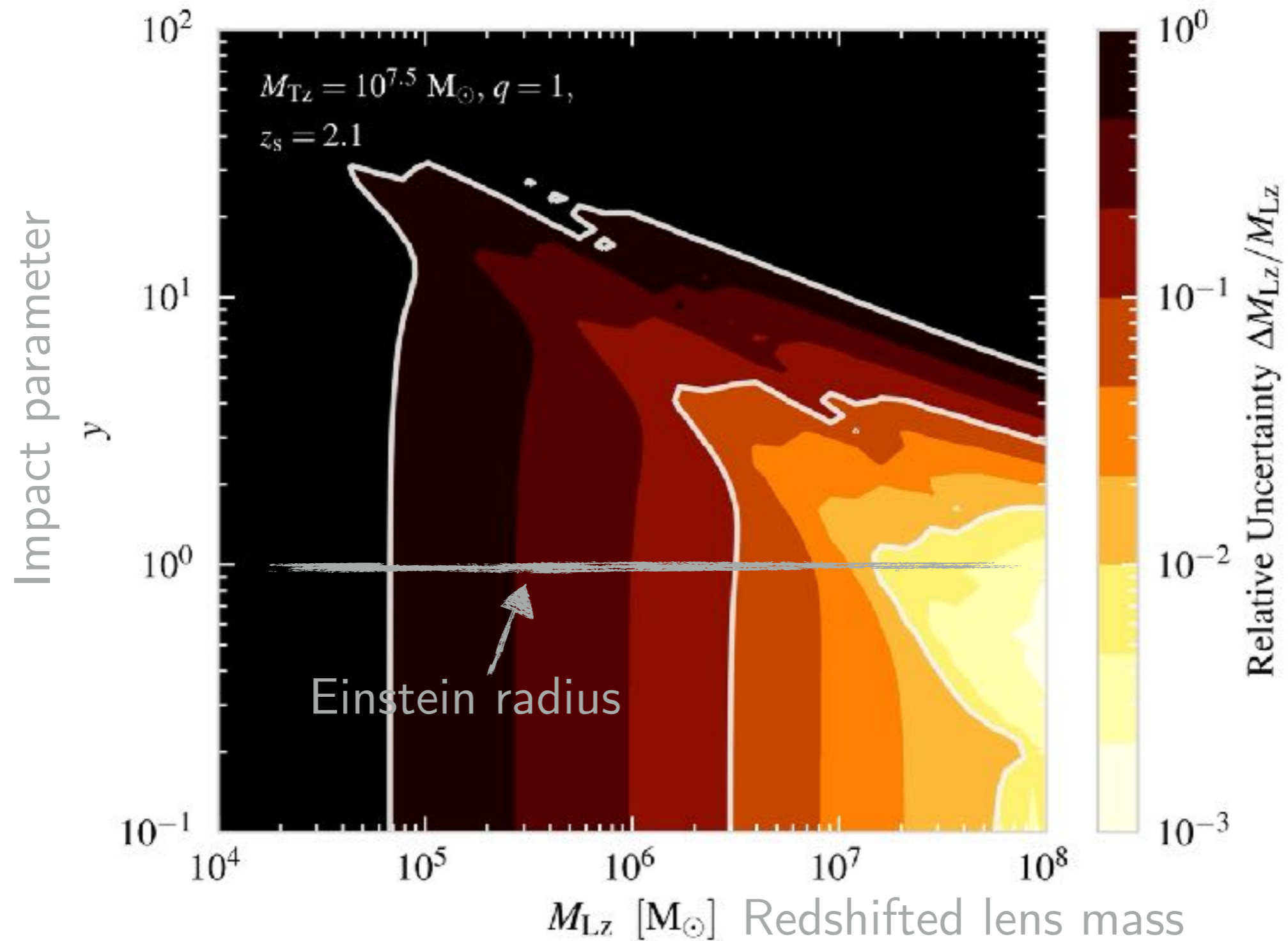


Probing compact objects ([Dai et al.'18](#), [Diego'19](#), [Tambalo et al.'22](#), ...), strong lensing + microlensing ([Seo et al.'21](#), [Mena et al.'22](#), ...), breaking mass-sheet degeneracy ([Cremonese, Ezquiaga, Salzano'21](#)), solving diffraction integral ([Feldbrugge&Turok'20](#), [Tambalo et al.'22](#))

