[Gustav Klimt]

Gravitational Wave Lensing: Current Searches and Future Prospects

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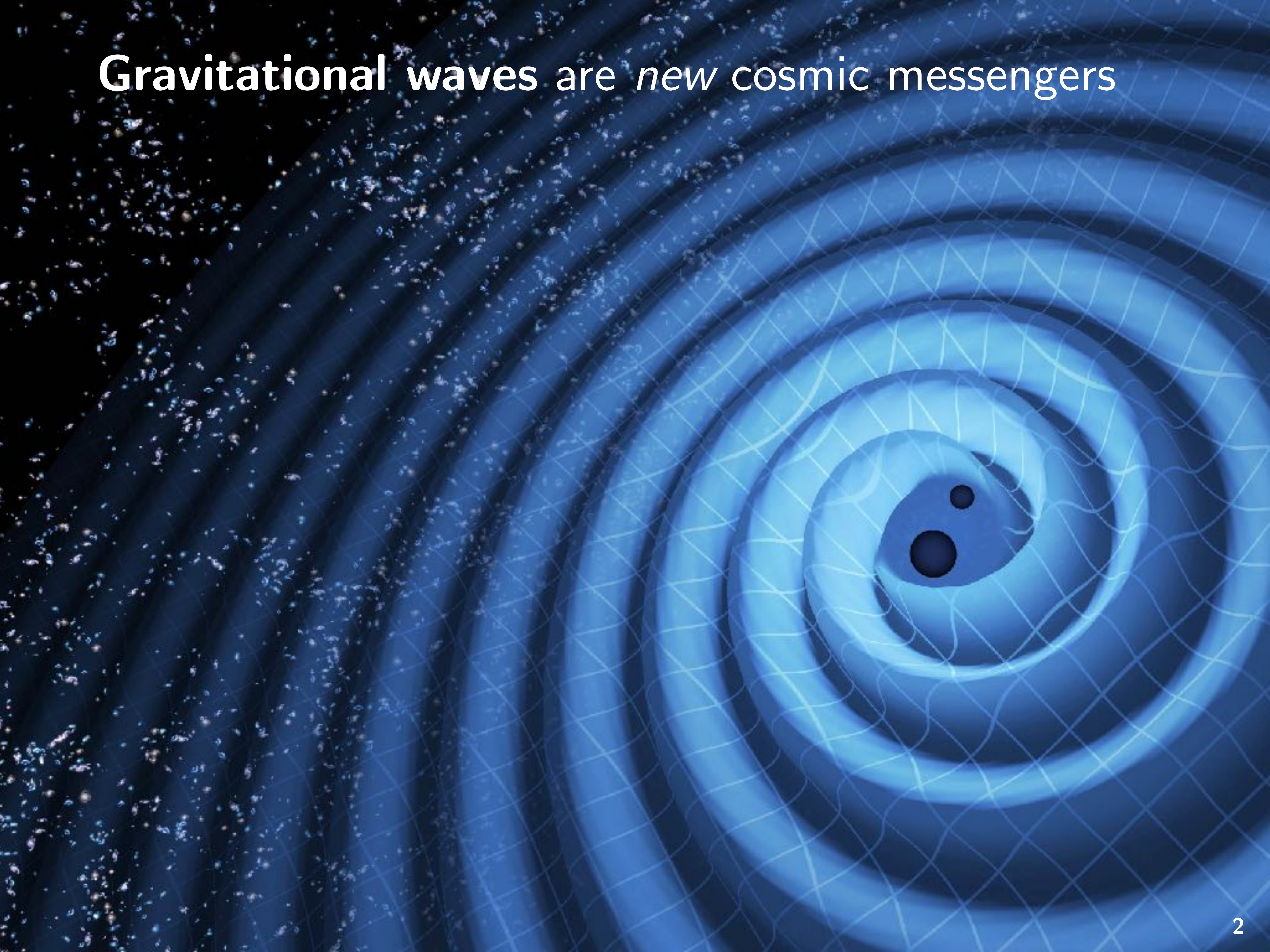
VILLUM FONDEN

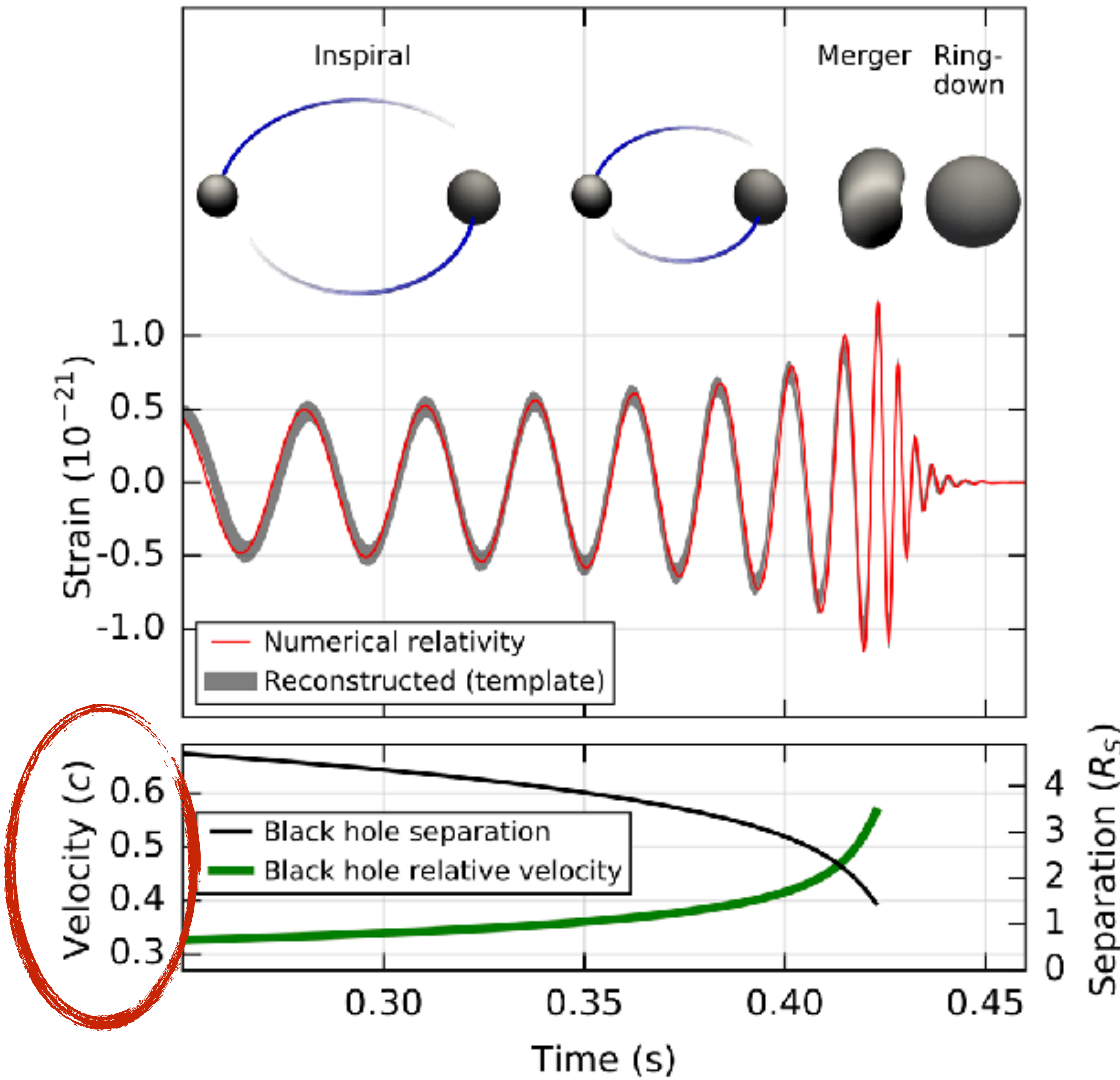
KØBENHAVNS UNIVERSITET

Gravitational waves are *new* cosmic messengers

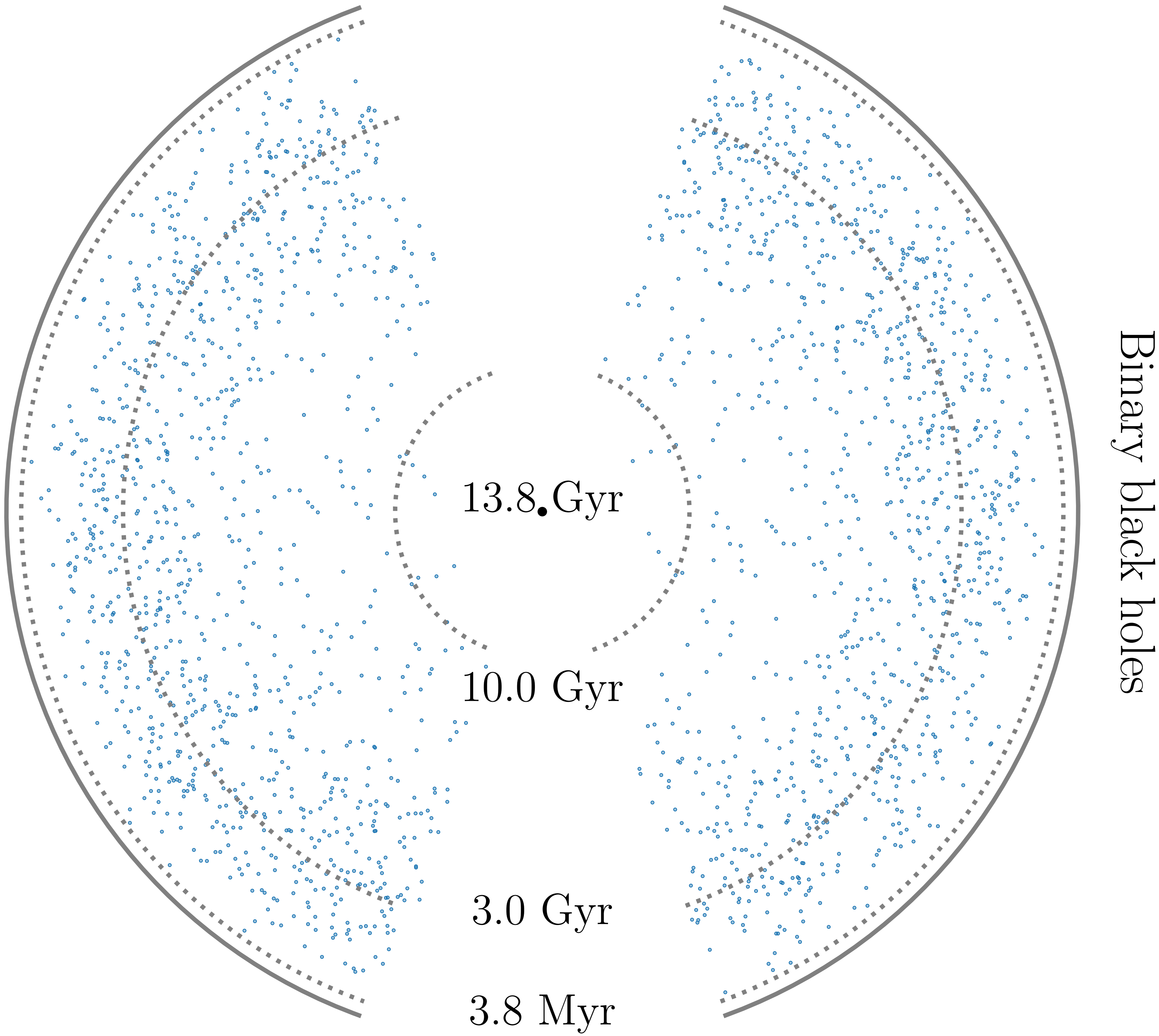


Gravitational waves from stellar-mass **binary black holes**

Strong-field gravity

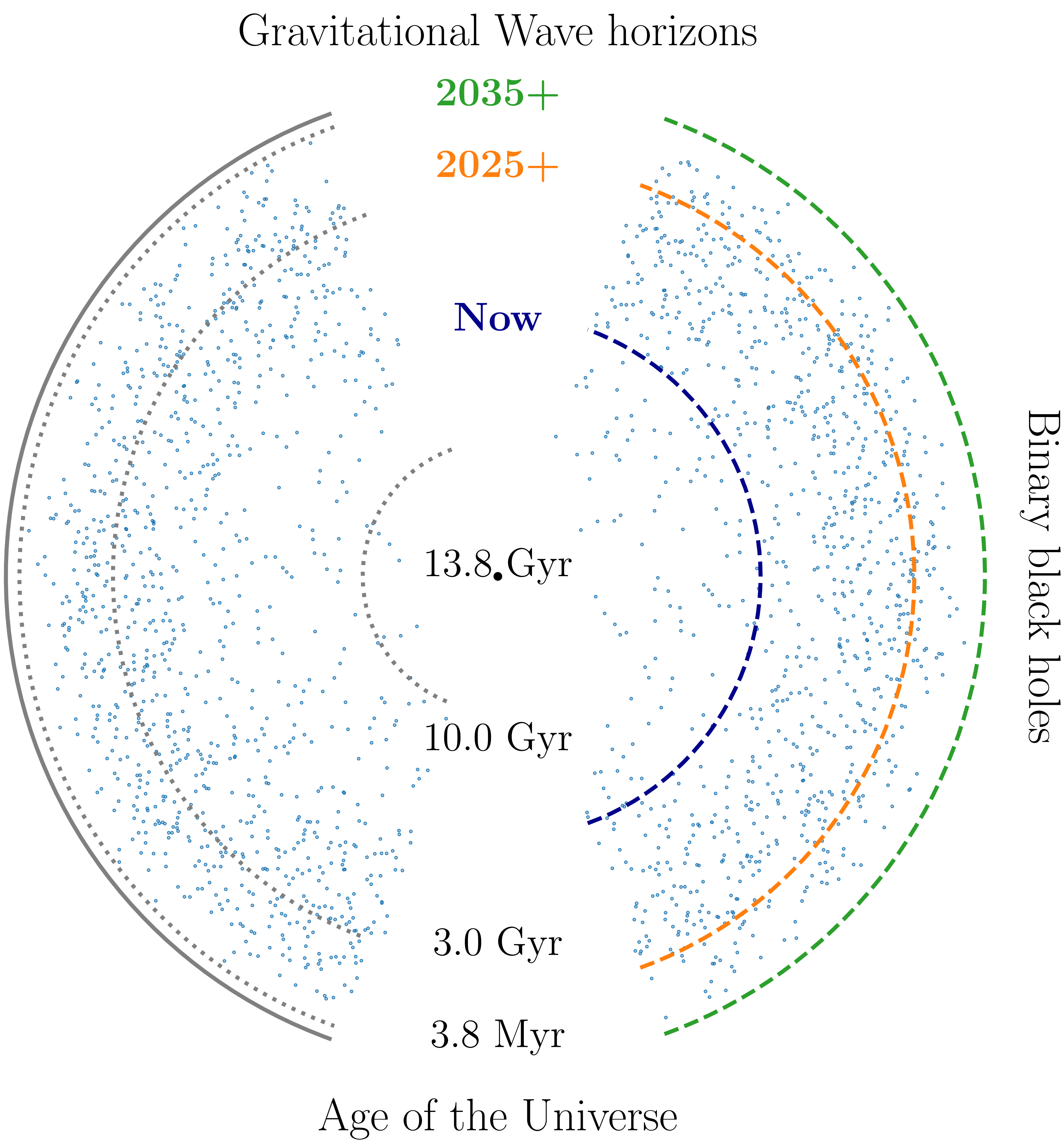


[First detection, GW150914]

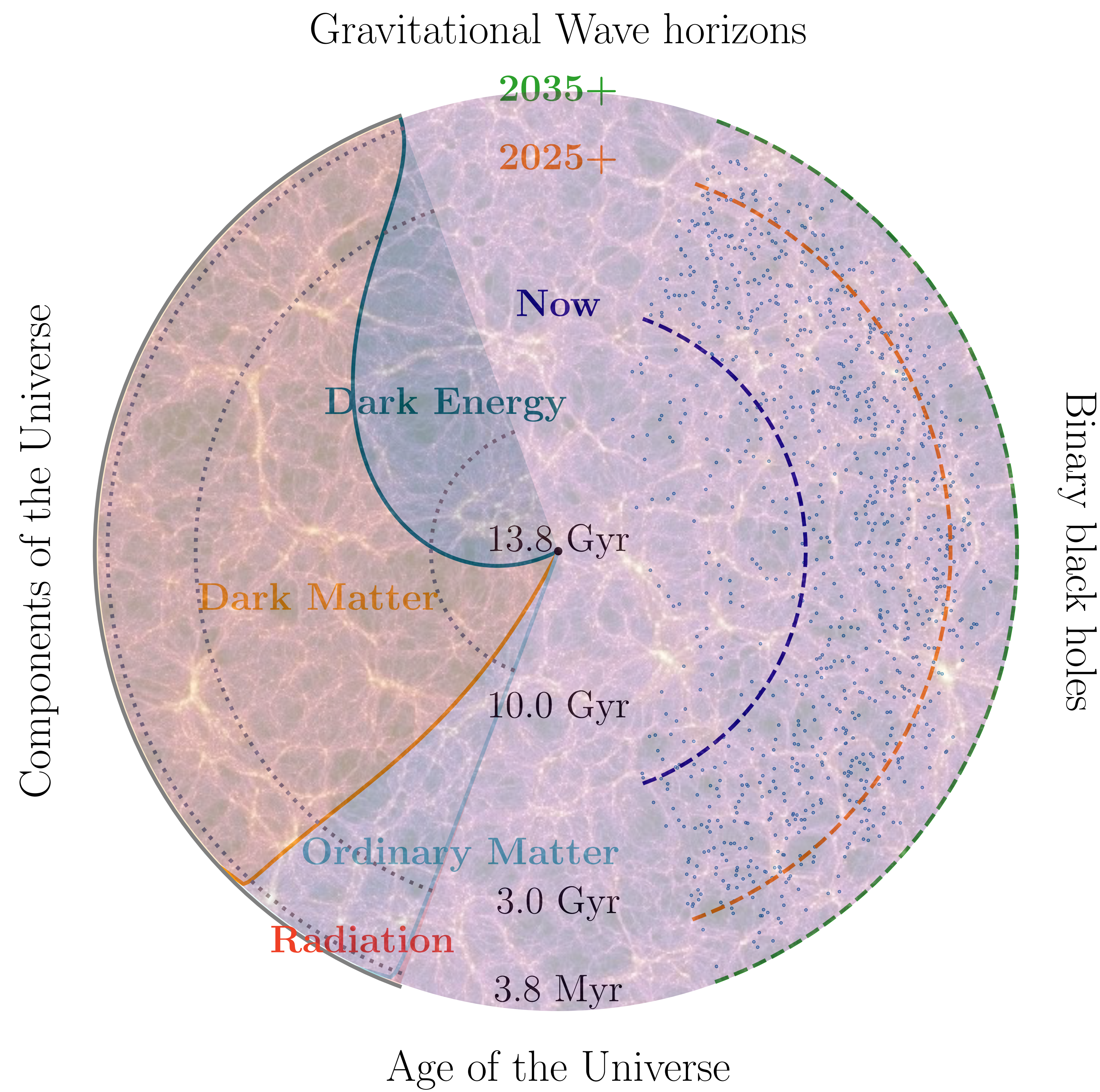


Age of the Universe

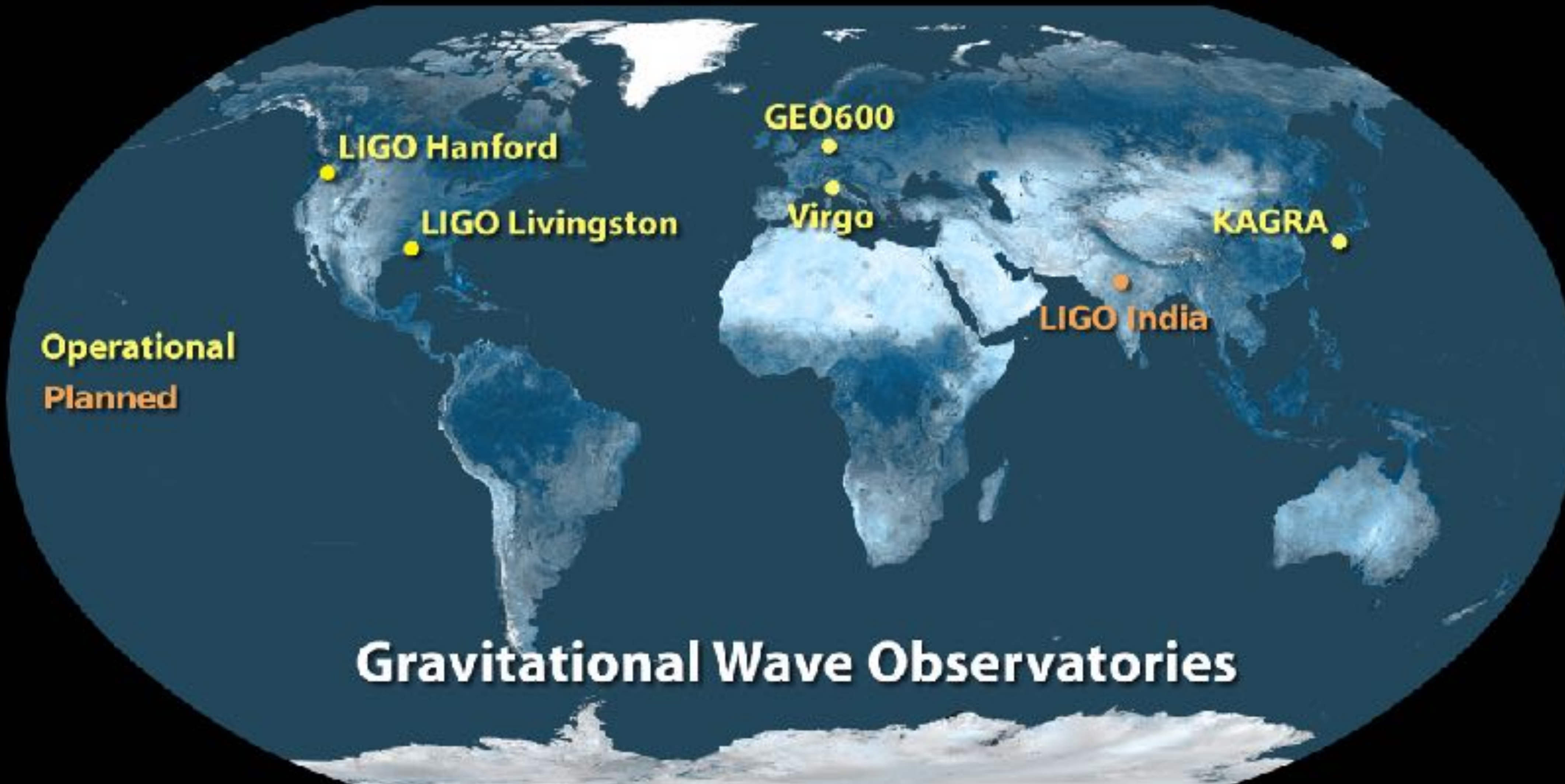
*stellar mass binary black holes



Gravitational Wave horizons
2035+
2025+
Now
13.8 Gyr
10.0 Gyr
3.0 Gyr
3.8 Myr
Age of the Universe
Binary black holes


Gravitational Wave horizons
2035+
2025+
Now
Dark Energy
13.8 Gyr
Dark Matter
10.0 Gyr
Ordinary Matter
3.0 Gyr
Radiation
3.8 Myr
Components of the Universe
Binary black holes
Age of the Universe

The era of gravitational wave astronomy is **here**!