# Wave effects: LISA



# Increased optical depth



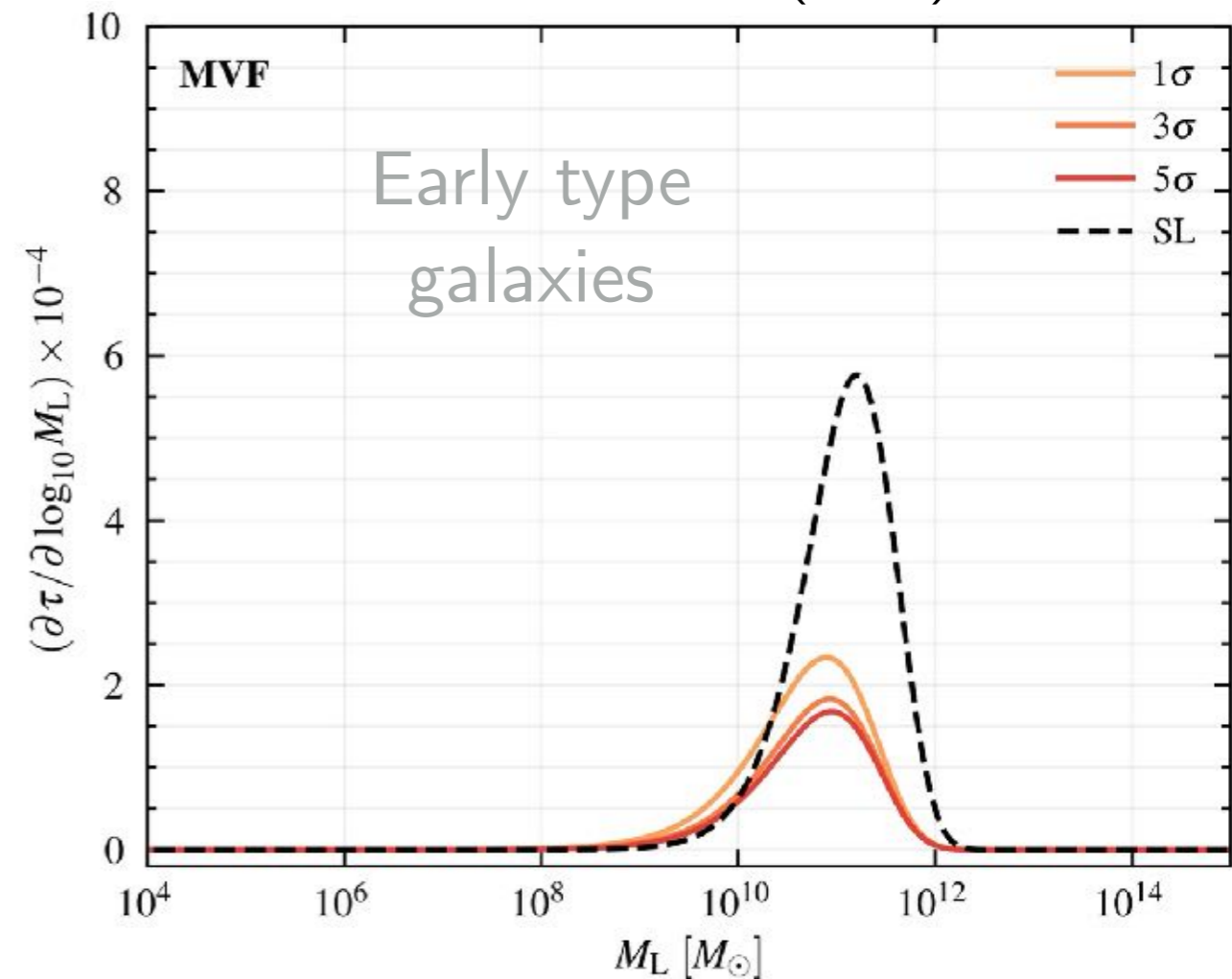
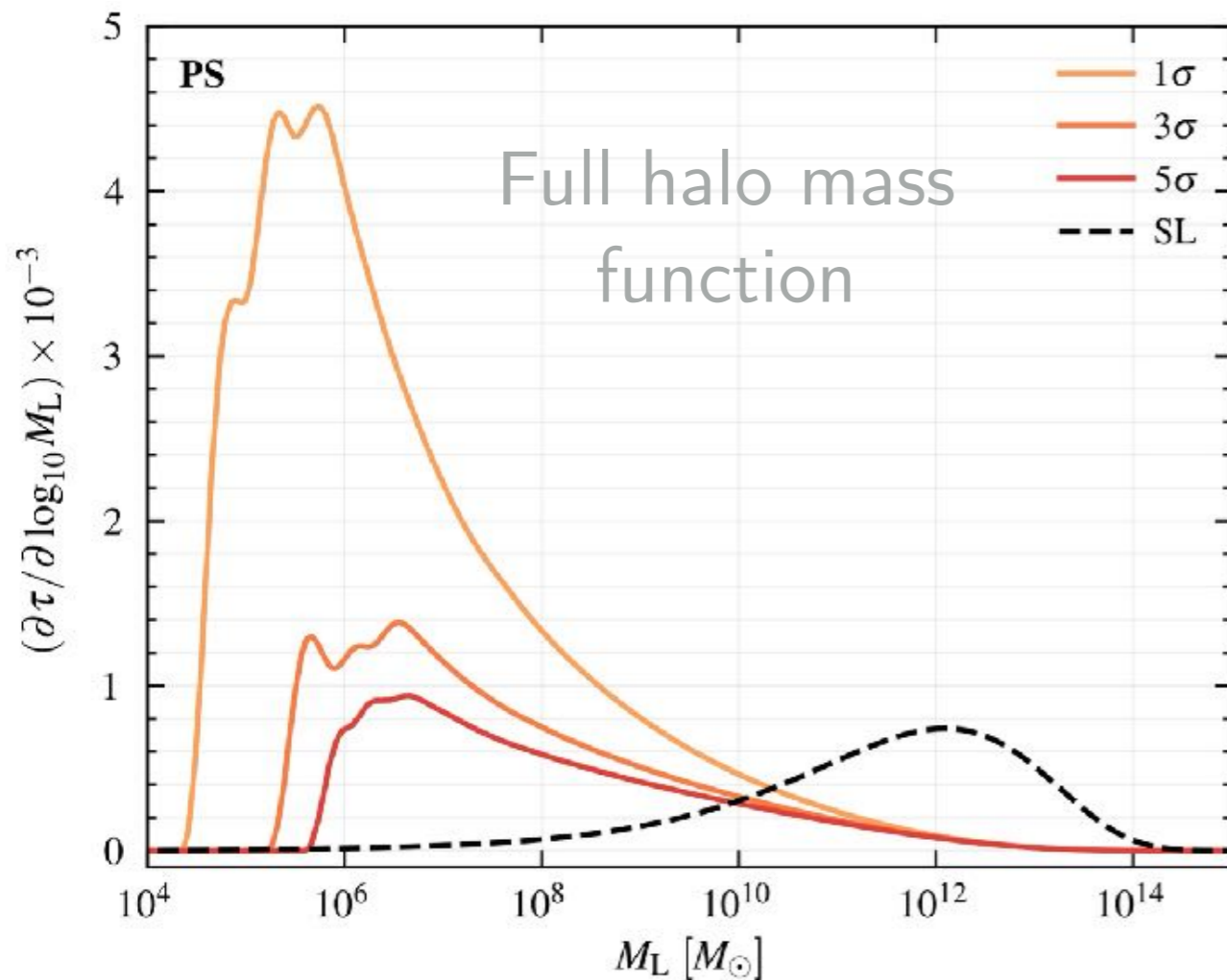


# Probing dark matter sub-halos



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- Probability of wave optics lensing:  
few percent and larger than strong lensing

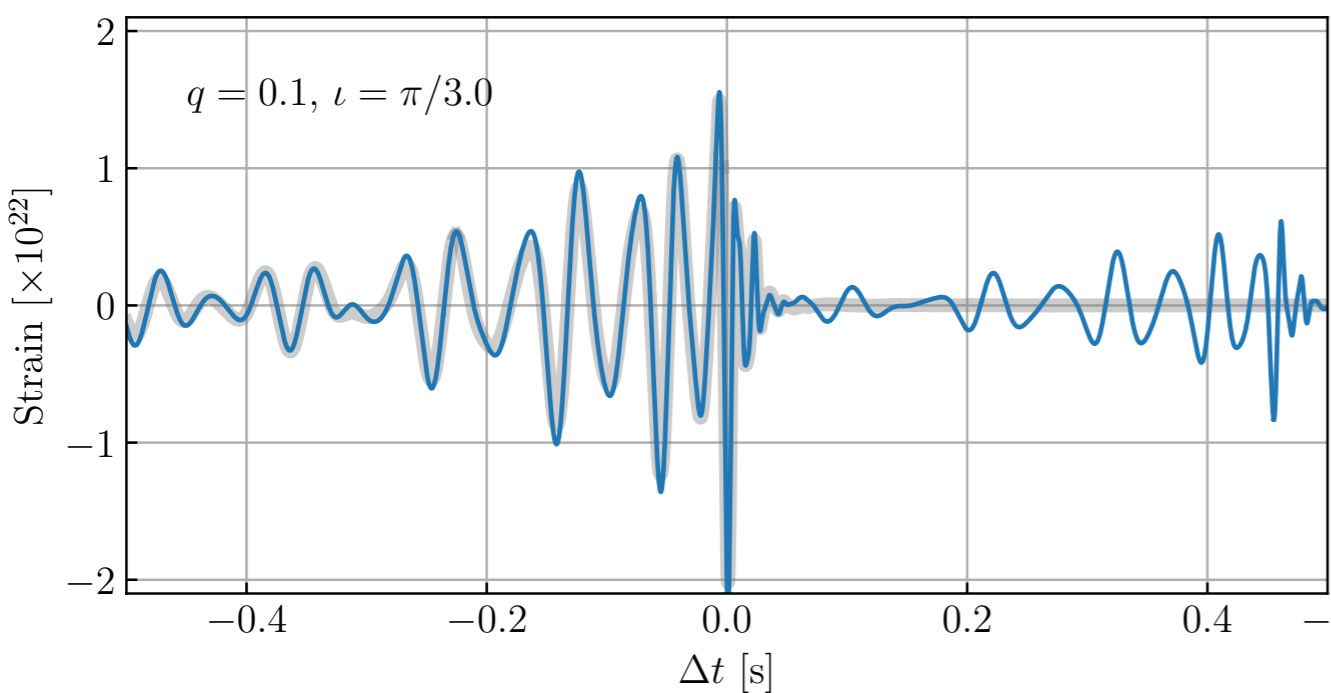
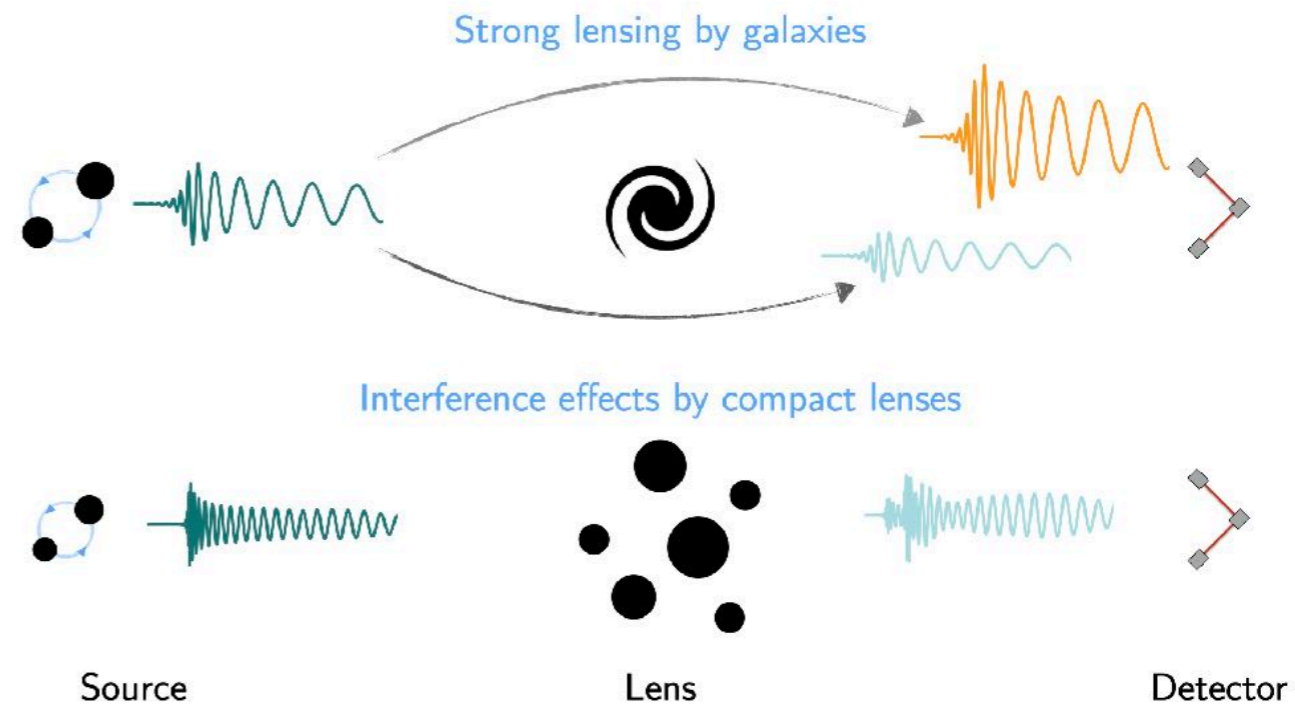
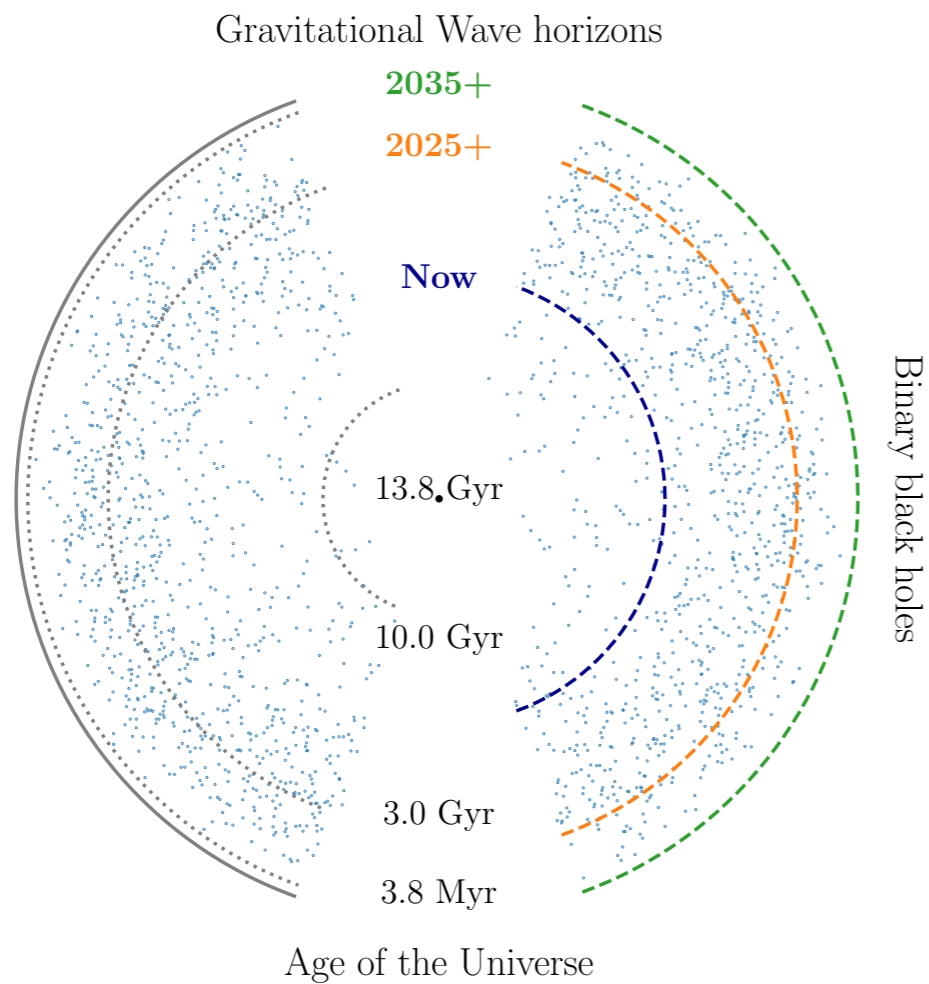