[Hanford, US]

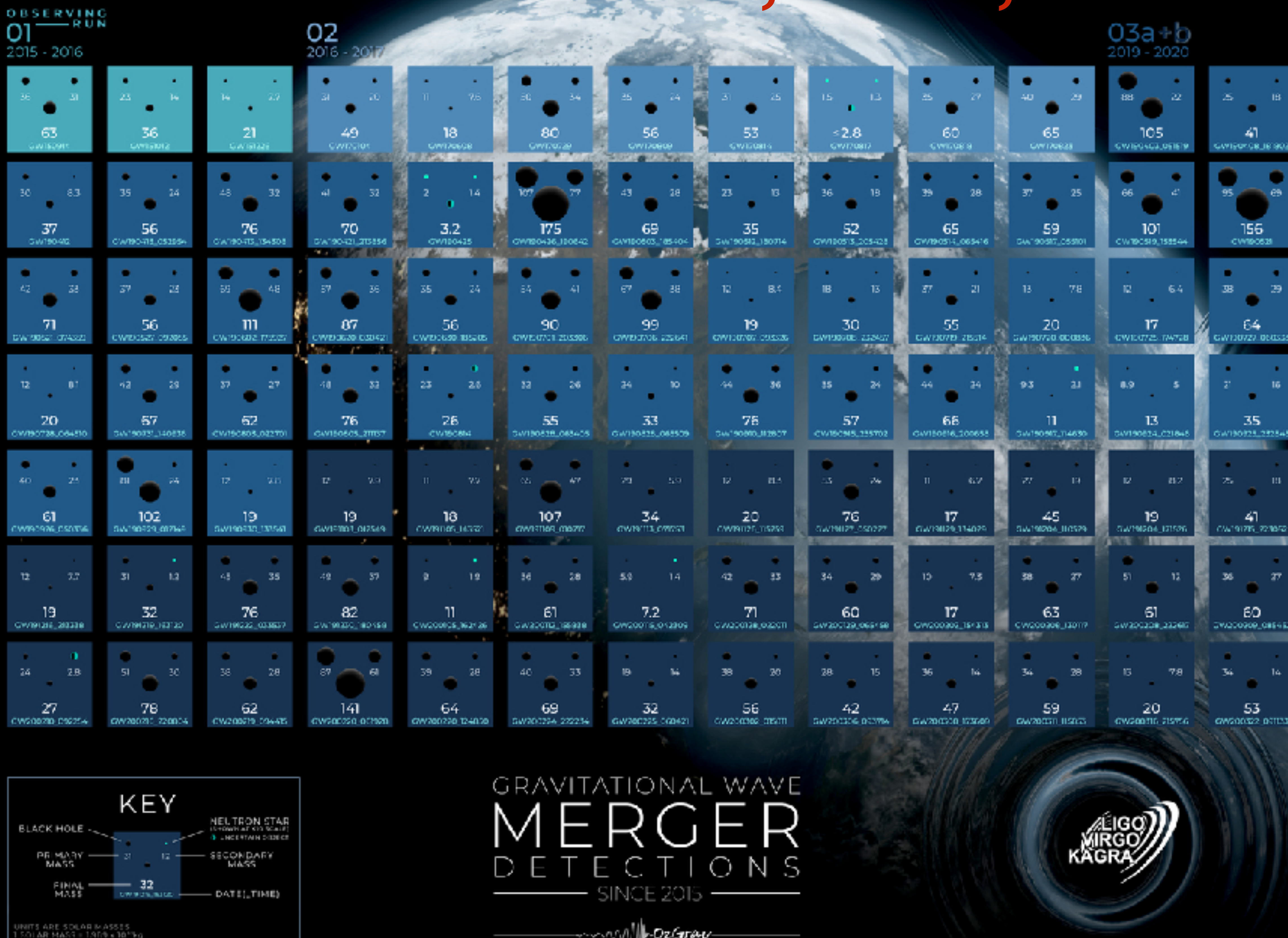
[Livingston, US]

[Virgo, Italy]

[KAGRA, Japan]

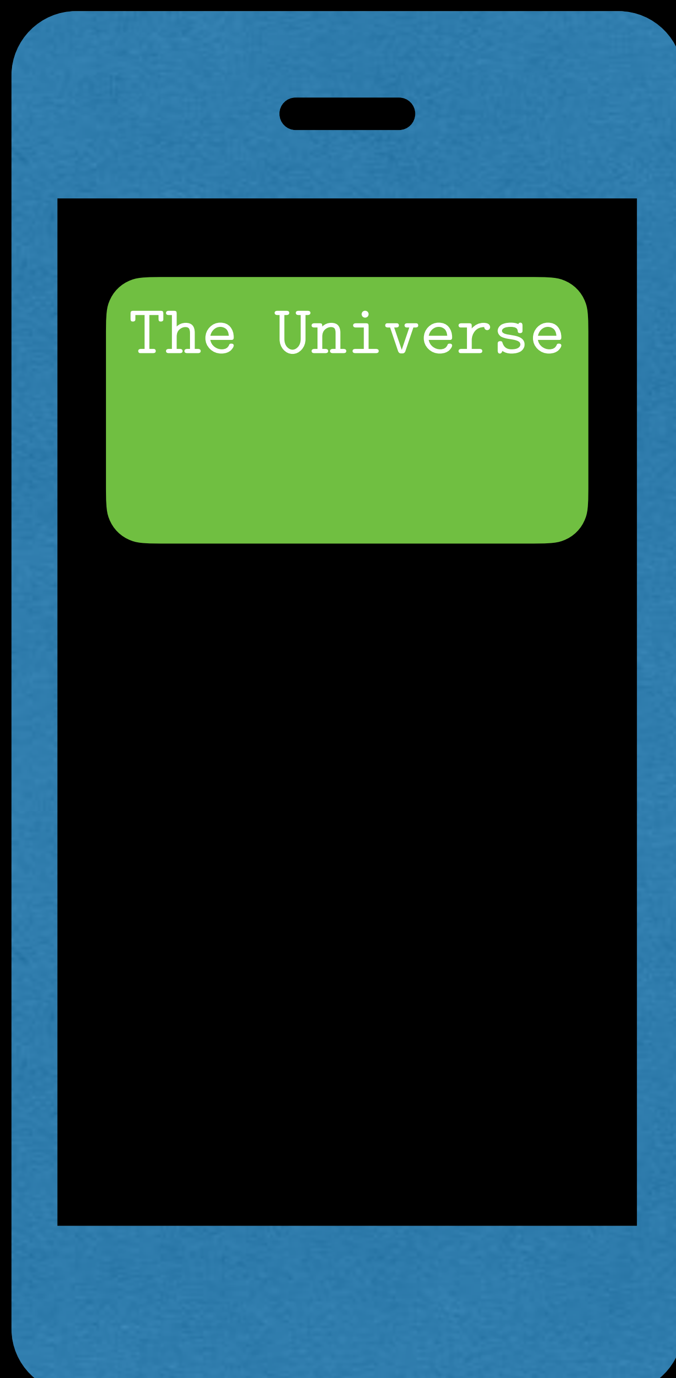
The era of gravitational wave astronomy is **here**!

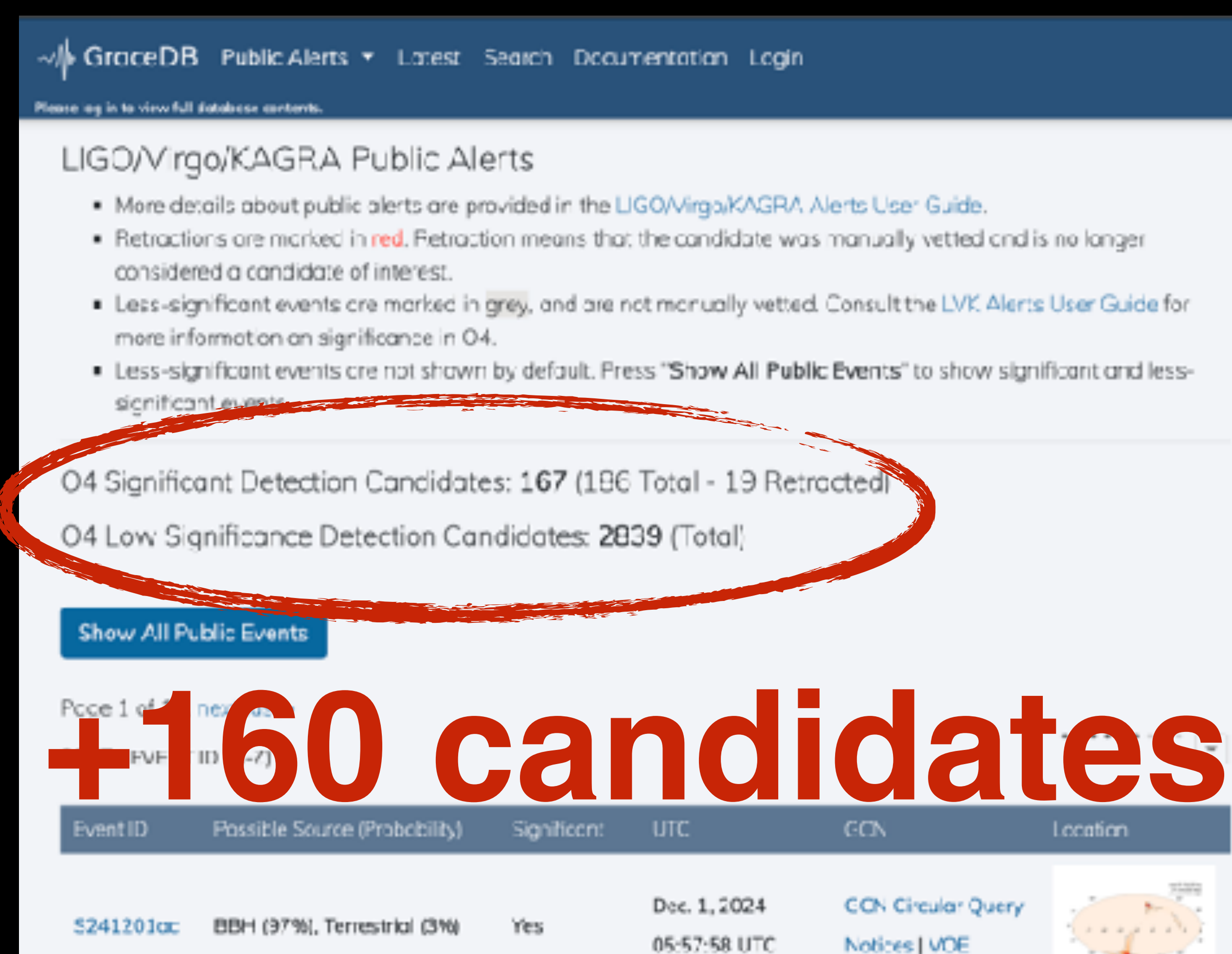
~100 events: BBH, BNS, NSBH



O4 is happening!

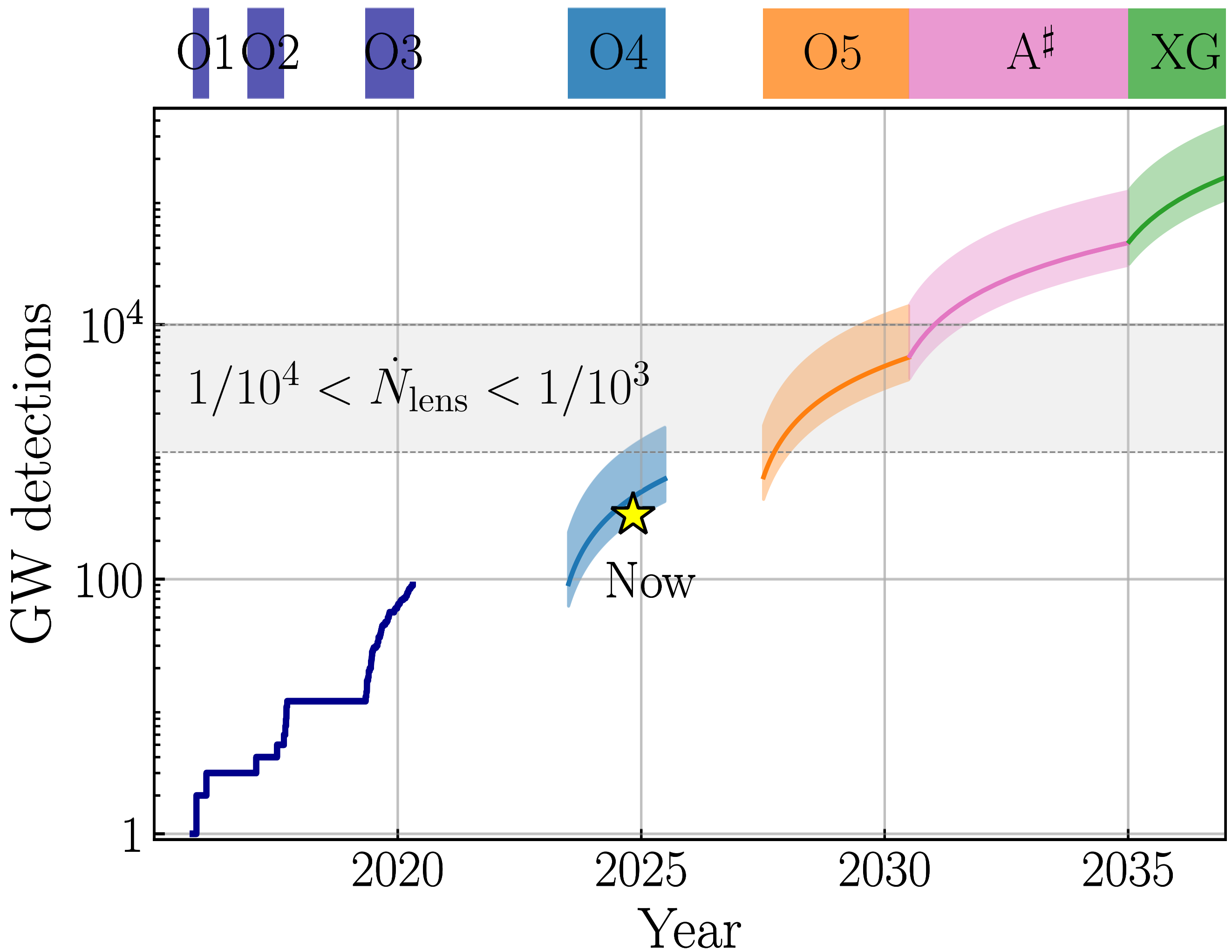
https://gracedb.ligo.org/superevents/public/O4/#

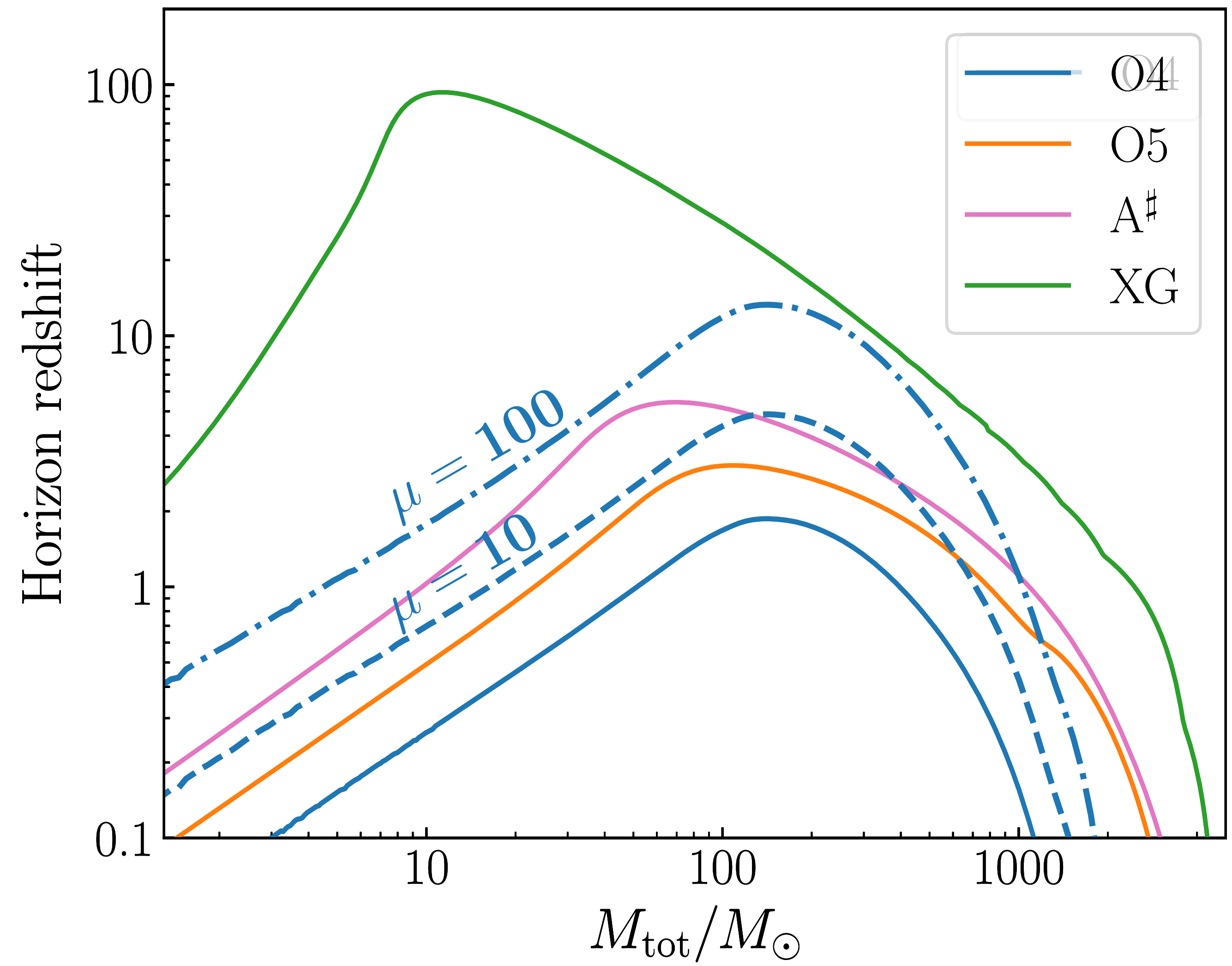




Gravitational wave lensing:

First detection *approaching*, expanding *horizons*





The plan

[this is an overview. Ask the experts in the room!]

[ezquiaga.github.io/slides/ezquiaga_vienna_24.pdf]

0. Motivation: gravity, astrophysics, cosmology

1. Gravitational waves are Standard Sirens

Waveforms from first principles, understood selection function

2. A crash-course on gravitational lensing

The diffraction integral, stationary phase approximation, repeated gravitational wave chirps

3. Current searches

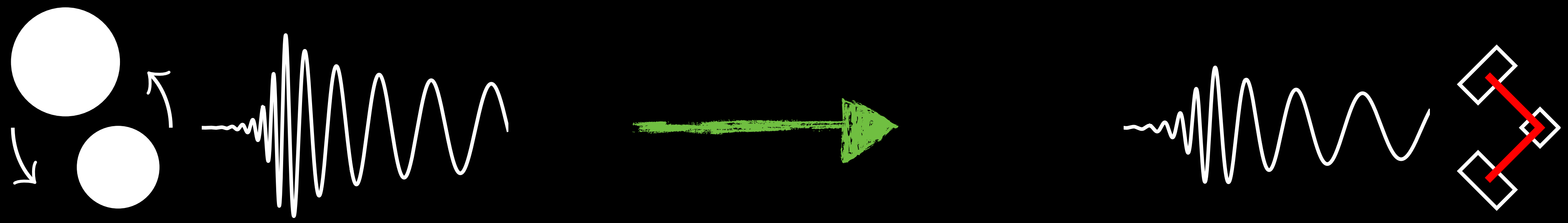
Multiple chirps, distorted waveforms, type II events, highly magnified gravitational waves

4. Future prospects

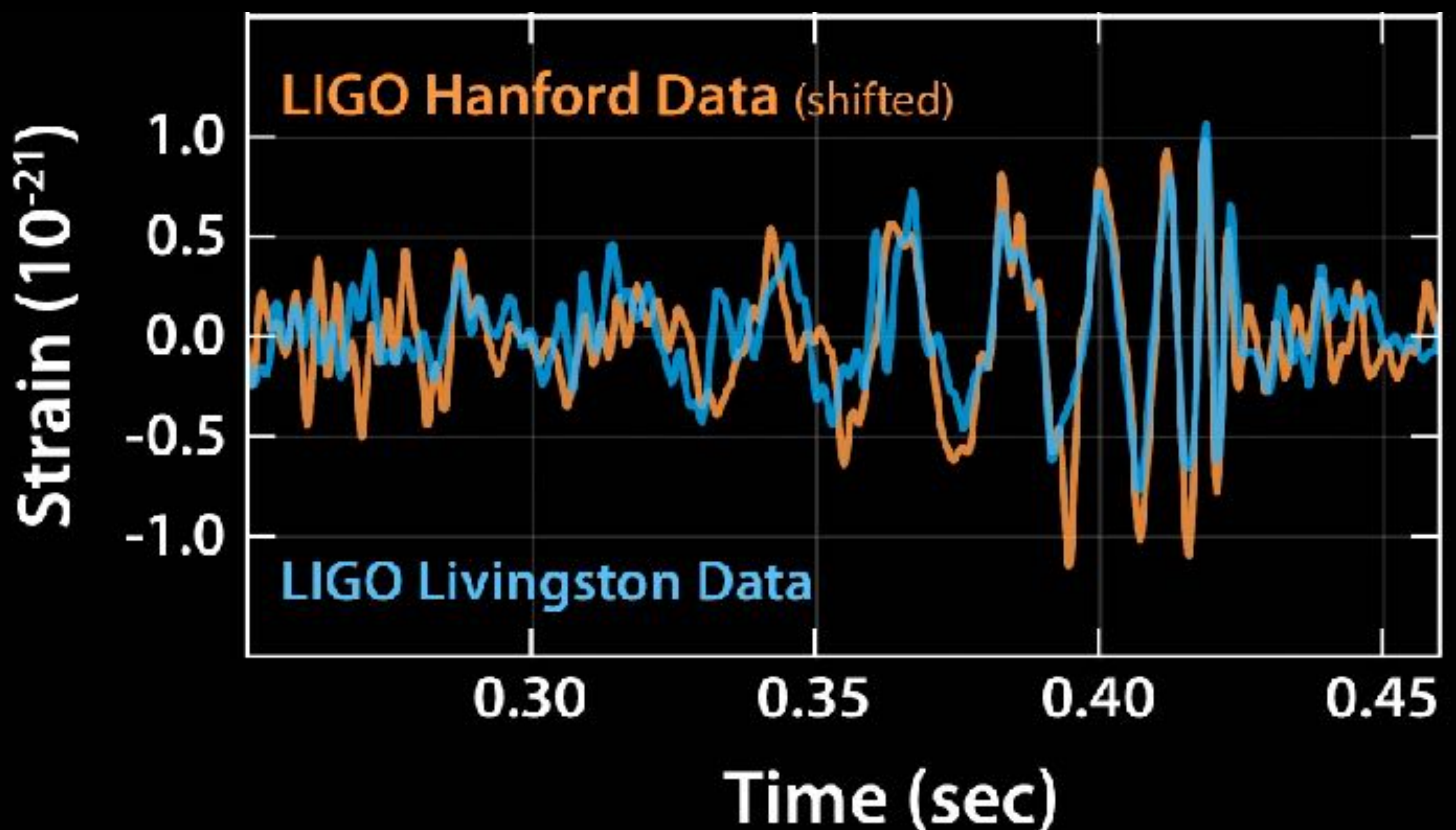
Substructure, multi-messenger & wave optics, source & lens populations, false violations of general relativity

1. Gravitational waves are standard sirens

Gravitational waves are standard sirens



[general relativity predicts waveform]