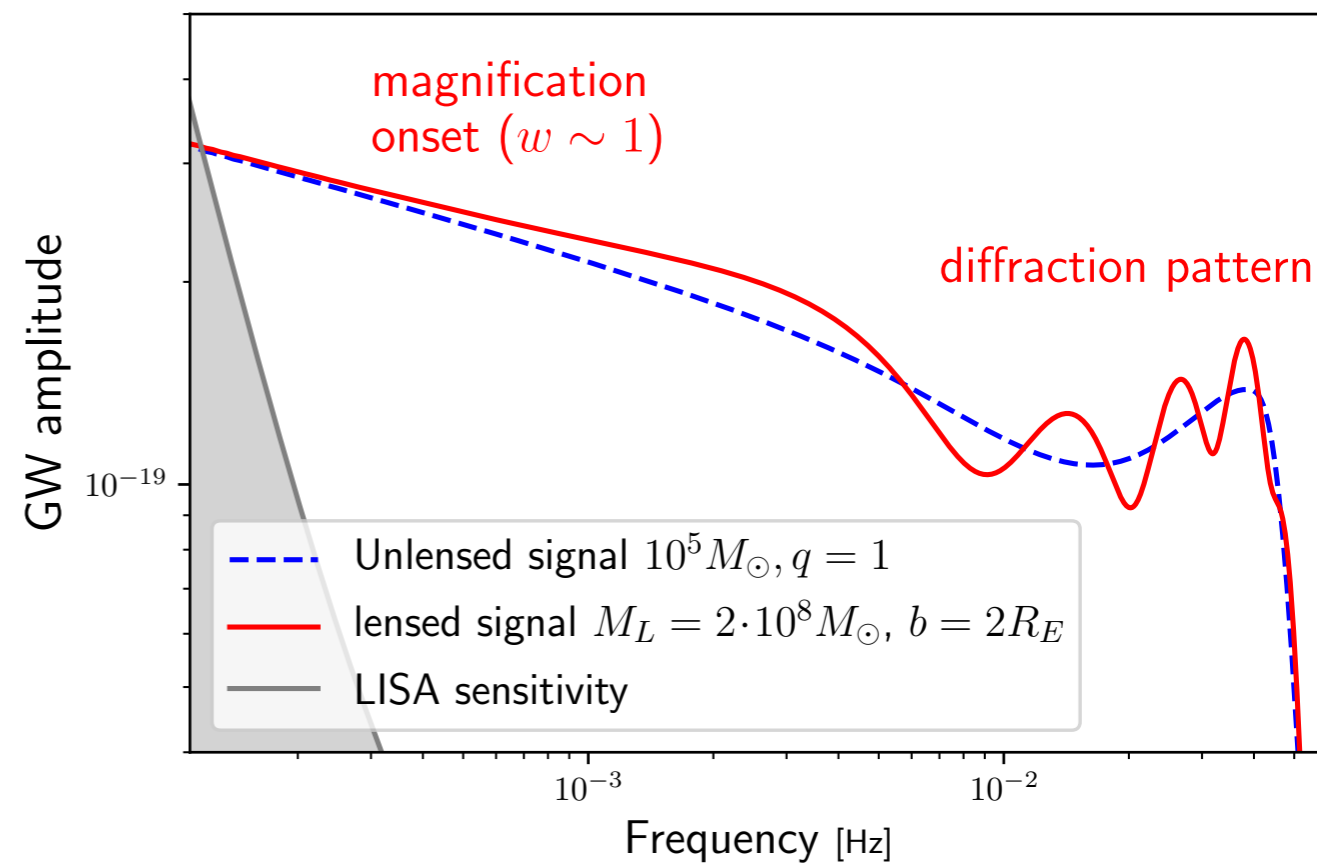
# Synergies

- **Cross correlate with strong lensing catalogs** (I'd love to know where to find them and how to use them!)
- **Targeted follow-ups** for promising candidates
- **Joint lensing inference** to improve cosmology/gravity tests
- **Constraints on optical depth** combining different probes