Gravitational waves are standard sirens

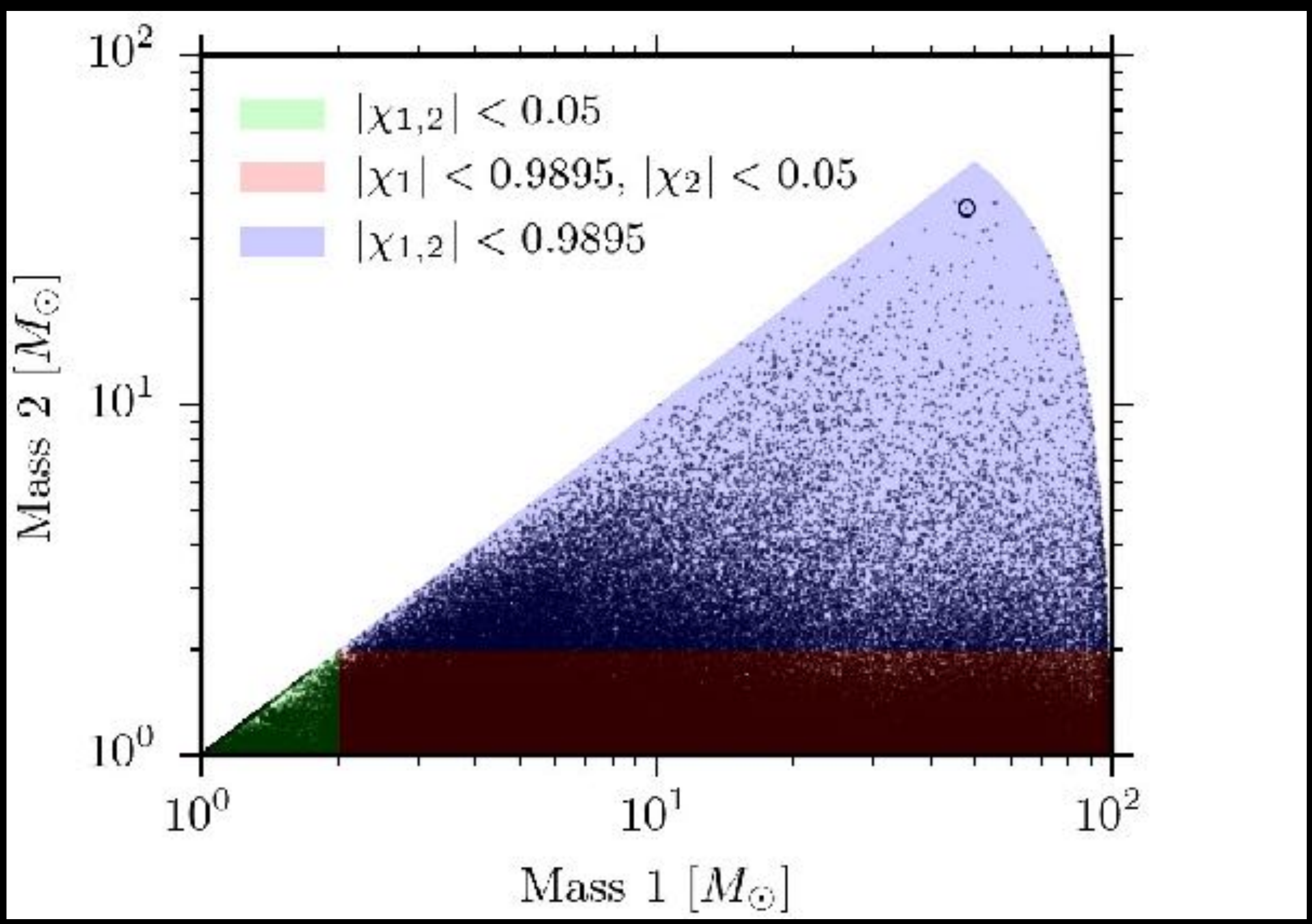
$$h_c(t_{\rm obs}) \sim \frac{\mathcal{M}_z^{5/3}\, f_{\rm obs}^{2/3}}{d_{\rm L}^{\rm gw}}$$

Gravitational waves are standard sirens

[well understood detectability]

[cosmo propagation]

[general relativity predicts waveform]



Gravitational waves are standard sirens

[well understood detectability]

[cosmo propagation]

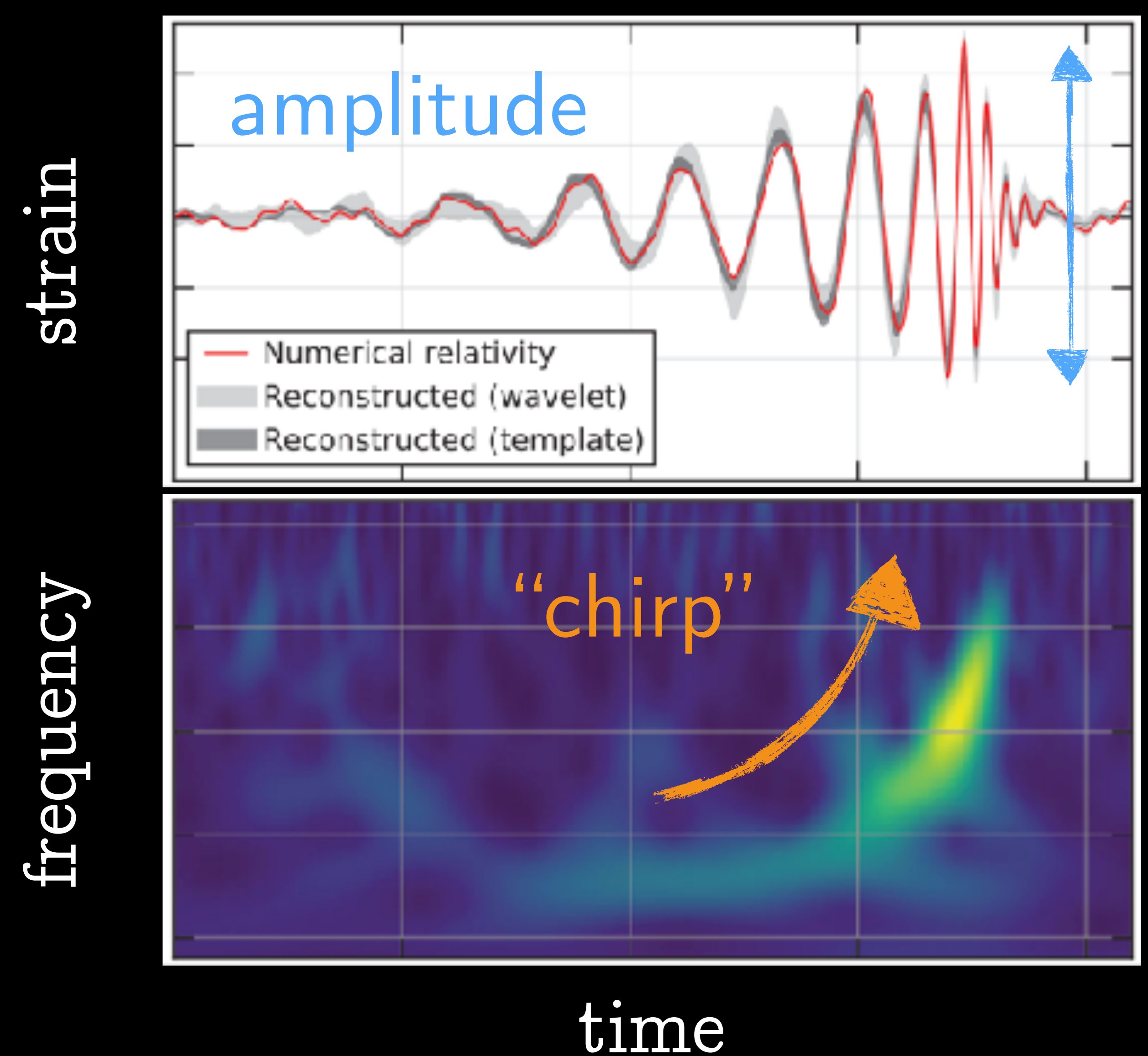
[general relativity predicts waveform]

$$d_L(z)$$

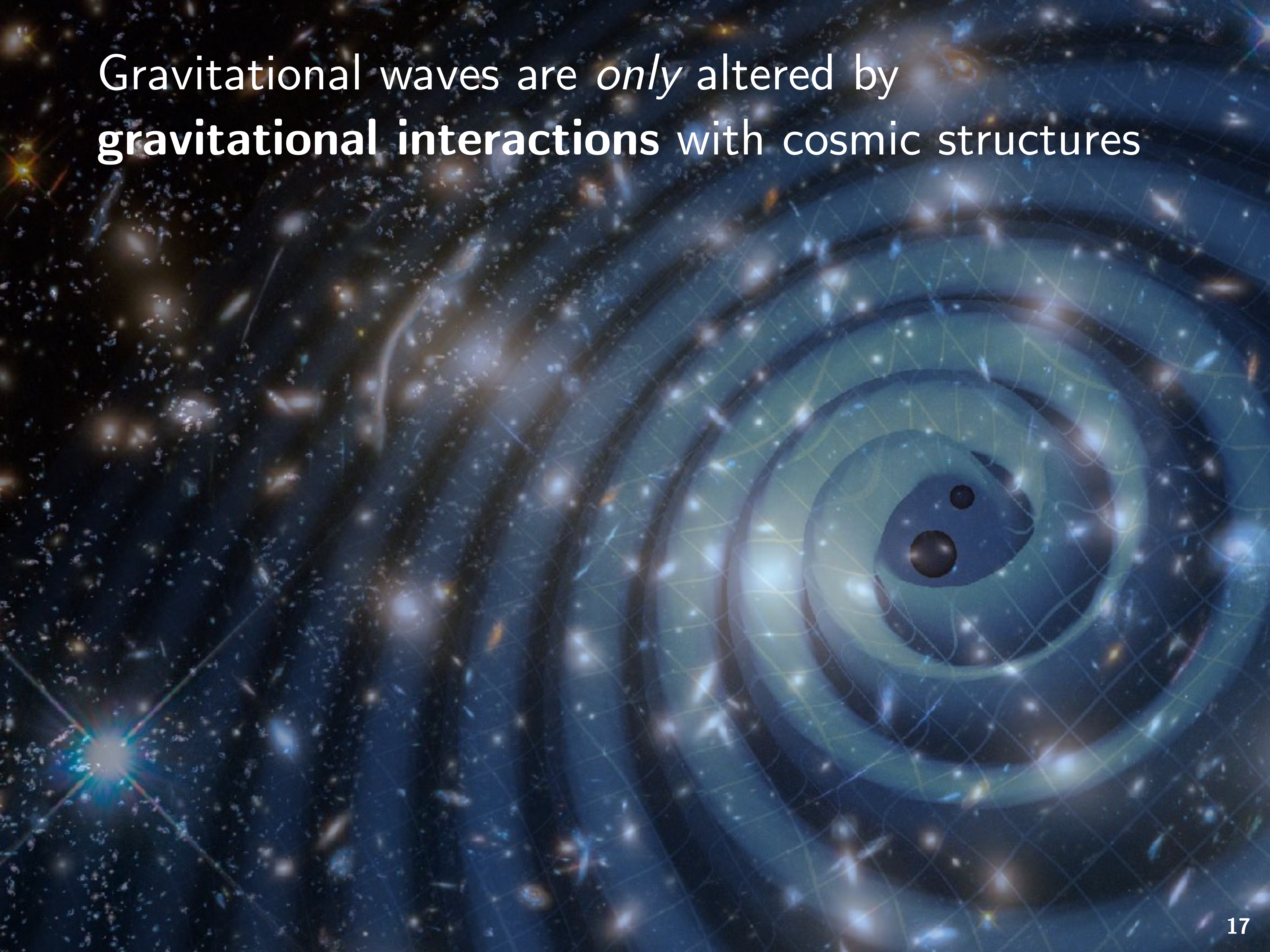
[GW Hubble diagram]

$$m_{\rm det} = (1+z)m$$

[Interplay with astrophysics]

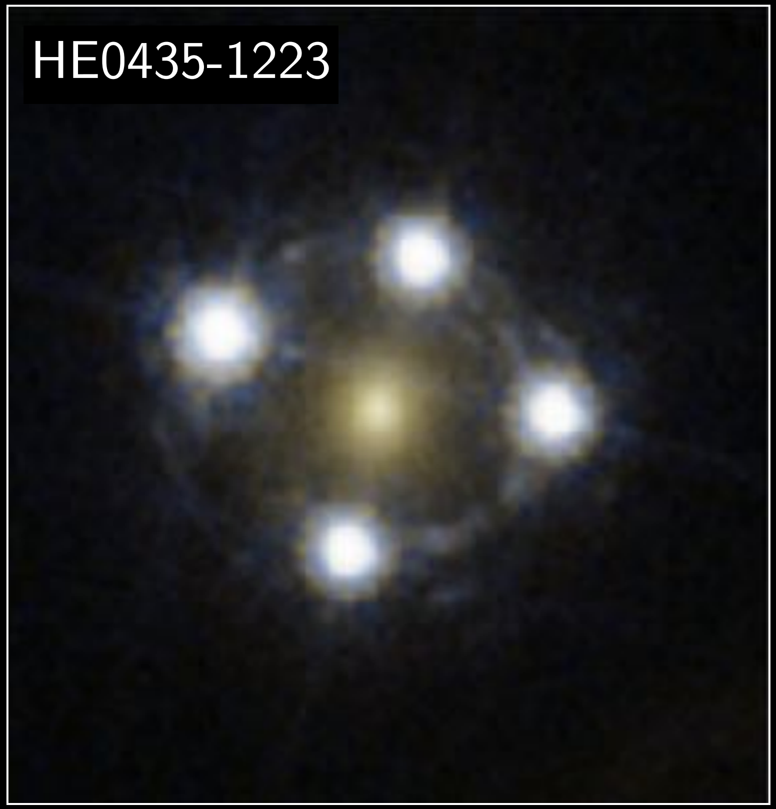


Gravitational waves are *only* altered by **gravitational interactions** with cosmic structures



2. A crash-course on gravitational lensing

Gravitational lensing - electromagnetic spectrum



[multiple images]

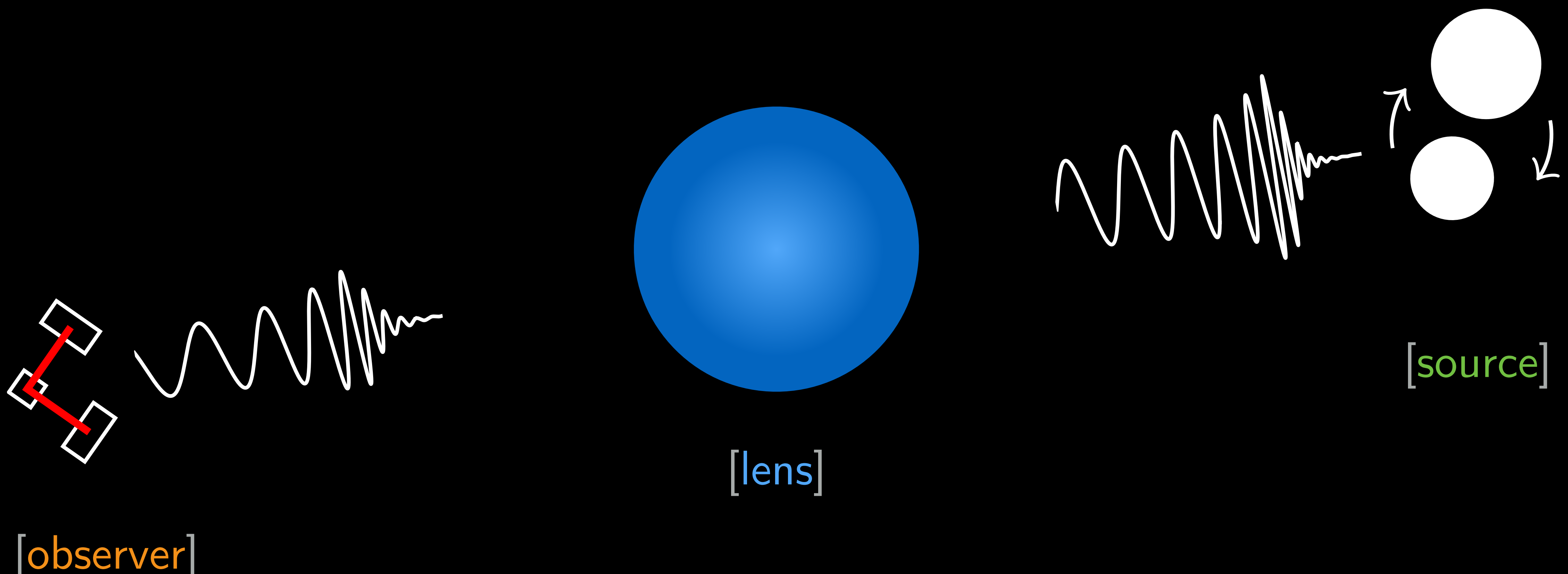
[arcs and rings]

Gravitational lensing

- Solve GW propagation on a curved background

$$\Box \bar{h}_{\mu\nu} + 2\bar{R}_{\alpha\mu\beta\nu}\bar{h}^{\alpha\beta} = 0$$

- We want to make a mapping between the source and the observer through the lens



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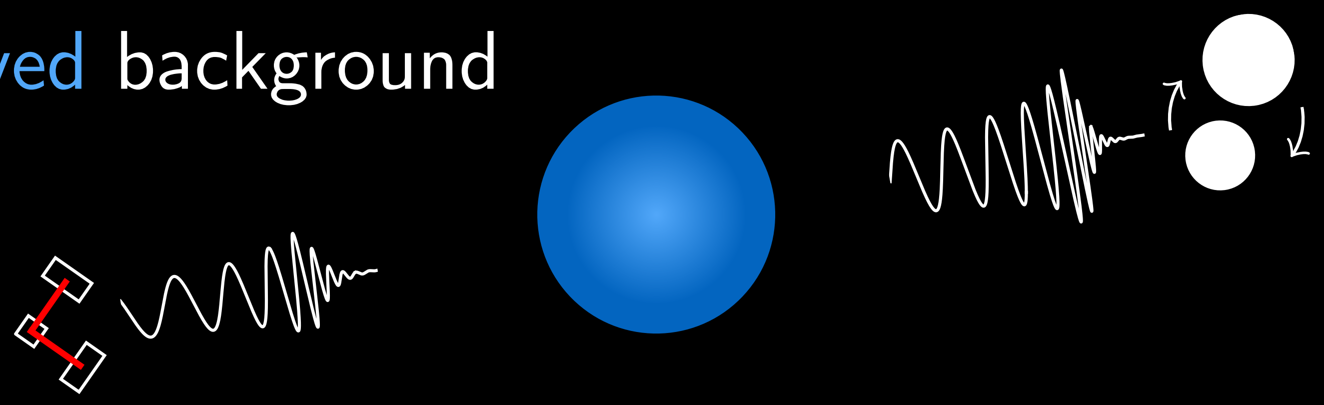


- Cosmological background + *gravitational potential*

$$ds^2 = a(\eta)^2\left(-(1+2\Phi)d\eta^2 + (1-2\Phi)d\vec{x}^2\right)$$

- Focus on *weak-field* limit

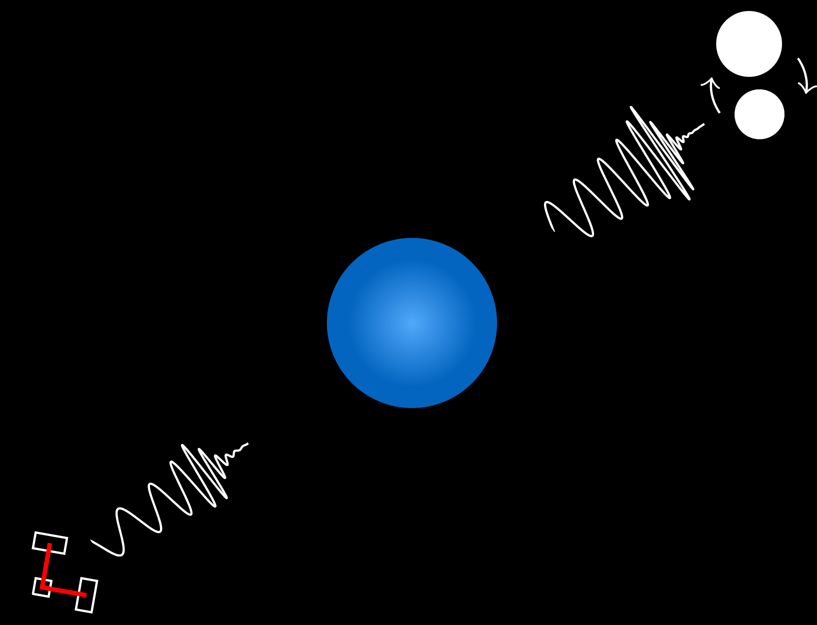
$$\Phi \sim r_{\rm Sch}/r \ll 1$$

- Equations simplify, same propagation for both *polarizations*

$$\nabla^2 h_A - (1-4\Phi)\partial_0^2 h_A = 0$$

[see Cusin's and Motohashi's talks for spin effects]

Gravitational lensing



- Within *weak-gravity*, solve in *Fourier* space:

$$\left(\nabla^2 + \omega^2\right) \tilde{h}_A = 4\Phi\omega^2 \tilde{h}_A \qquad R_L \ll D_L, D_{LS}$$

- For cosmological lenses, impose *thin lens* approximation.

- Integral solution: $h_L(\omega) = F(\omega, \theta_S) \cdot h(\omega)$

$$F(w, \vec{y}) = \frac{w}{2\pi i} \int \mathrm{d}^2 x \;\; \exp[iwT_d(\vec{x}, \vec{y})]$$

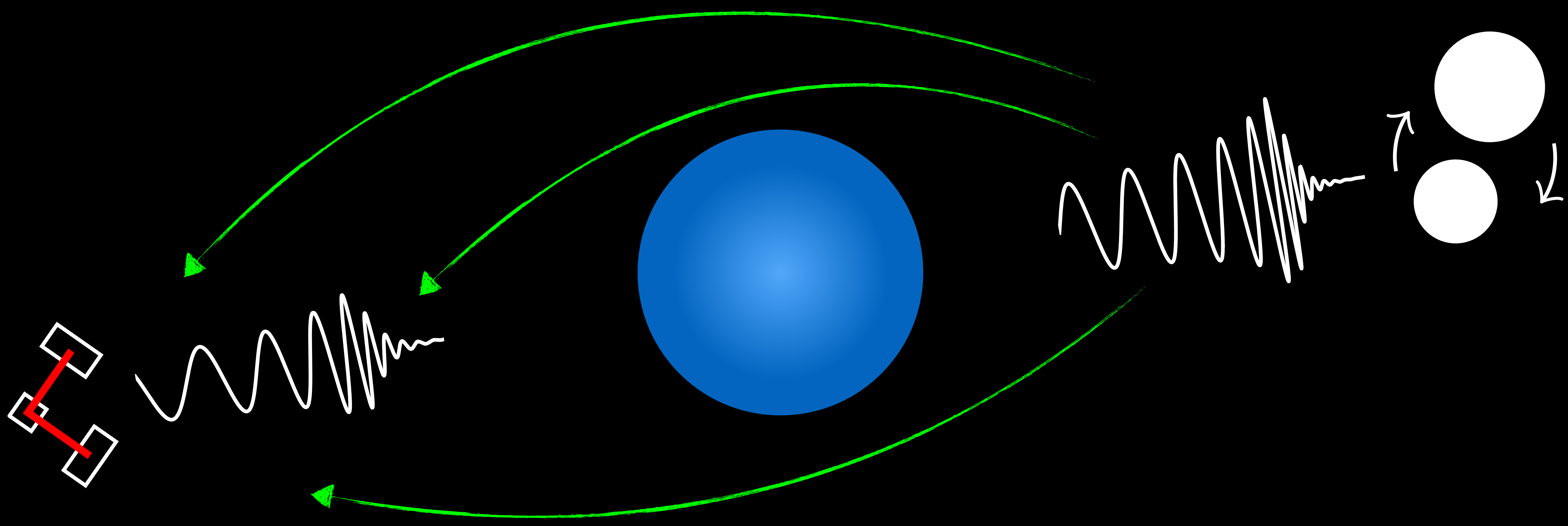
[Dimensionless variables] $\vec{x} \equiv \vec{\theta}/\theta_*\,, \quad \vec{y} \equiv \vec{\theta}_S/\theta_*\,, \quad w \equiv \tau_D \theta_*^2 \omega$

$$T_d \equiv t_d/\tau_D\theta_*^2 \qquad \tau_D \equiv (1+z_L)D_L D_S/cD_{LS}$$

Gravitational lensing

$$h_L(\omega) = F(\omega, \theta_S) \cdot h(\omega)$$

$$F(w, \vec{y}) = \frac{w}{2\pi i} \int \mathrm{d}^2 x \;\; \exp[iwT_d(\vec{x}, \vec{y})]$$



Stationary Phase Approximation

- Solve integral in the limit of *highly oscillatory* integrand

$$F(w,\vec{y}) = \frac{w}{2\pi i}\int \mathrm{d}^2x \;\; \exp[iwT_d(\vec{x},\vec{y})]$$

- Stationary points define the images:

$$\left.\frac{\partial t_d}{\partial \theta_a}\right|_{\vec{\theta}=\vec{\theta}_j} = 0$$

$$T_d(\vec{\theta}) \approx T_d(\vec{\theta}_j) + \frac{1}{2}\sum_{(a,b)=1}^{2} \delta\theta_a \delta\theta_b \frac{\partial^2 T_d(\vec{\theta}_j)}{\partial\theta_a \partial\theta_b} + \cdots$$

- Hessian matrix determines magnifications

$$\mu(\theta_j) = 1/\det(T_{ab}(\theta_j))$$

$$T_{ab} \equiv \tau_D^{-1} \partial^2 t_d / \partial\theta_a \partial\theta_b$$

Multiple chirps

$\Delta t_d \cdot \omega \gg 1$

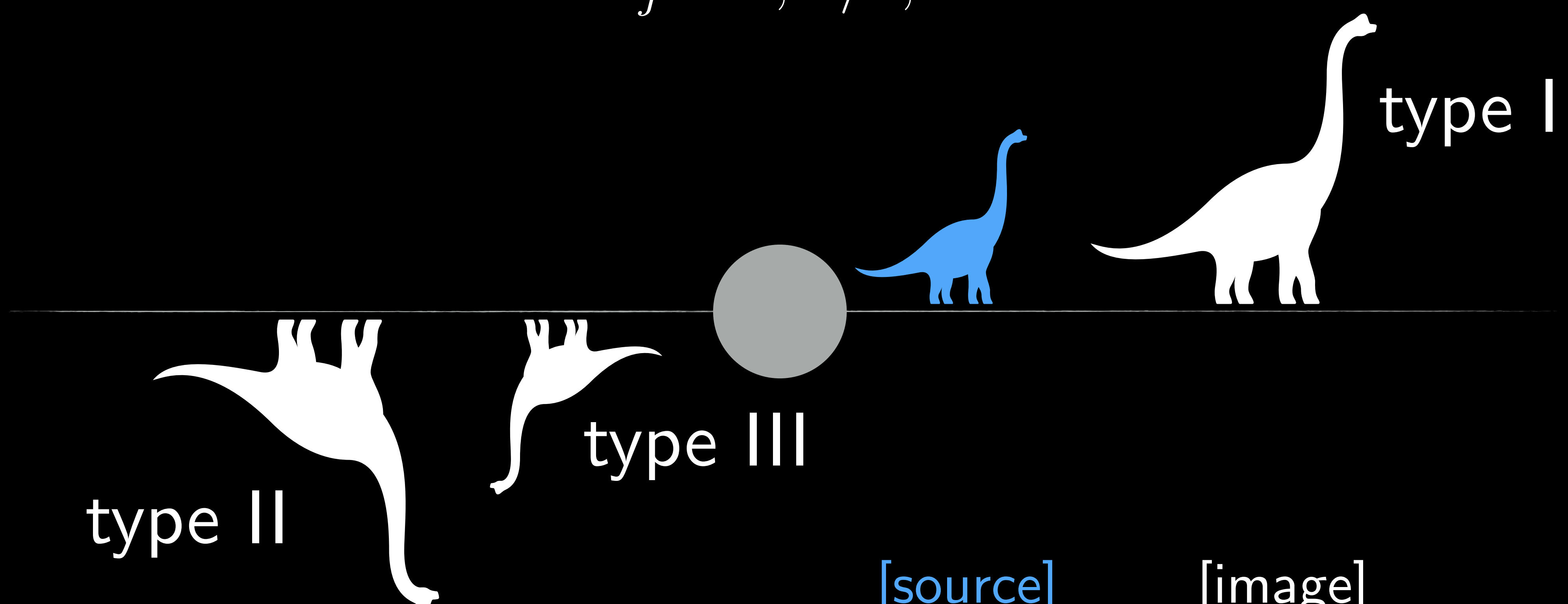
$$h_L(\omega) = F(\omega, \theta_S) \cdot h(\omega)$$

$$F \approx \sum_j |\mu_j|^{1/2} \exp\left(i\omega t_j - i\pi n_j\right)$$

Magnification
Time delay
Phase shift

- Lensed signals acquire a different phase shift

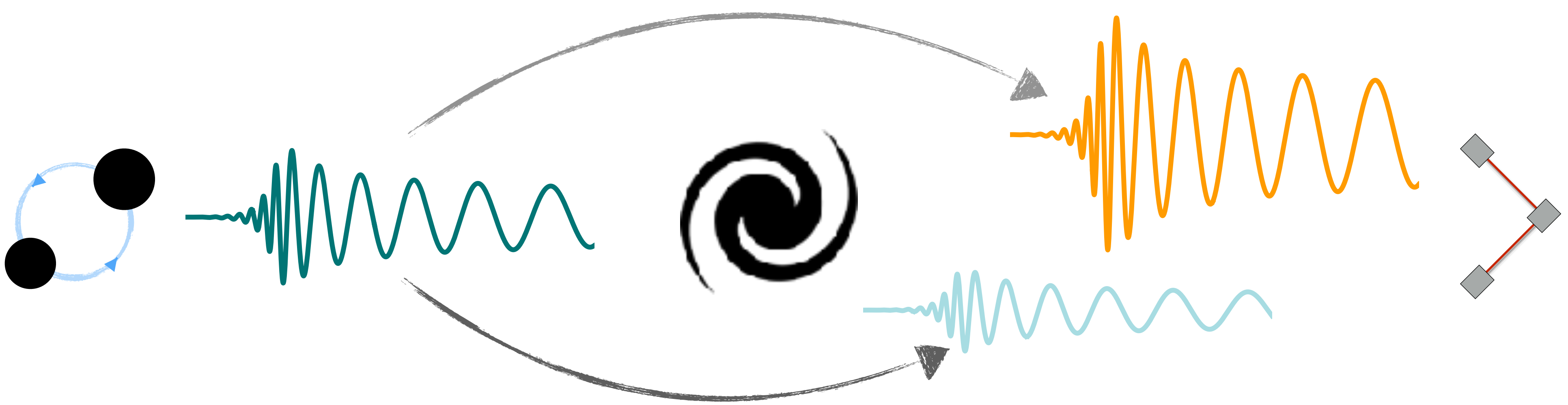
$$n_j = 0,\ 1/2,\ 1$$



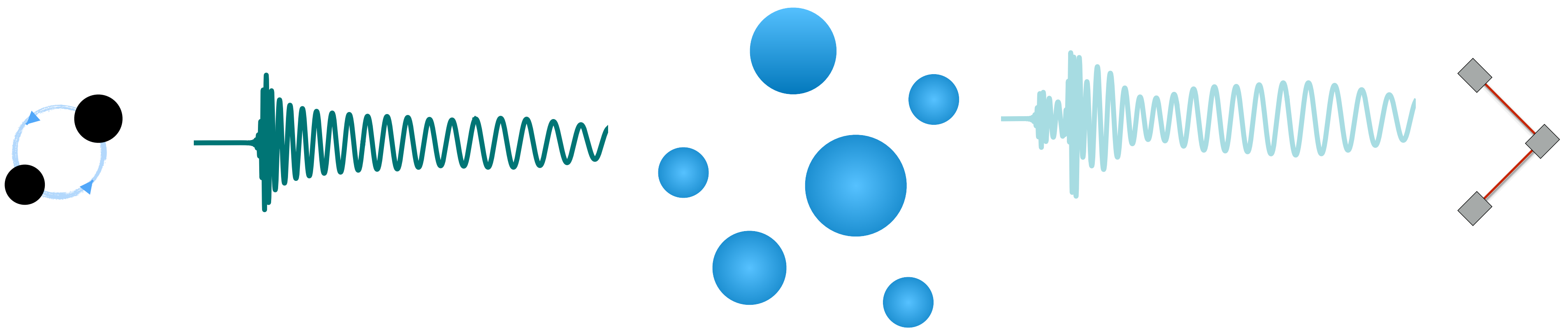
3. Current searches

Gravitational lensing - gravitational wave spectrum

Repeated chirps due to strong lensing

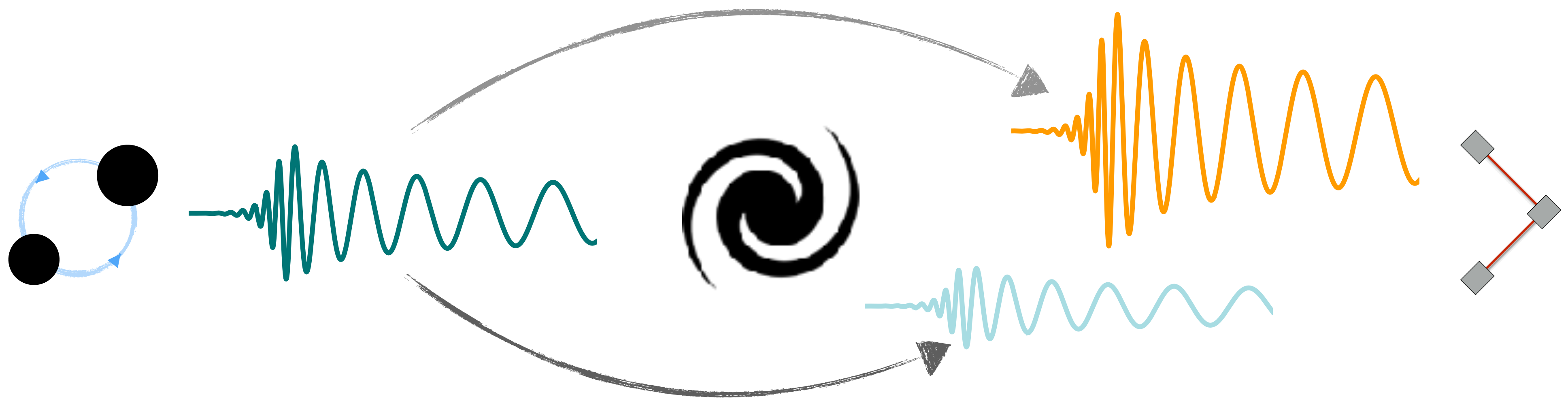


Waveform *distortions* by substructures



Source Lens Detector

Repeated chirps due to strong lensing



- The properties of the *j*-th chirp

$$d_L^j = d_L/\sqrt{|\mu_j|} \qquad t_{\rm ref}^j = t_{\rm ref} + \Delta t_j$$

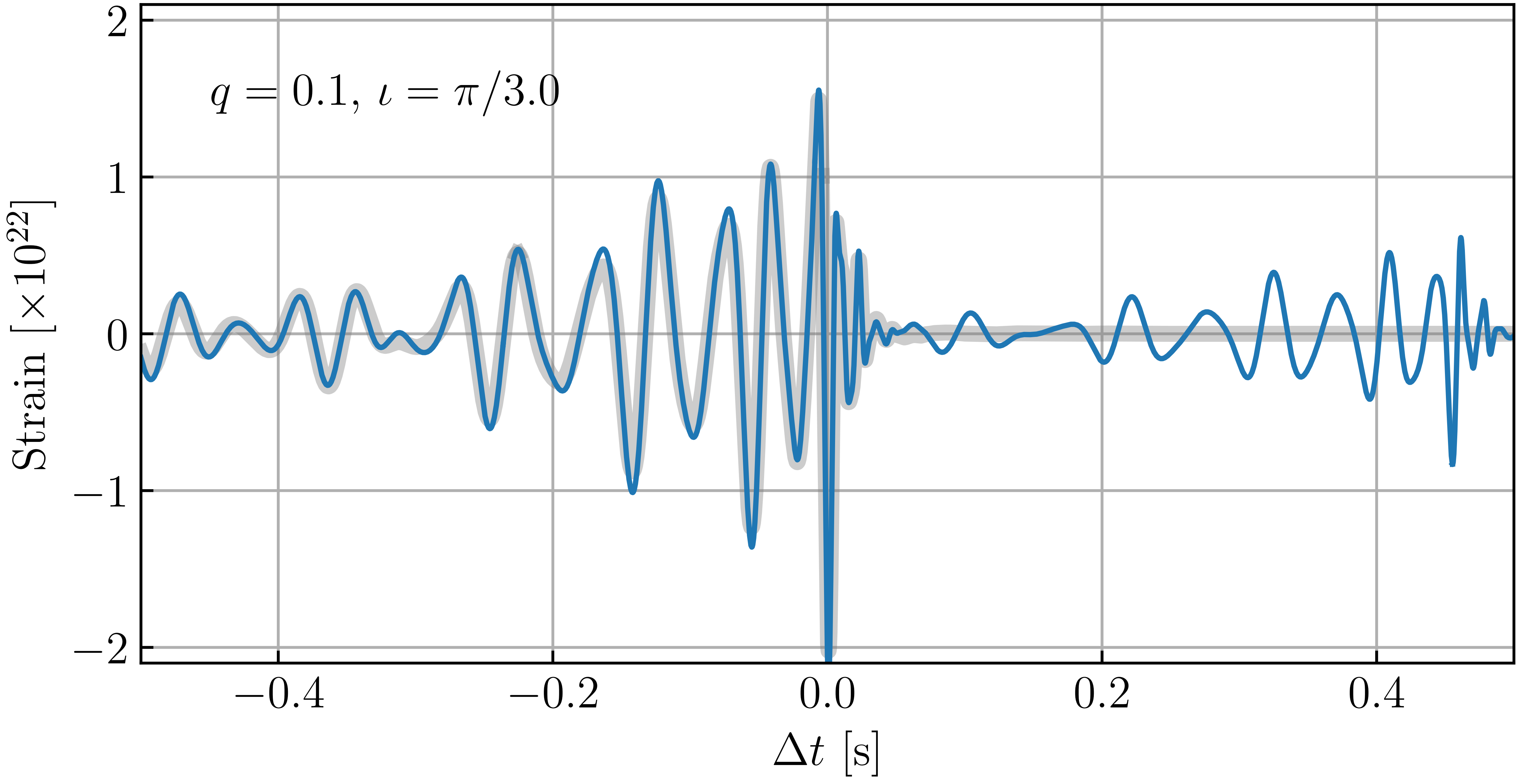
$$m_{\rm det}^j = m_{\rm det} \qquad \phi_{\rm ref}^j = \phi_{\rm ref} - \pi/2$$

- If not identified as lensed, a *magnified* events appears *closer* and *more massive*

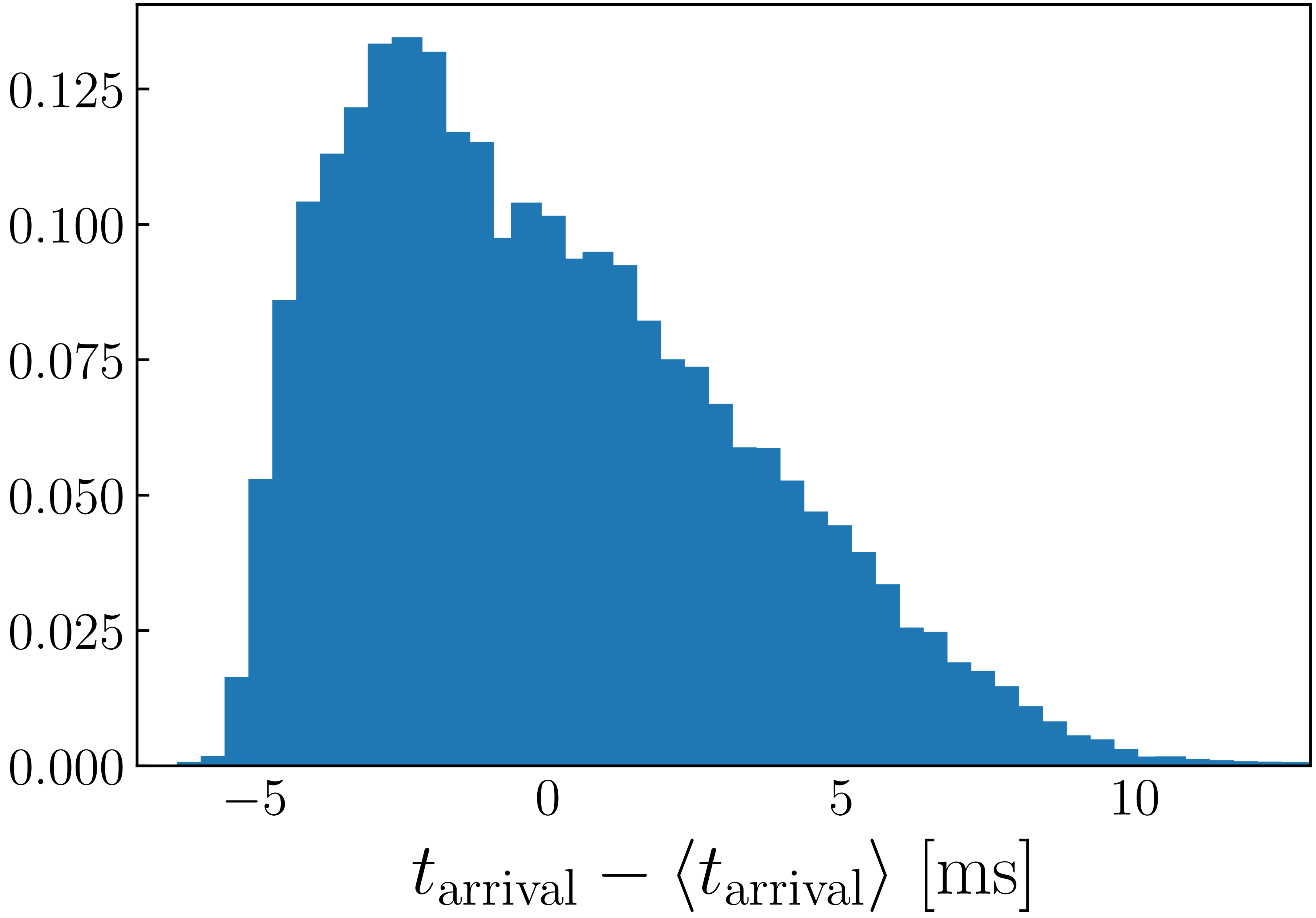
$$m_{\rm src}^j = m_{\rm det}/(1 + z(d_L^j))$$

[see Chen's talk]

Repeated, *coherent* signals

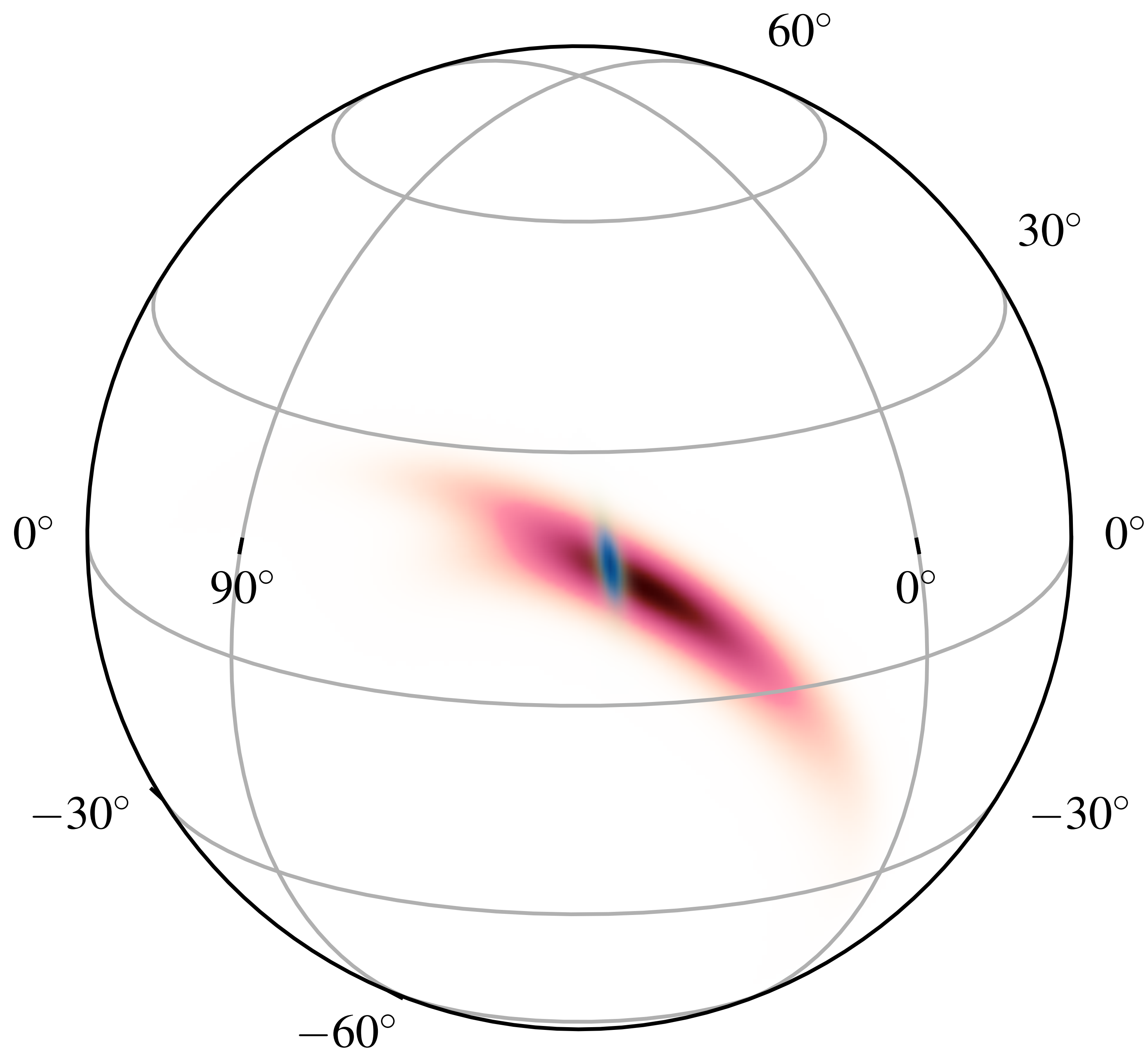


Precise timing

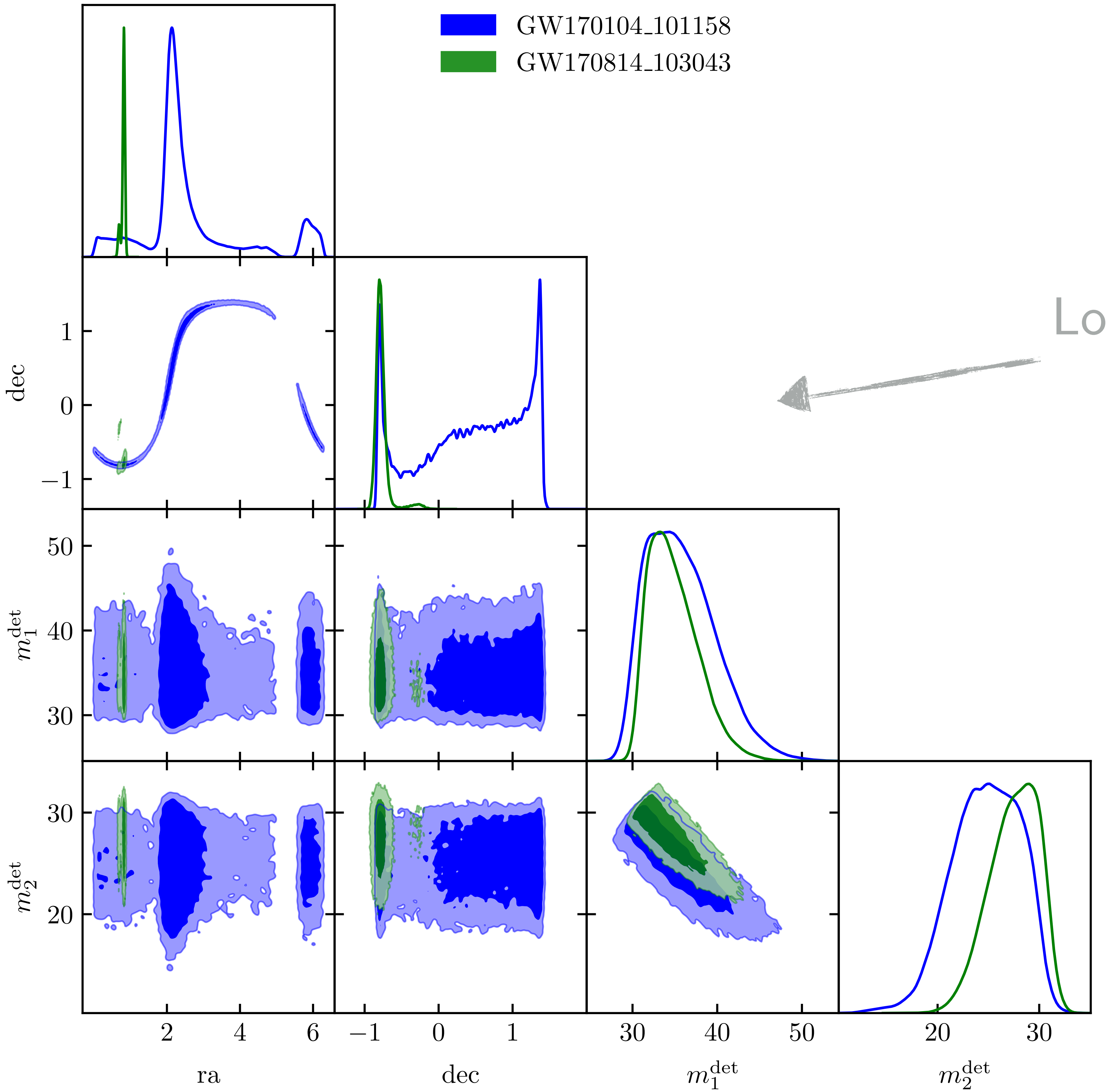


Poor sky localization

$$\theta_E \sim 1'' \sqrt{\frac{M}{10^{12} M_\odot}} \sqrt{\frac{1\text{Gpc}}{D}}$$