**Multiple images, type II distortion, expected soon!**



**Wave effects, PBH, sub-halos**



Medfinansieret af Den Europæiske  
Unions Connecting Europe-facilitet

# Join us!

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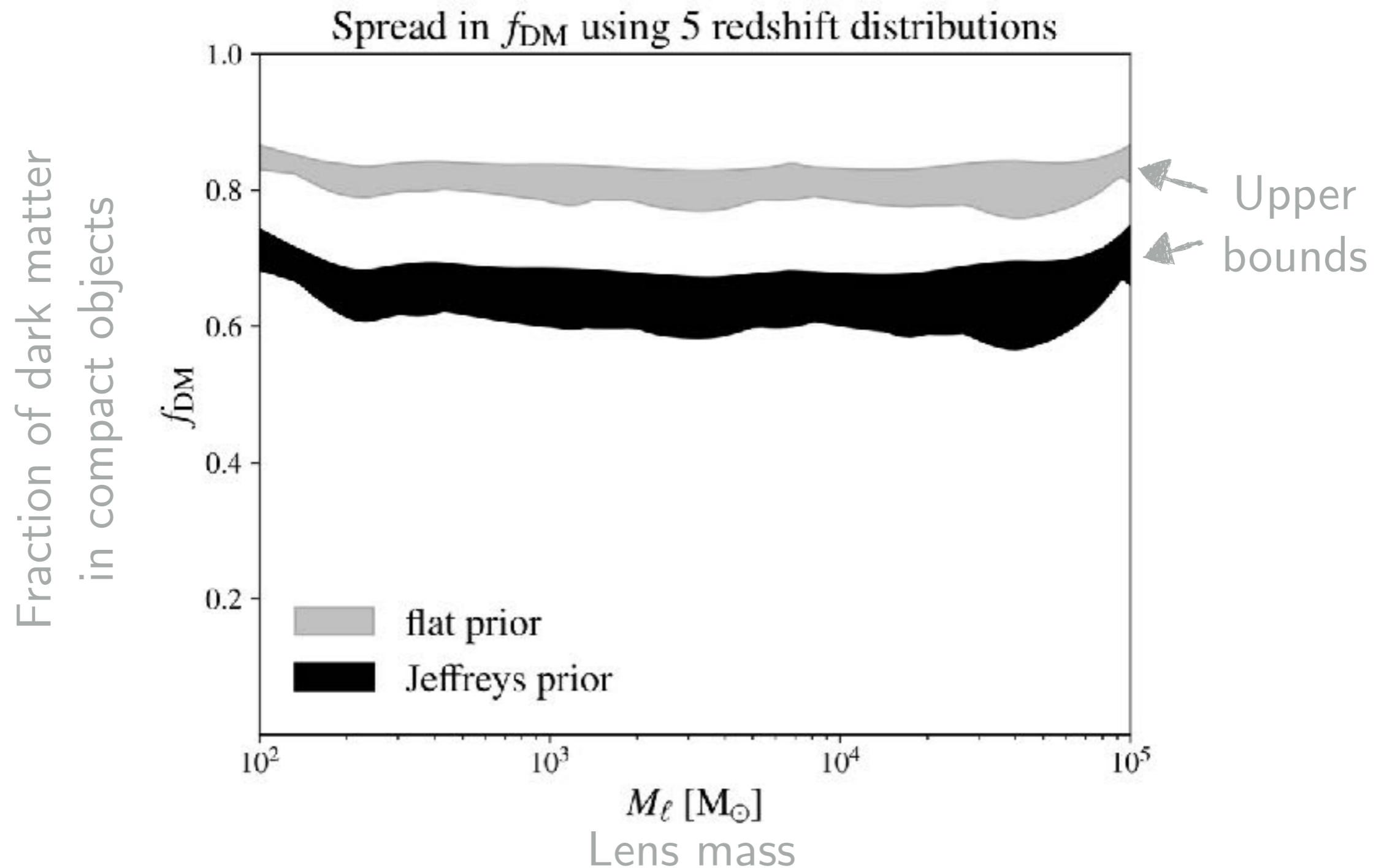