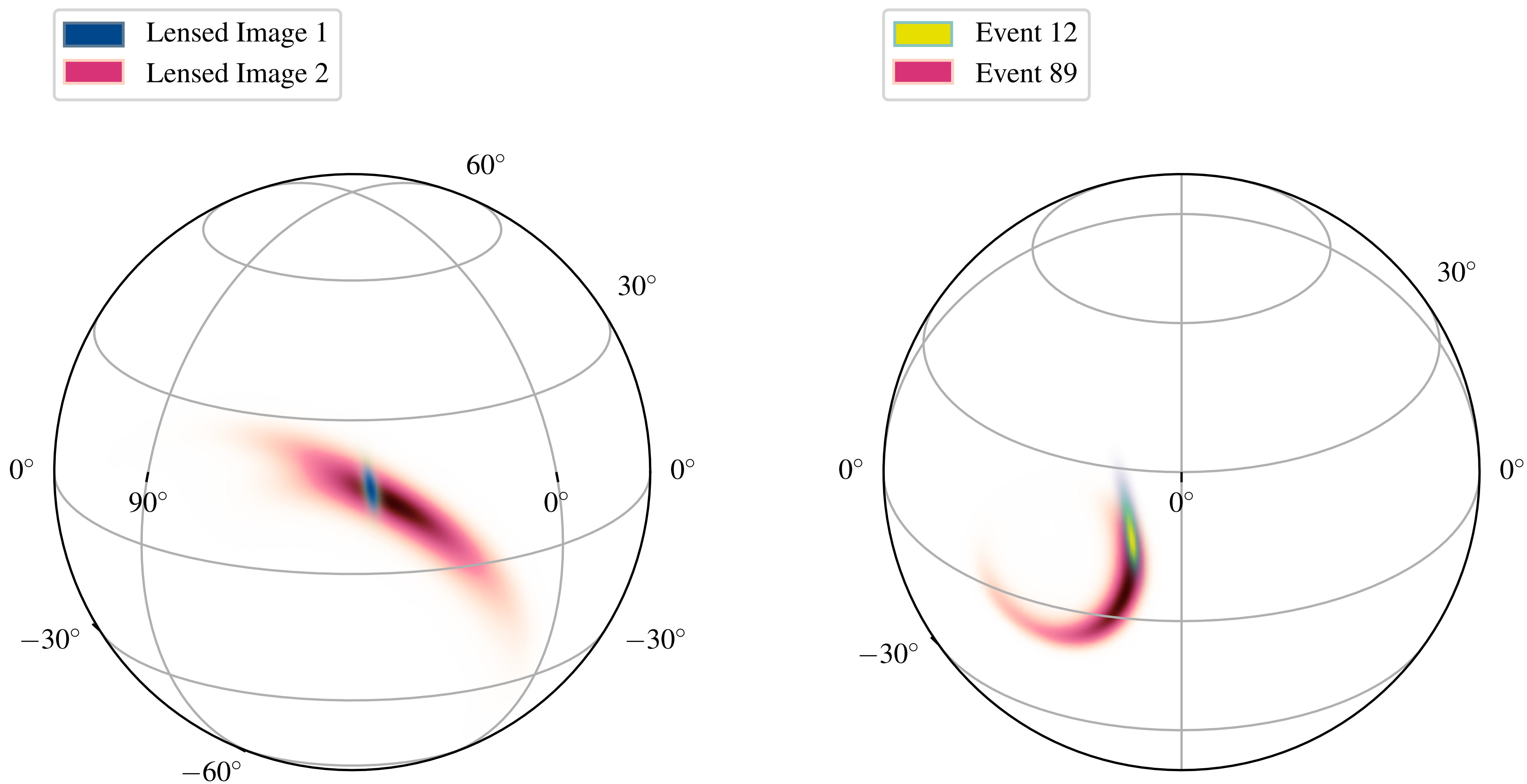


Searching for repeated chirps



Look for events with similar properties: masses, sky positions, spins...

Searching for repeated chirps: false alarms



$$N_{\mathrm{false\,alarm}} \sim N^2$$

[*Çalışkan*, **Ezquiaga**, Hannuksela and Holz; PRD'22]

Searching for repeated chirps

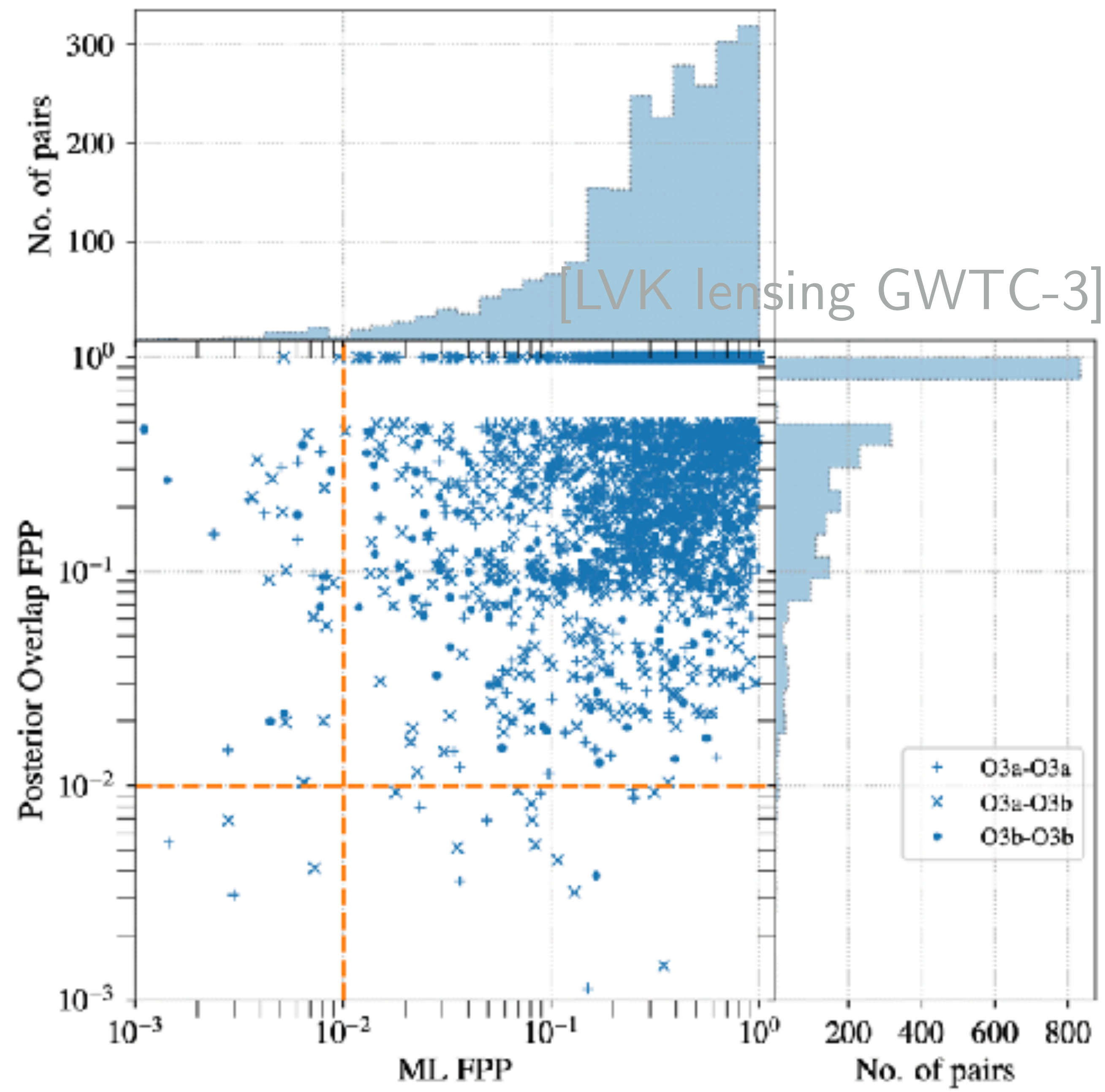
- Given the large number of pairs, need quick methods to identify promising pairs
- Compute the posterior overlap

  [Haris et al.; 2018]

- Use machine learning (ML) summary statistic

  [Goyal et al.; 2021]

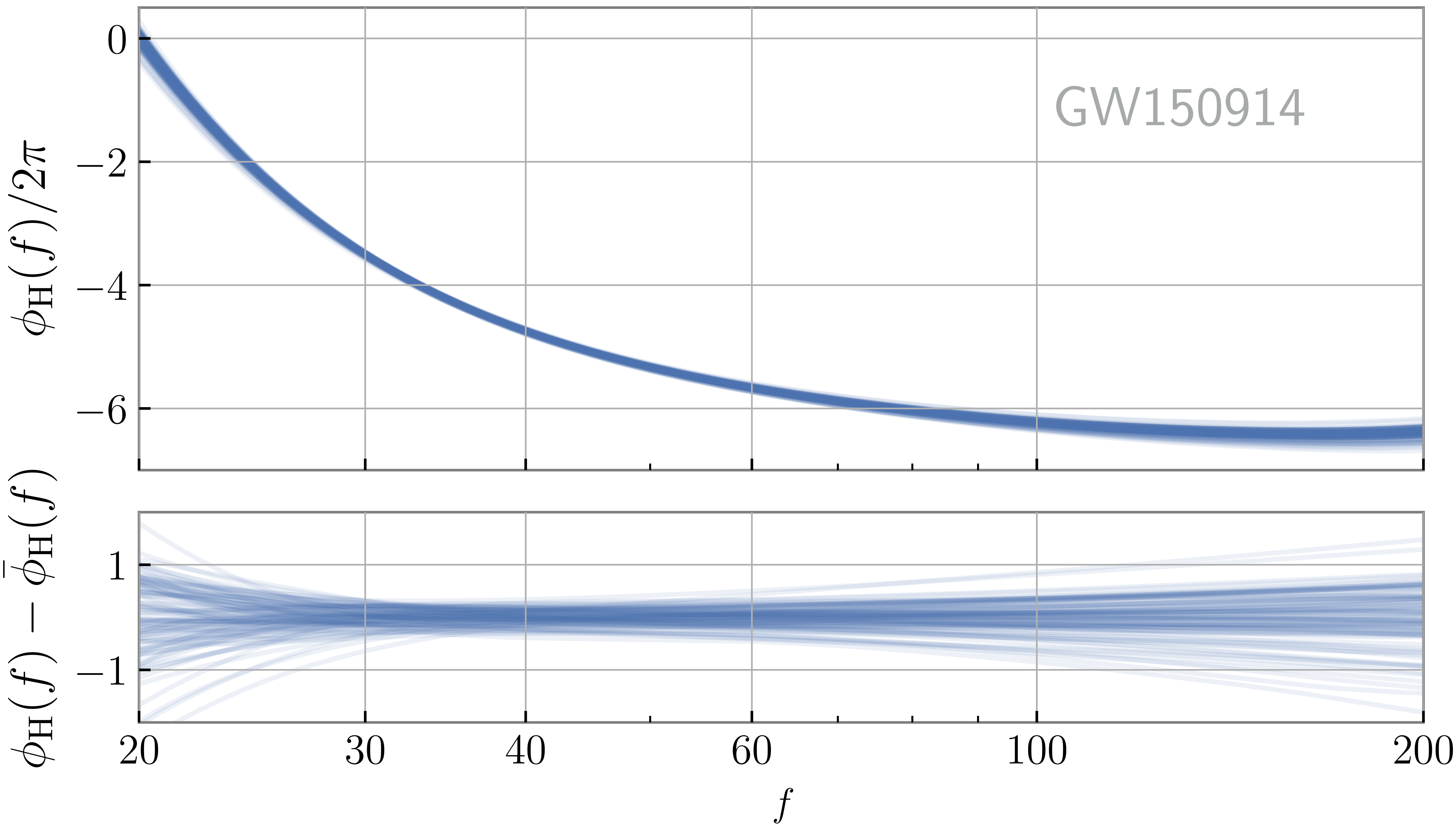
  [see Li's talk]



[LVK lensing GWTC-3]

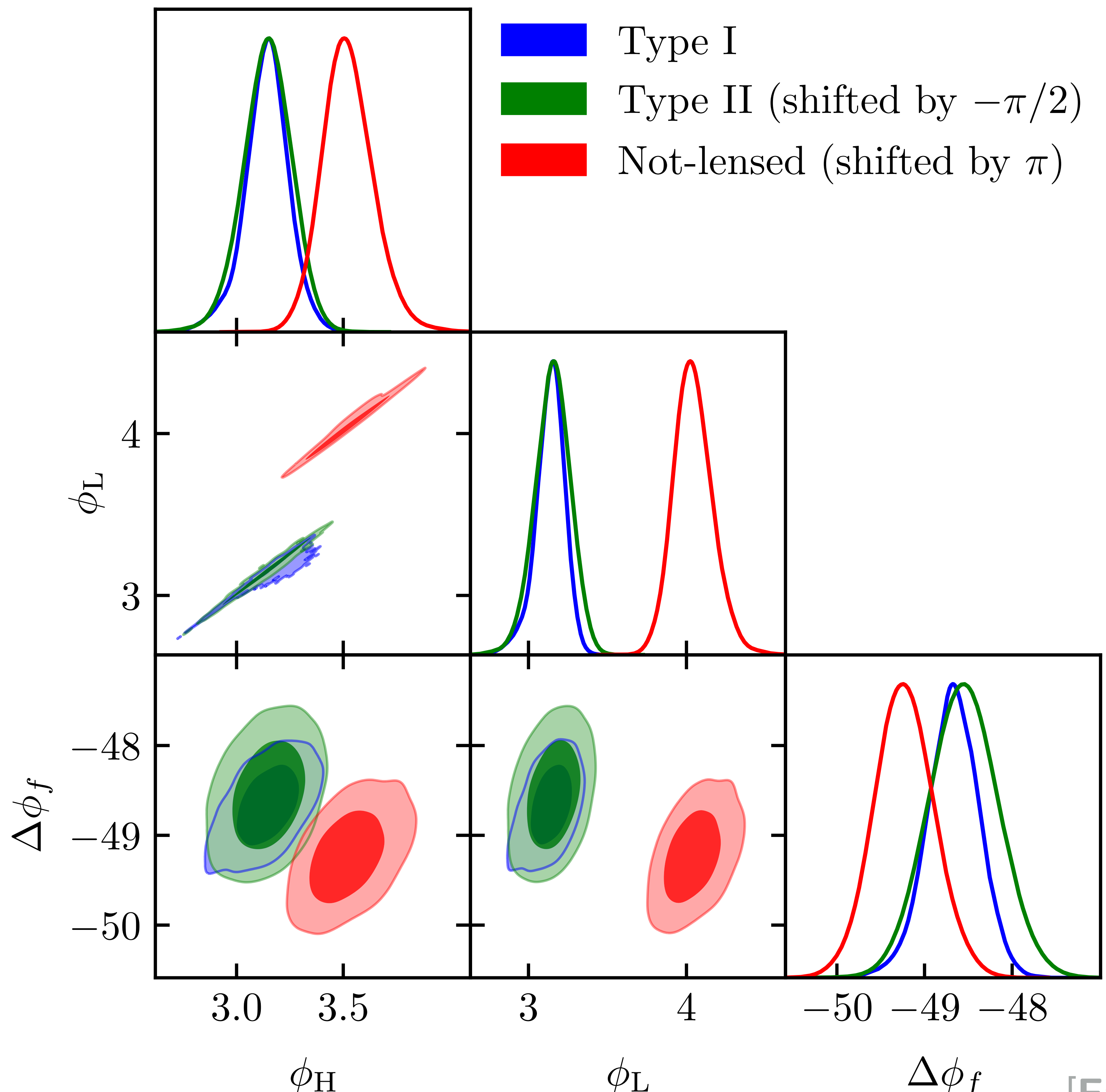
FPP= false-positive probability

Fight false alarms: **phase consistency**



[**Ezquiaga**, Hu, Lo; PRD'23]

Fight false alarms: **phase consistency**

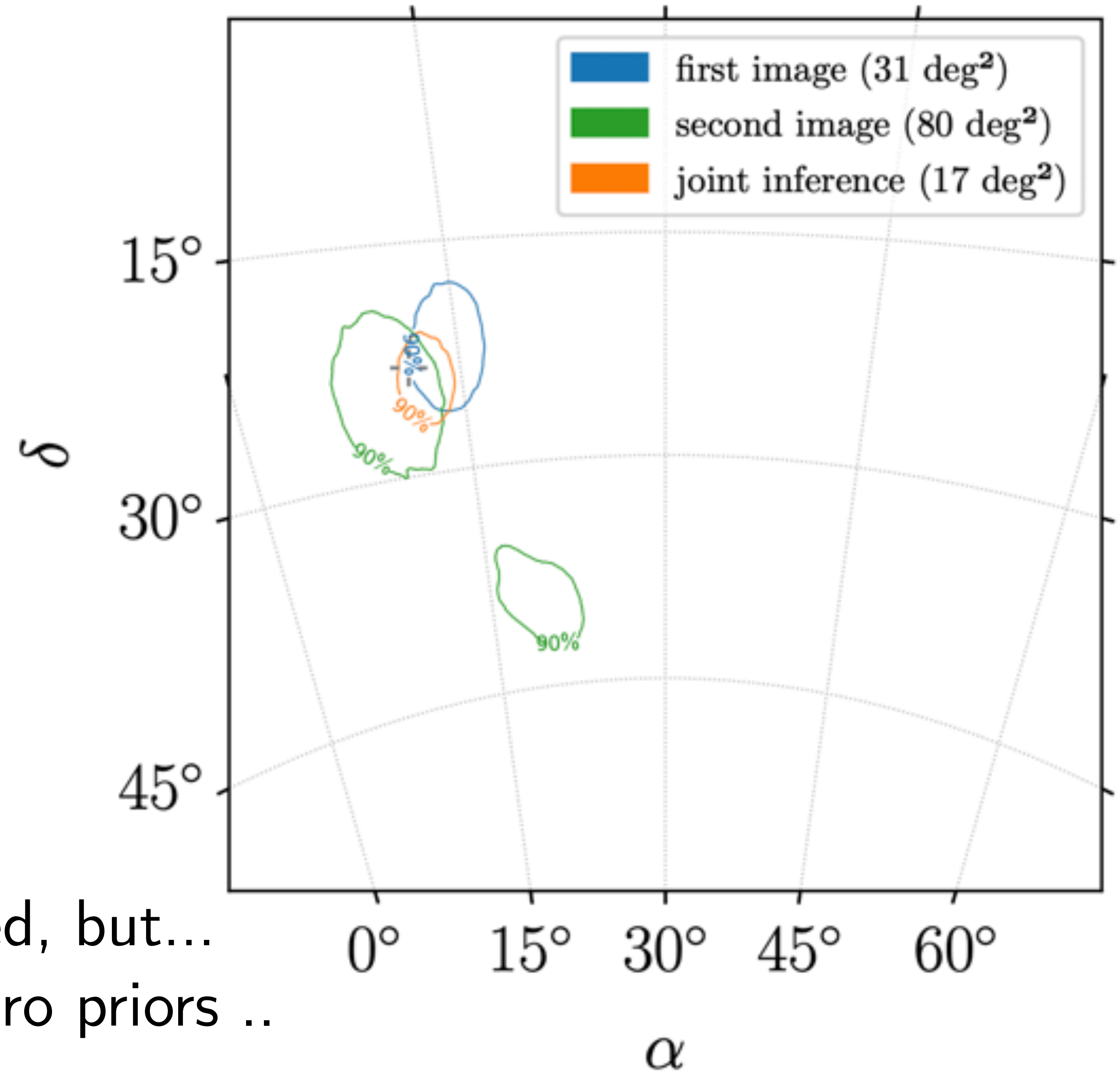


[**Ezquiaga**, Hu, Lo; PRD'23]

Joint parameter estimation

- Infer the parameters of the source under the *lensing hypothesis* using data from *multiple* events

- Allows for Bayesian model comparison
  - Measure consistency of events: coherence ratio ($C^L{}_U$)
  - With source/lens populations priors, compute Bayes factor ($B^L{}_U$)

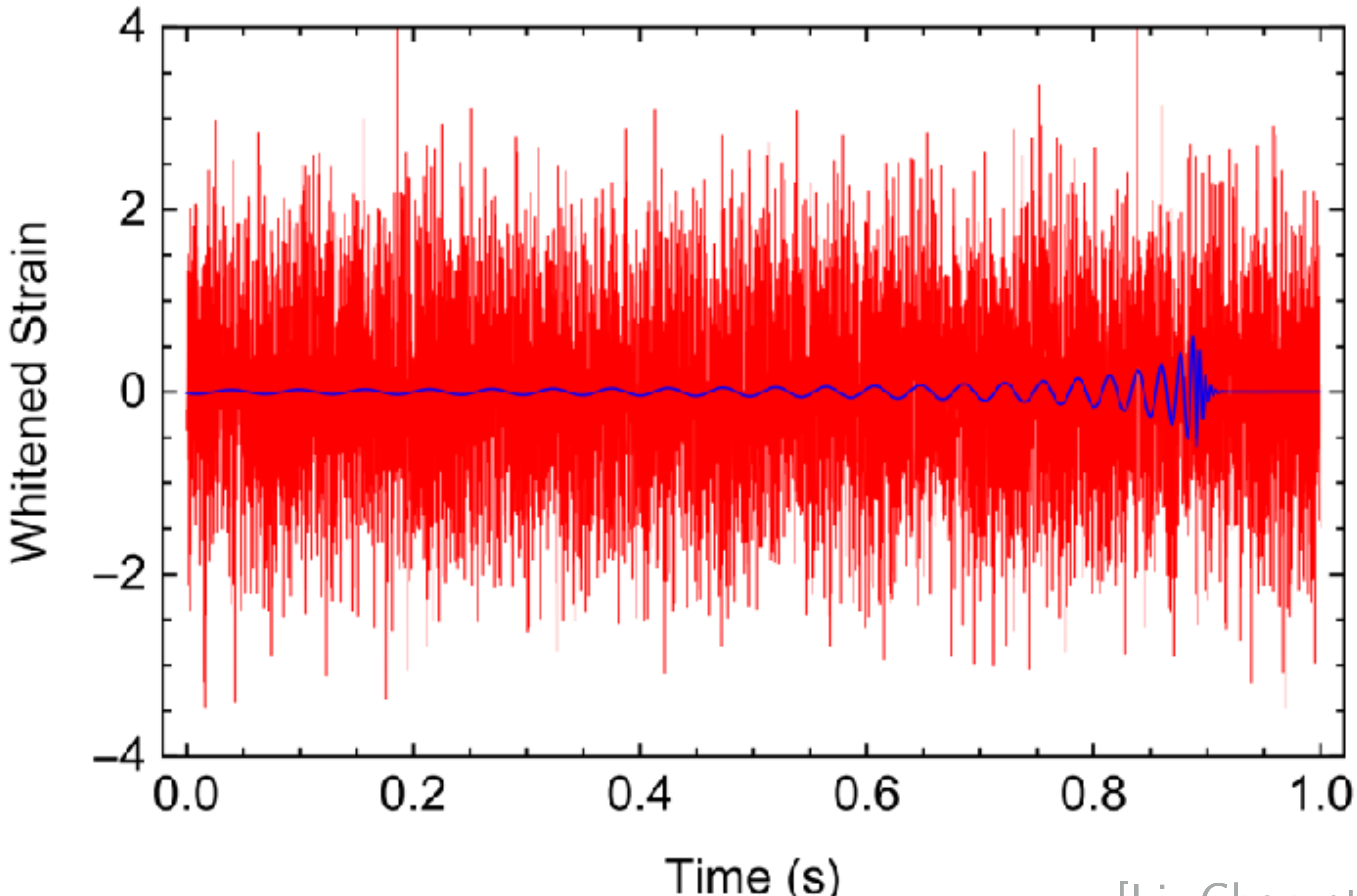


- This is in principle all we need, but... computationally costly ... astro priors ..

[Janquart et al.; golum] [Lo & Magaña; hanabi]

Sub-threshold searches

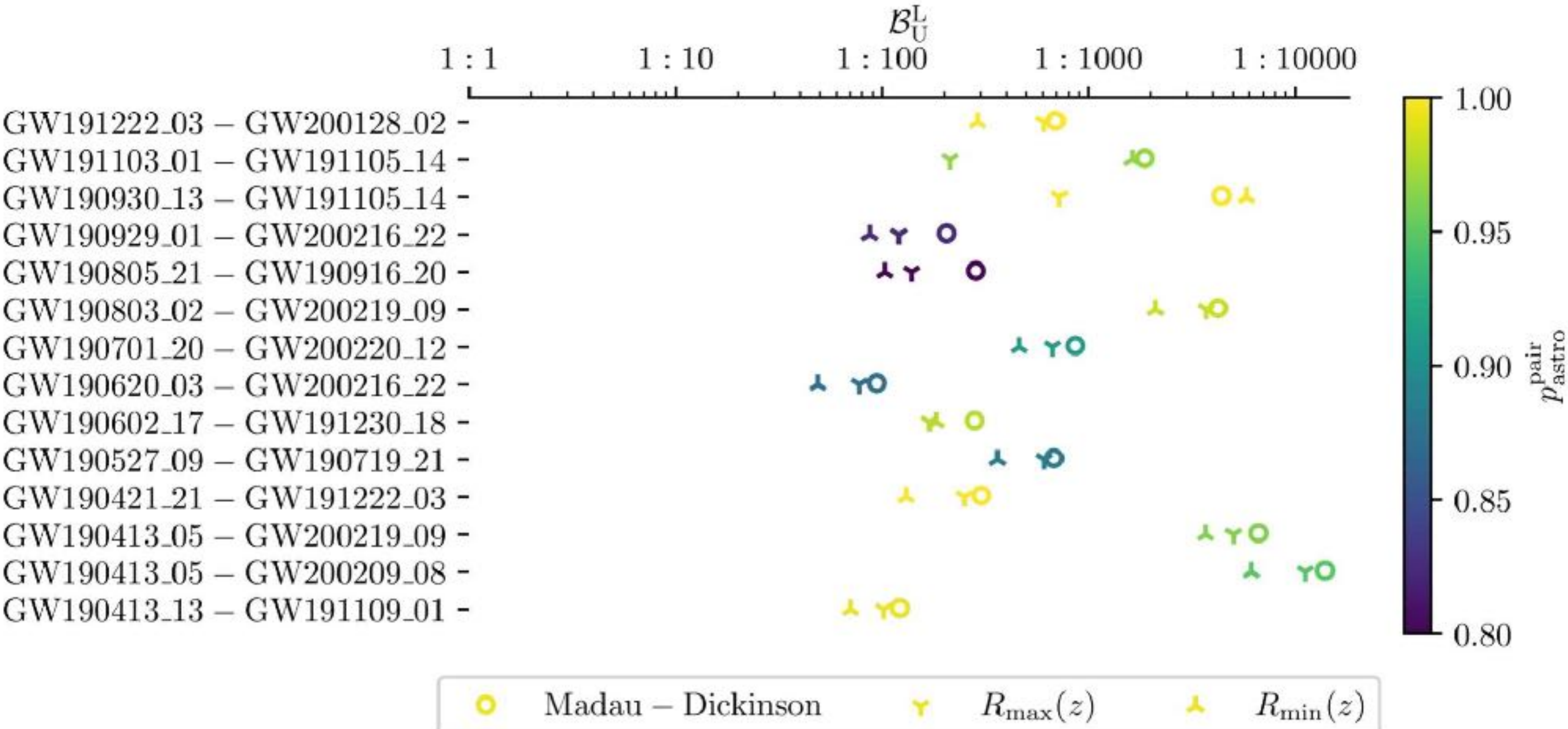
- Demagnified events could be under the noise (sub-threshold event)
- *Targeted* searches following super-threshold events reduce template bank and increase sensitivity



[Li, Chan et al.; tesla]

GWTC-3 results

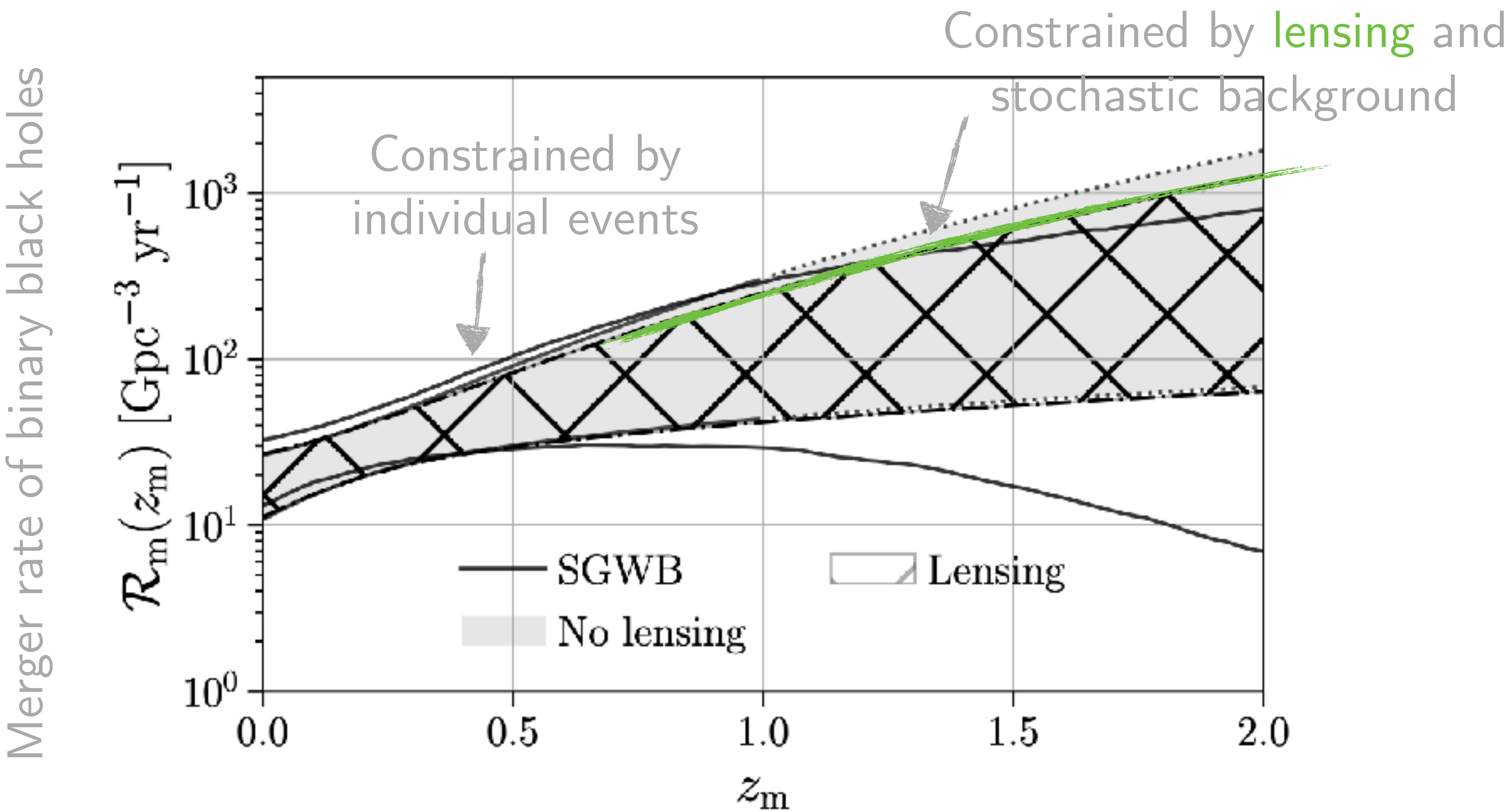
- No evidence of repeated chirps in the data



[LVK lensing GWTC-2] [LVK lensing GWTC-3]

GWTC-3 results

- *Upper bound* on binary black hole merger rate



[LVK lensing GWTC-2] [LVK lensing GWTC-3]

Phase shifts & higher modes

- A gravitational wave is a superposition of *frequency modes*

$$h = \sum_{\ell,m\geq 0} \mathcal{A}_{\ell m} \cos[m(\Omega \Delta t + \varphi_c) - \chi_{\ell m}]$$

- A lensed signal of **type I** has different amplitude and arrival time

$$h_{\mathrm{I}} = \sum_{\ell,m\geq 0} |\mu_{\mathrm{I}}|^{1/2} \mathcal{A}_{\ell m} \cos[m(\Omega \Delta t_{\mathrm{I}} + \varphi_c) - \chi_{\ell m}]$$

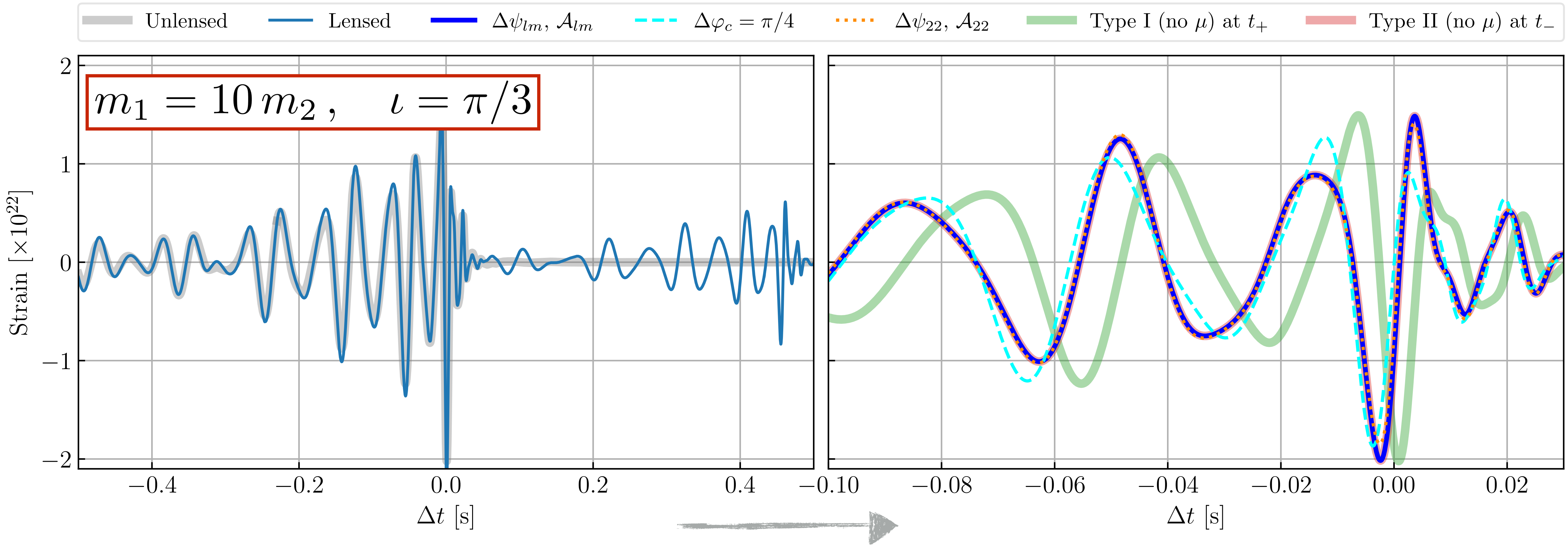
- A lensed signal of **type II** has also a phase shift

$$h_{\mathrm{II}} = \sum_{\ell,m\geq 0} |\mu_{\mathrm{II}}|^{1/2} \mathcal{A}_{\ell m} \cos\left[m\left(\Omega \Delta t_{\mathrm{II}} + \varphi_c\right) - \chi_{\ell m} + \frac{\pi}{2}\right]$$

[Dai & Venumadhav; 2017] [**Ezquiaga** *et al.*; PRD'20]

Waveform distortions in **type II** images

- Lensing imprints *small* but *characteristic* modifications in the signals that cannot be mapped to other astrophysical parameters



Zoom in the shape of each image

[**Ezquiaga** *et al.*; PRD'20]

Caustics

- For point sources, there are singular points in the lens mapping

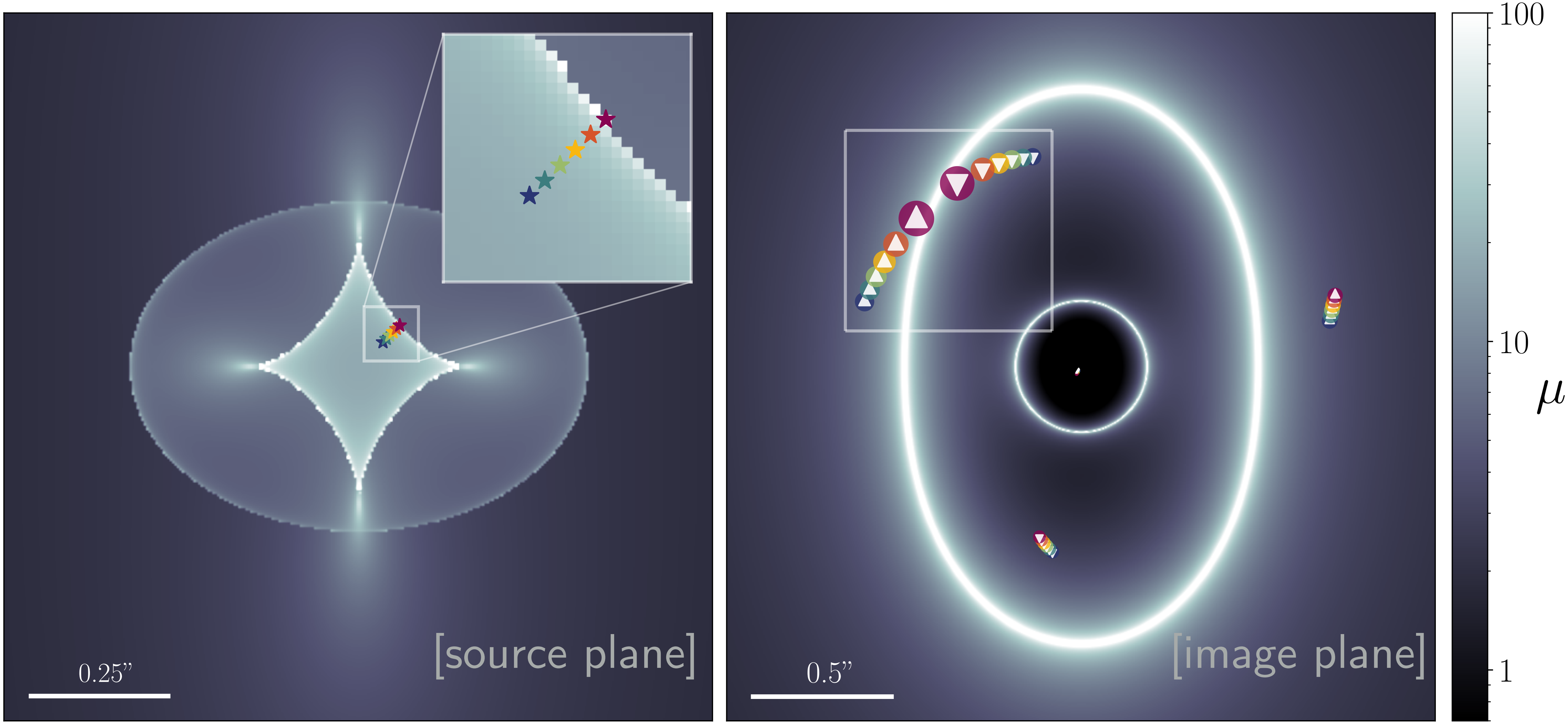
$$\det\left(\frac{\partial^2 T_d(\theta_j)}{\partial\theta_a\partial\theta_b}\right) \to 0 \quad \Rightarrow \mu(\theta_j) \to \infty$$

- Caustics exhibit *universal* behaviors (described by catastrophe theory)

$$\mu_\pm \sim 1/\sqrt{\Delta\theta_S} \sim \Delta t^{-1/3}$$

- SPA is broken when approaching to a caustic
- Maximum magnification set by diffraction

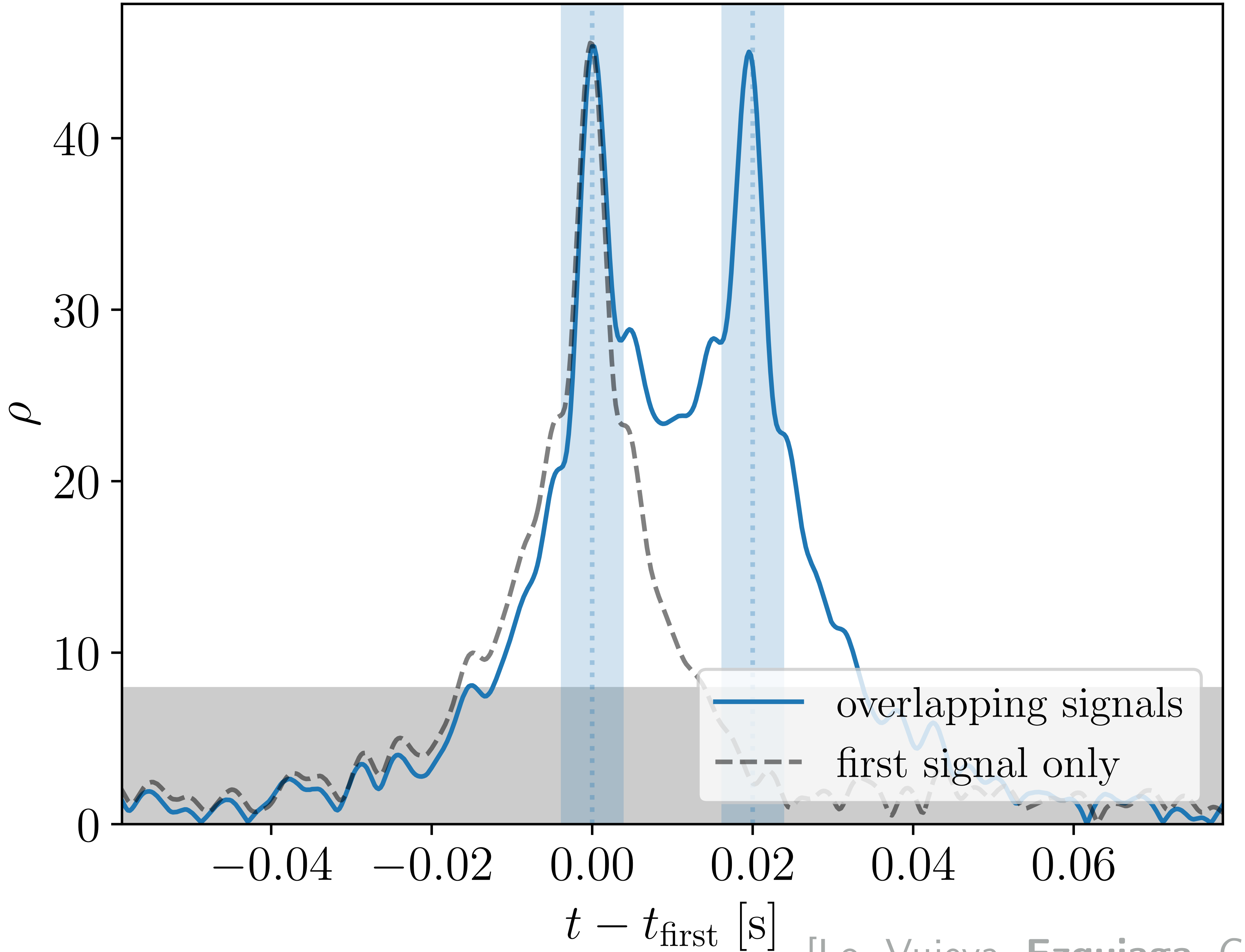
Approaching a (fold) caustic



[galaxy lens with a cored singular isothermal ellipsoid density profile]

[Lo, Vujeva, **Ezquiaga**, Chan; 2024]

Highly magnified, overlapping signals



[Lo, Vujeva, **Ezquiaga**, Chan; 2024]

Wave optics

$\Delta t_d \cdot \omega$

- Time delay scales with the lens mass

$$\Delta t_d(y=1) \simeq 4 \left(\frac{(1+z_L)M_L}{100 M_\odot} \right) \mathrm{ms}$$