[ezquiaga.github.io/joinus](https://ezquiaga.github.io/joinus)





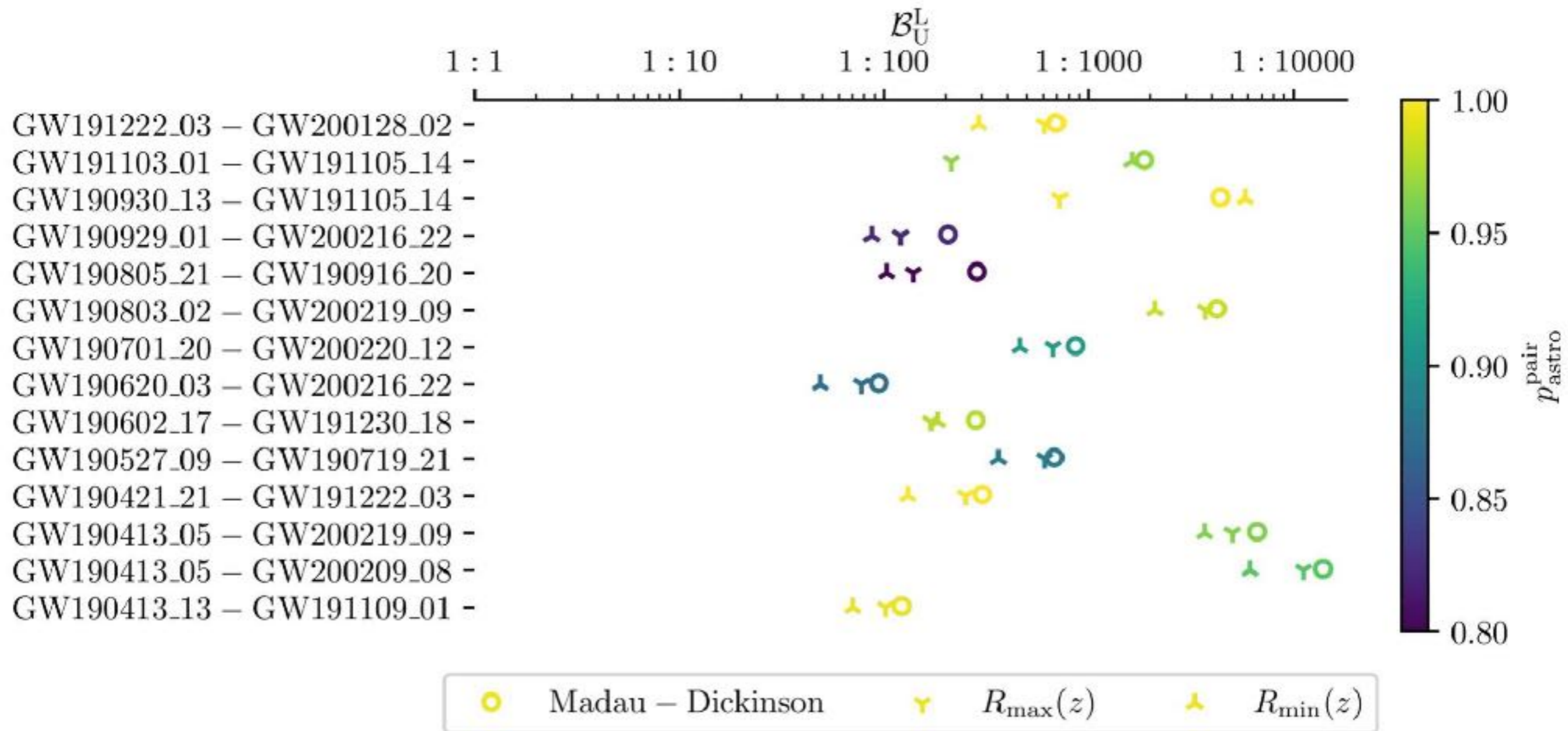
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LVK (incl. **Ezquiaga**); *Search GW lensing full O3* ([arXiv 2304.08393](#), [science summary](#))

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LVK (incl. **Ezquiaga**); *Search GW lensing full O3* ([arXiv 2304.08393](https://arxiv.org/abs/2304.08393), [science summary](#))