[point mass lens]

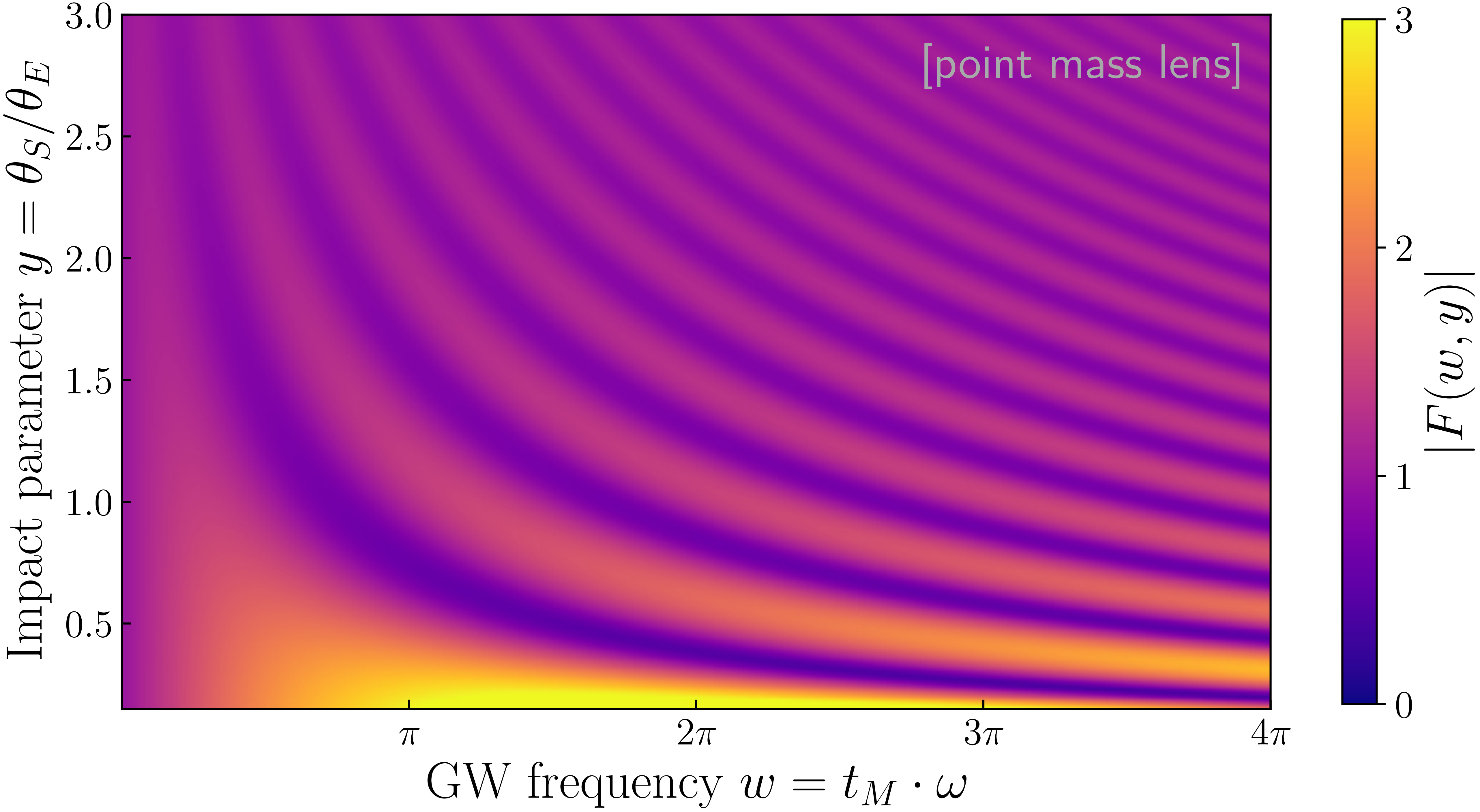
- GW frequency scales with binary mass *(has astrophysical size!)*

$$f \sim \frac{1}{2\pi} \frac{1}{2t_{\mathrm{Sch}}} \sim 800\mathrm{Hz} \left(\frac{10 M_\odot}{M} \right)$$

- Wave optics regime: $\Delta t_d \cdot \omega \sim 1$

- Low-frequency limit has small lensing $\omega \to 0 \quad \Rightarrow \quad F \to 1$

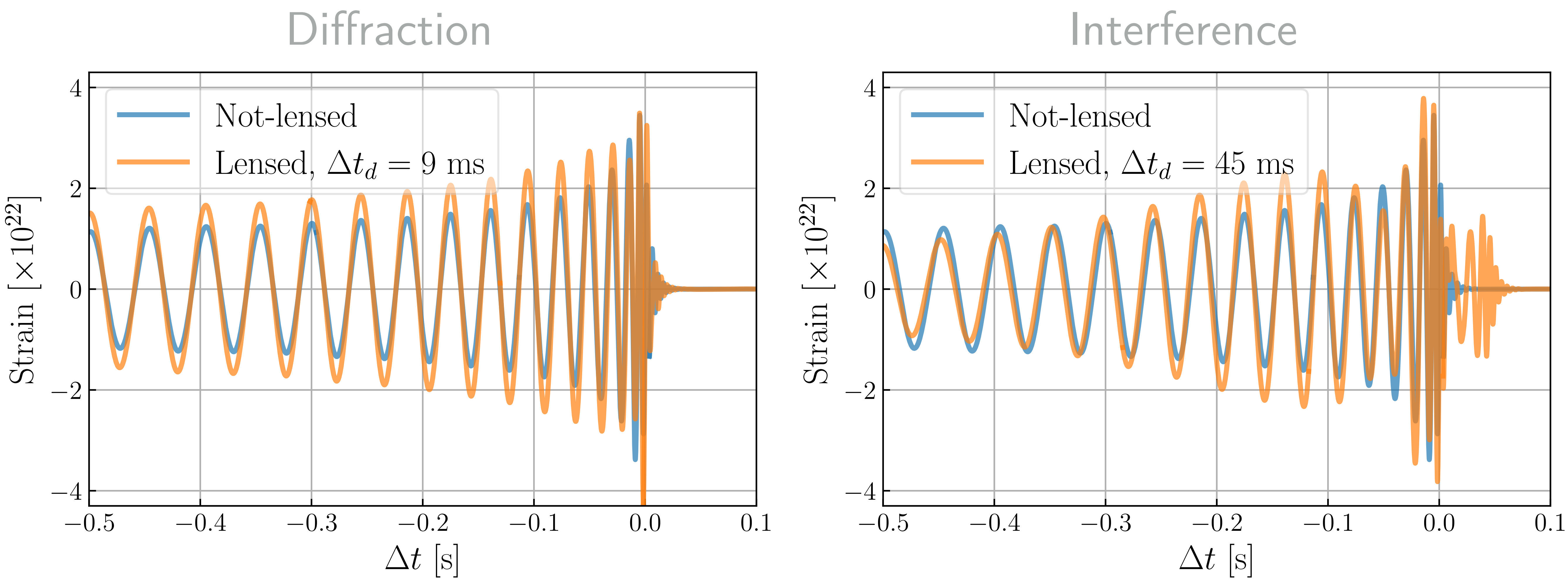
Wave optics: **diffraction**



[see Ubach's talk]

E.g. compact (point) lenses

$$\Delta t_d(y=1) \simeq 4 \left(\frac{(1+z_L)M_L}{100 M_\odot} \right) \text{ms}$$



- Most lens models require solving the diffraction integral numerically. Great recent progress [see Villarubia-Rojo's talk]

Parameter estimation of lensed signals

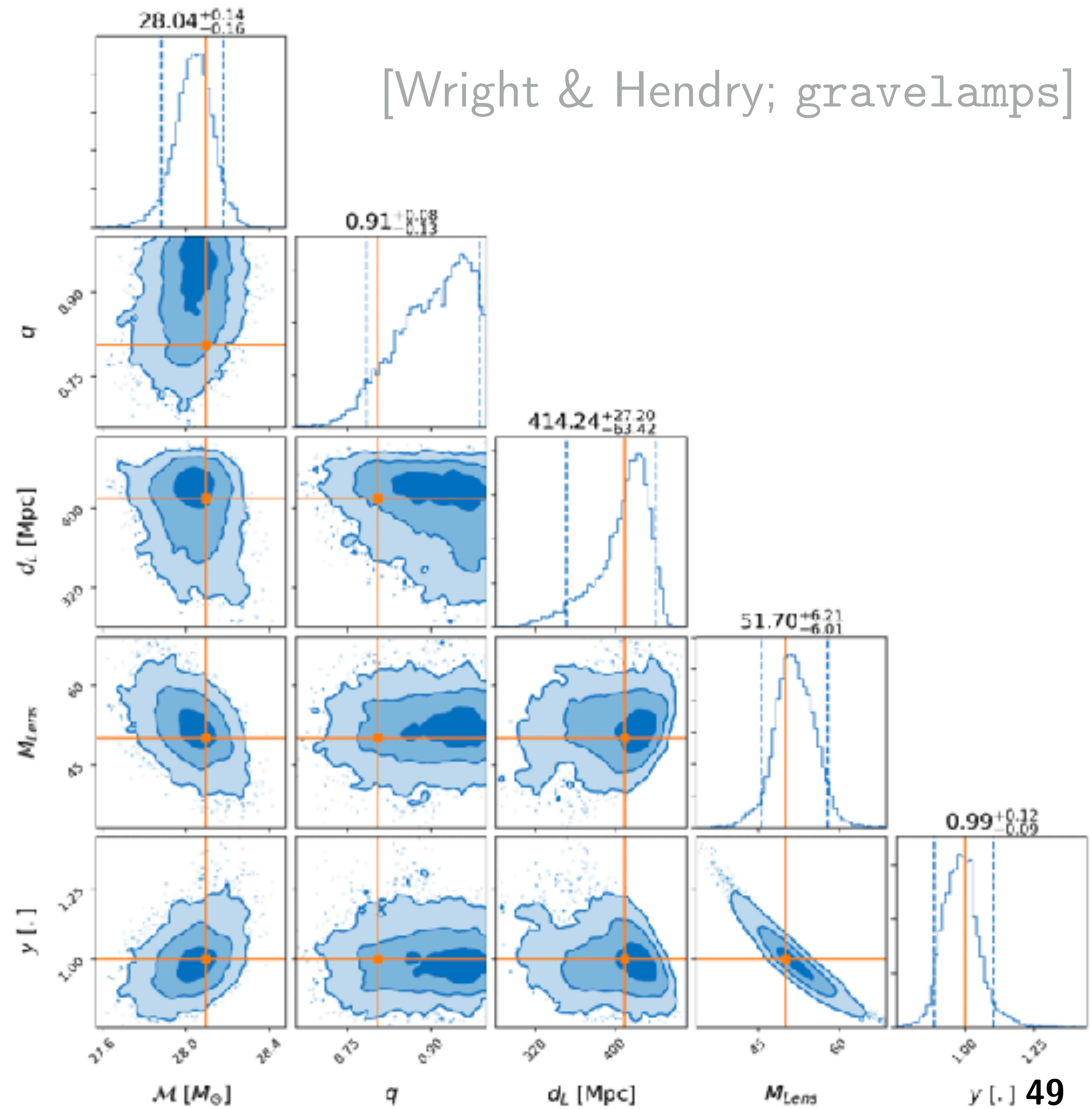
- Include lensed parameters in the inference

- Requires efficient models for the computation of the amplification function.

- Allows to make Bayesian model comparison ($B^L{}_U$)

- Addressing possible *waveform systematics* and *noise artifacts* is crucial!

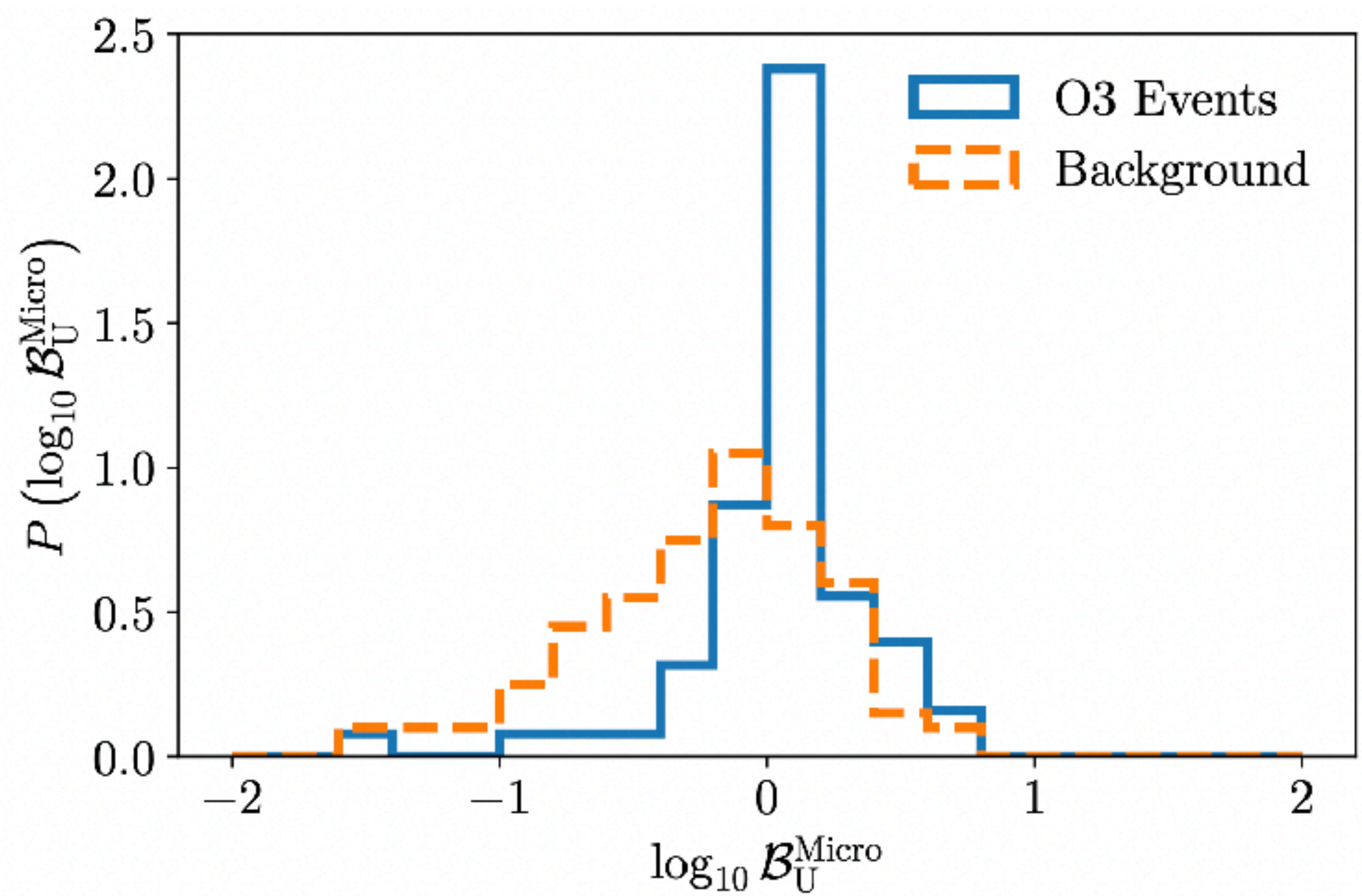
[Janquart et al.; MNRAS'23]



[Wright & Hendry; gravelamps]

GWTC-3 results

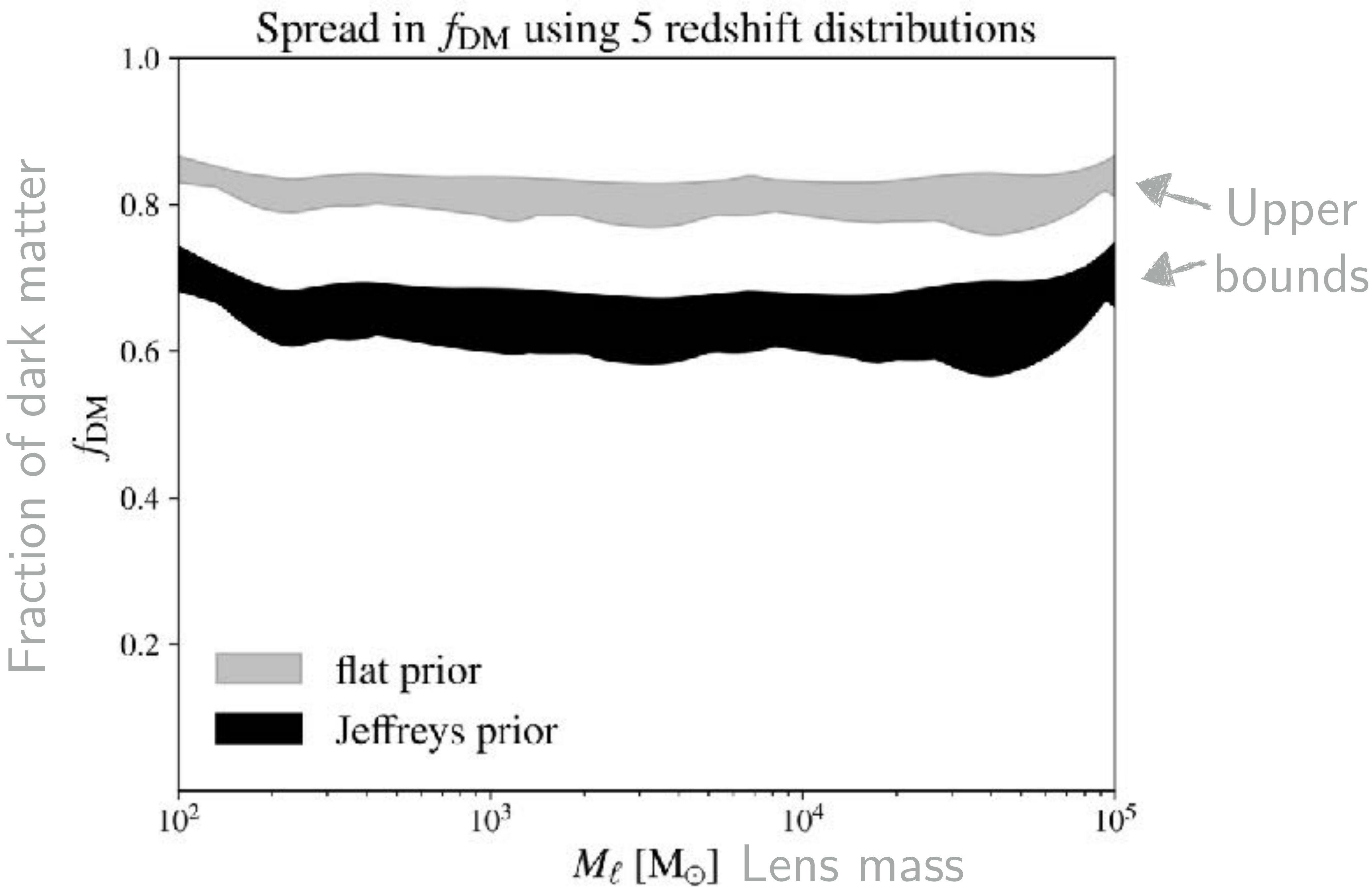
- No evidence of distorted waveforms by lensing ("microlensing")



[LVK lensing GWTC-2] [LVK lensing GWTC-3]

GWTC-3 results

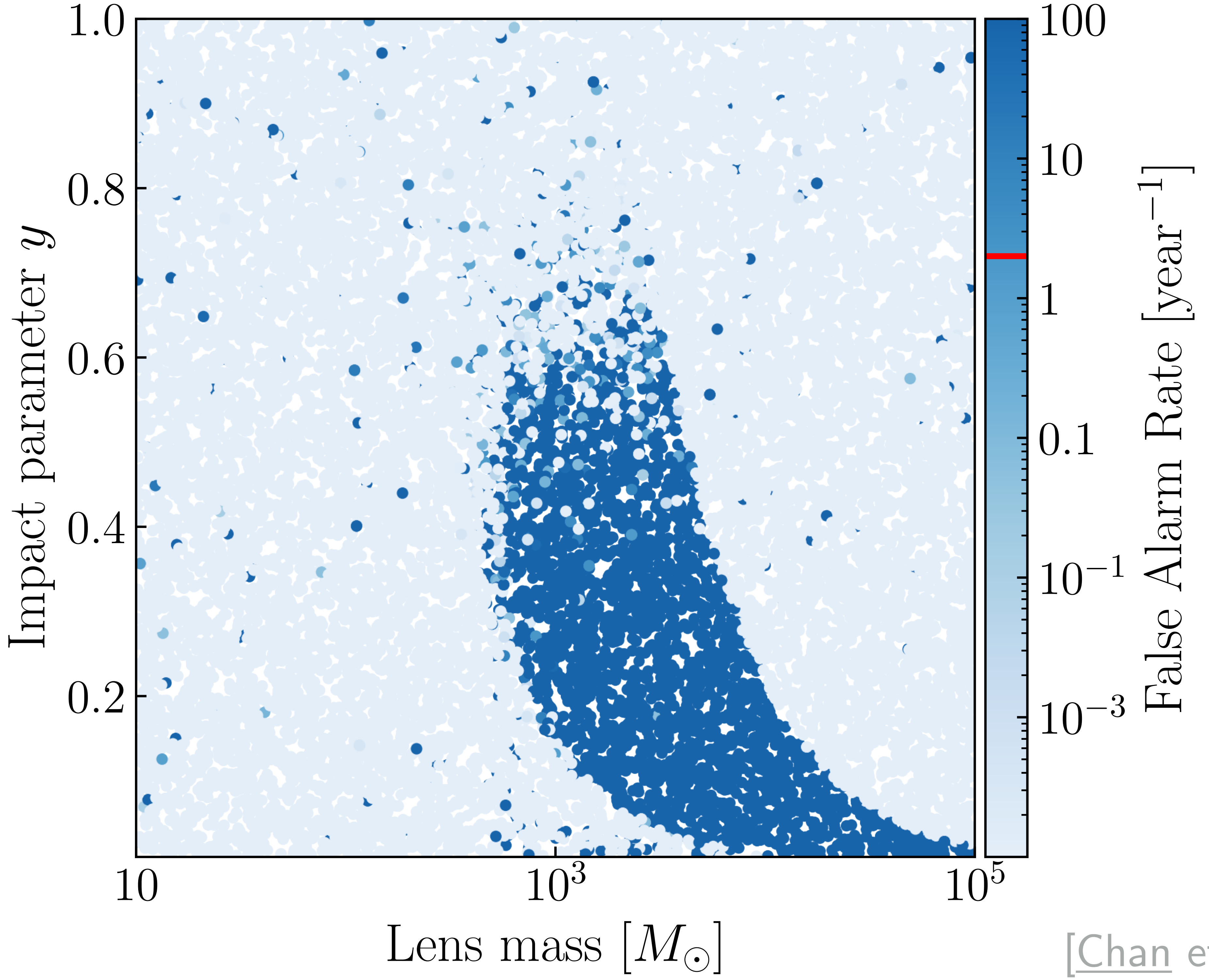
- *Upper bound* fraction of compact lenses (w.r.t. dark matter)



[LVK lensing GWTC-2] [LVK lensing GWTC-3]

Searching for lensed GWs

- Distorted waveforms could be missed by current searches!

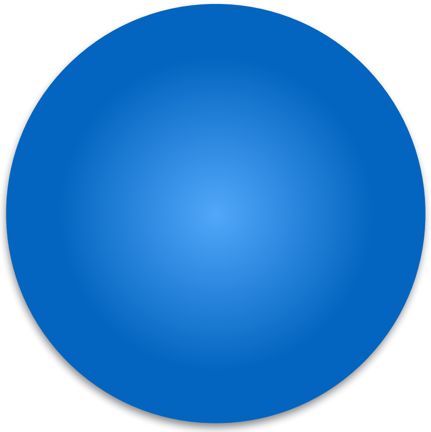


[Chan et al.; 2024]

4. Future prospects

Substructures

- Gravitational waves are effectively point sources. They are very sensitive to *small scales*

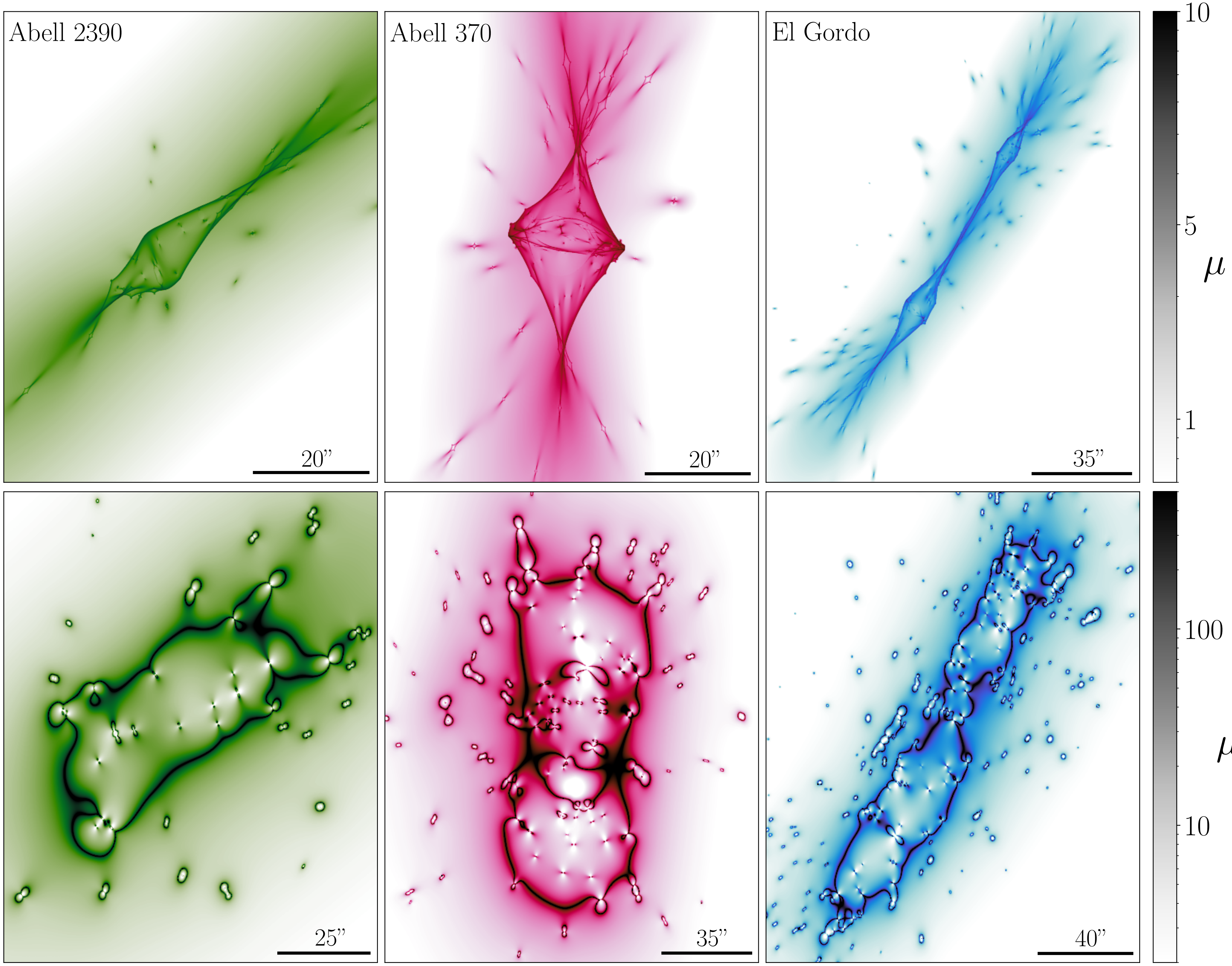


v.s.

[Singular Isothermal Sphere]

[Hubble Space Telescope]

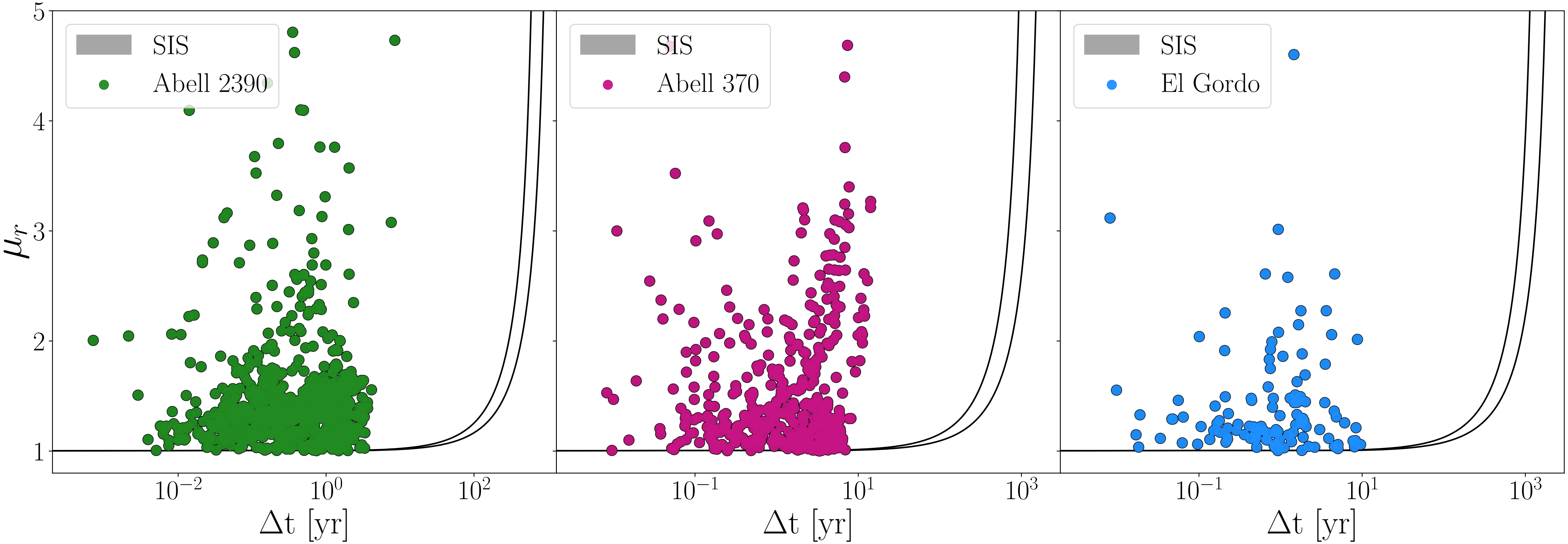
Substructures - clusters



[Vujeva, **Ezquiaga**, Lo, Chan; *to appear*]

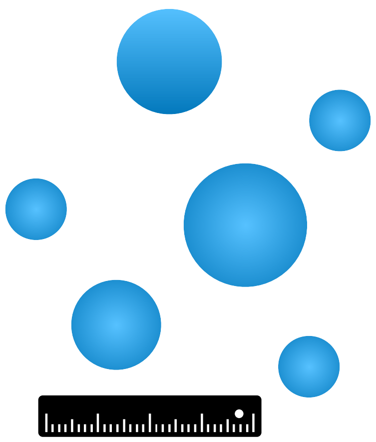
Substructures - clusters

- In real clusters, relative magnifications and time delays change dramatically compared to singular isothermal sphere (SIS)

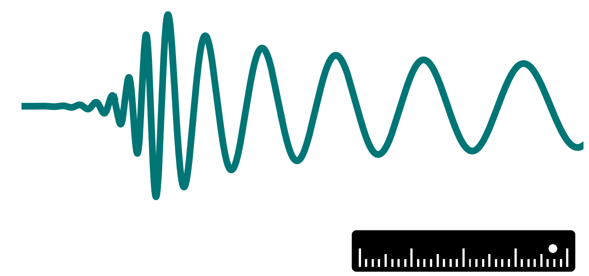


[Vujeva, **Ezquiaga**, Lo, Chan; *to appear*]

Substructures - subhalos

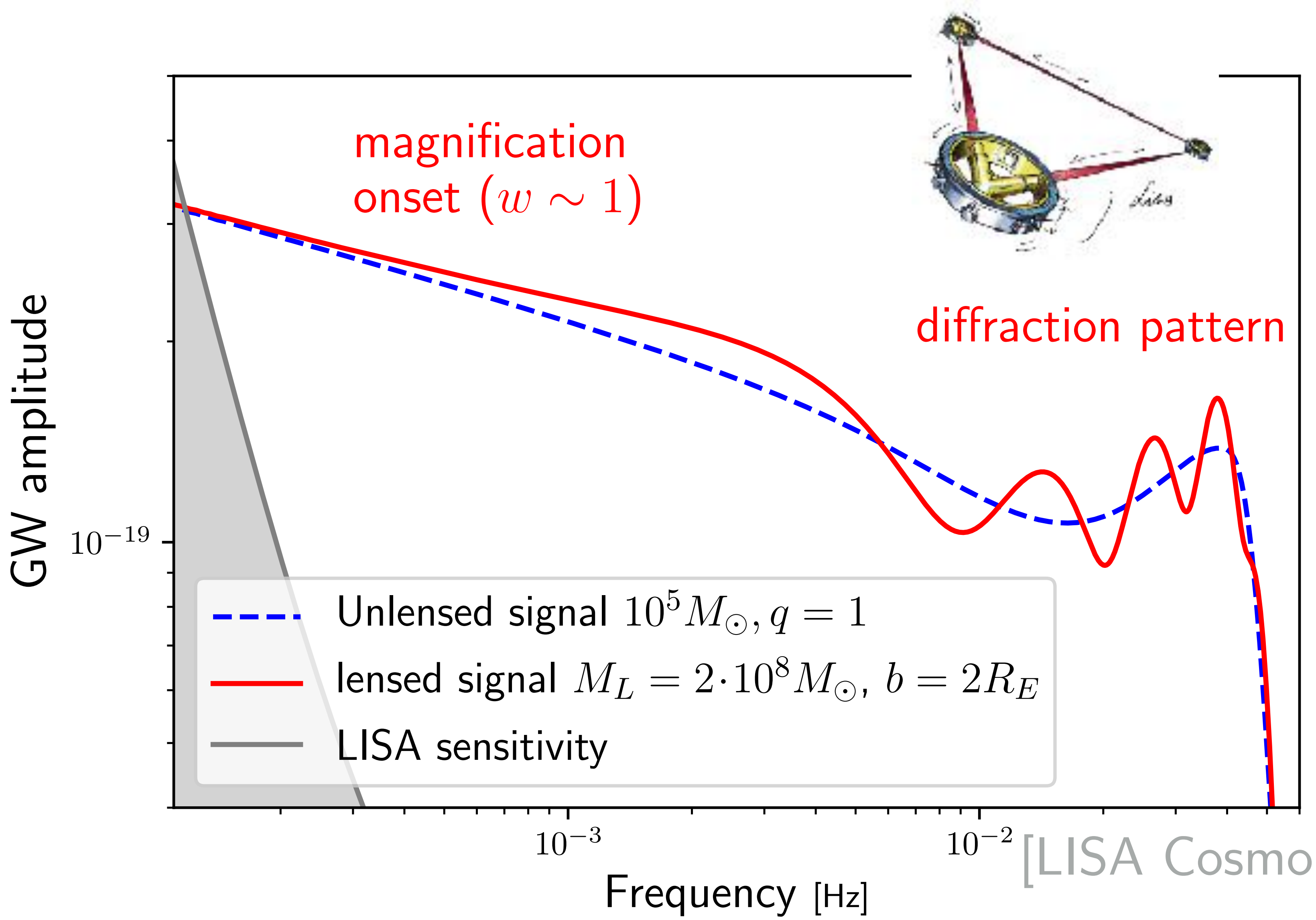


- Dark matter halos are made of smaller halos



- Gravitational waves could interfere!

$$\lambda_{\rm gw} \sim 10^3 {\rm km} \left(\frac{M_{\rm bbh}}{10 M_\odot} \right)$$

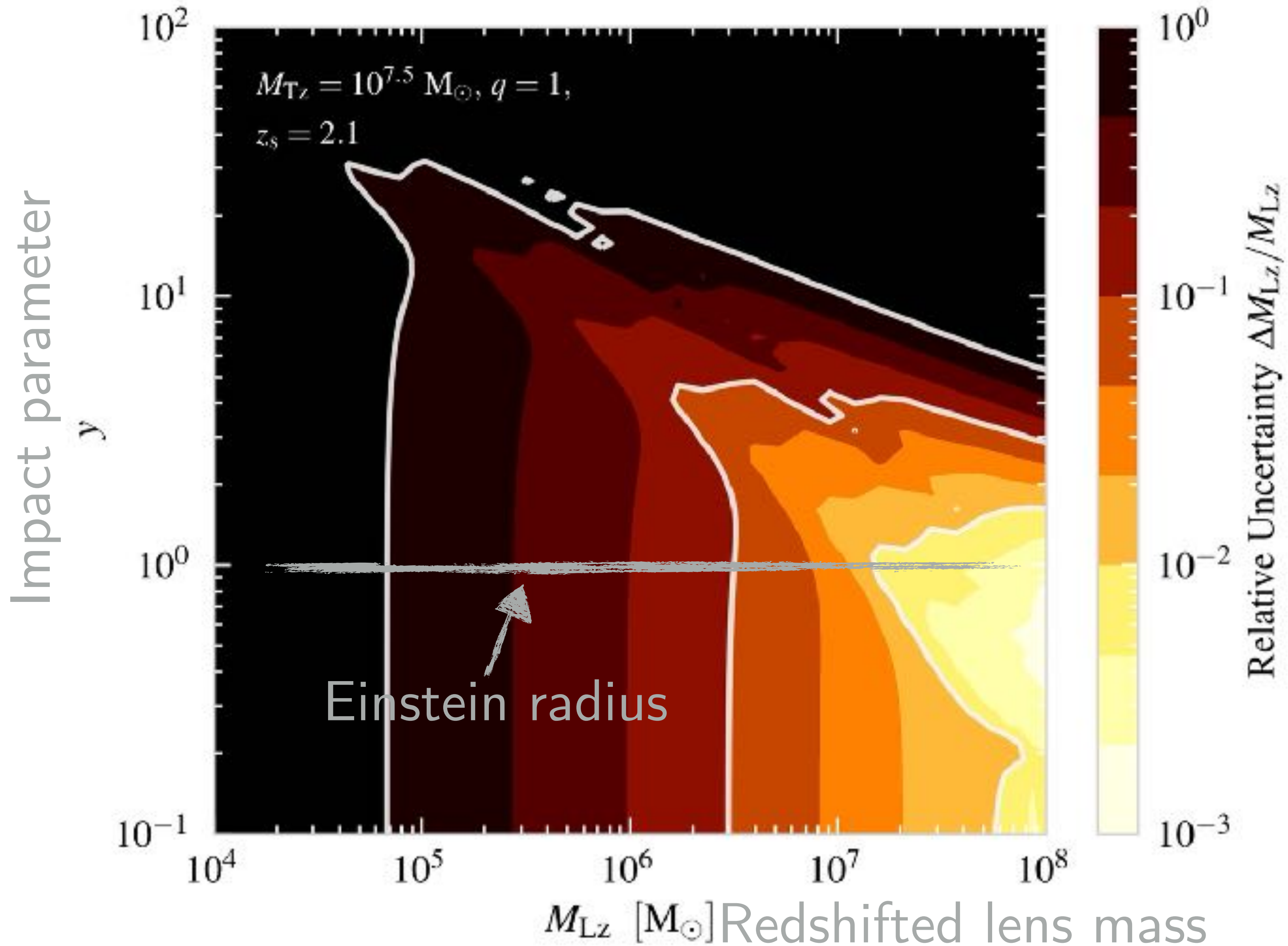


[LISA Cosmo white paper]

[see Goyal's talk]

Substructures - subhalos

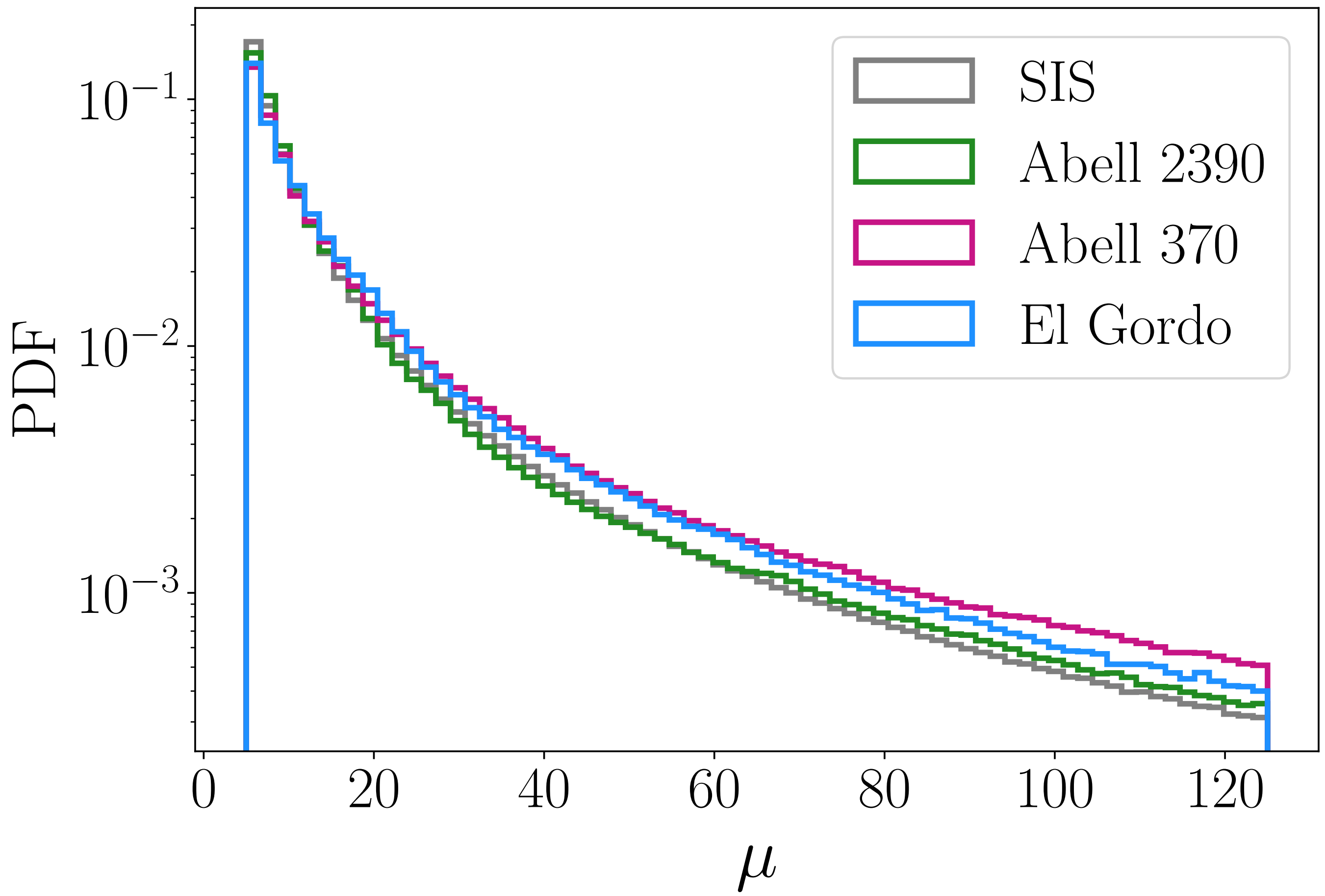
- *Sensitive to lensing beyond the Einstein radius!*



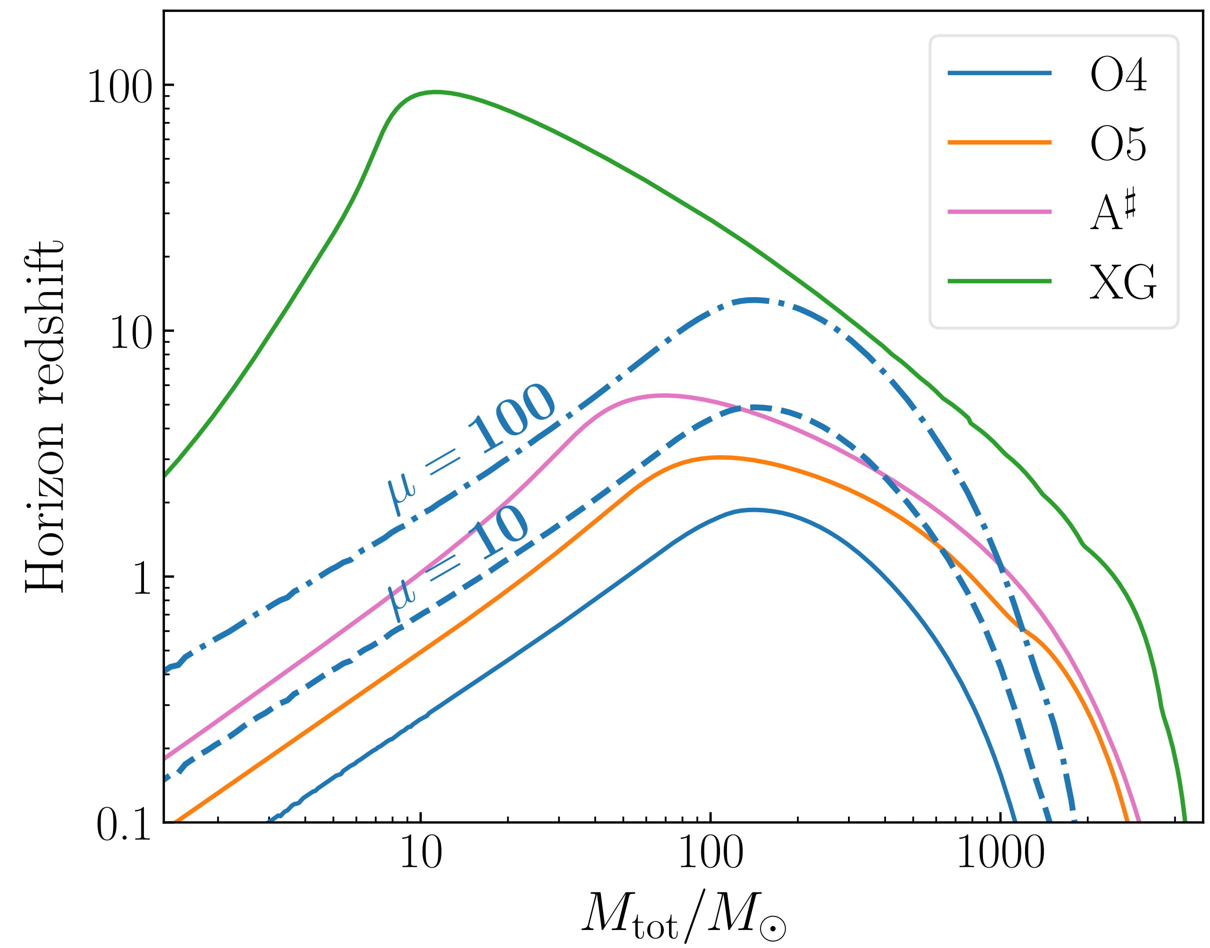
[Çalışkan *et al.*; PRD'23]

Highly magnified gravitational waves

- Substructures can enhance high magnification tail
- Even more sensitive to small lenses close to the caustics!



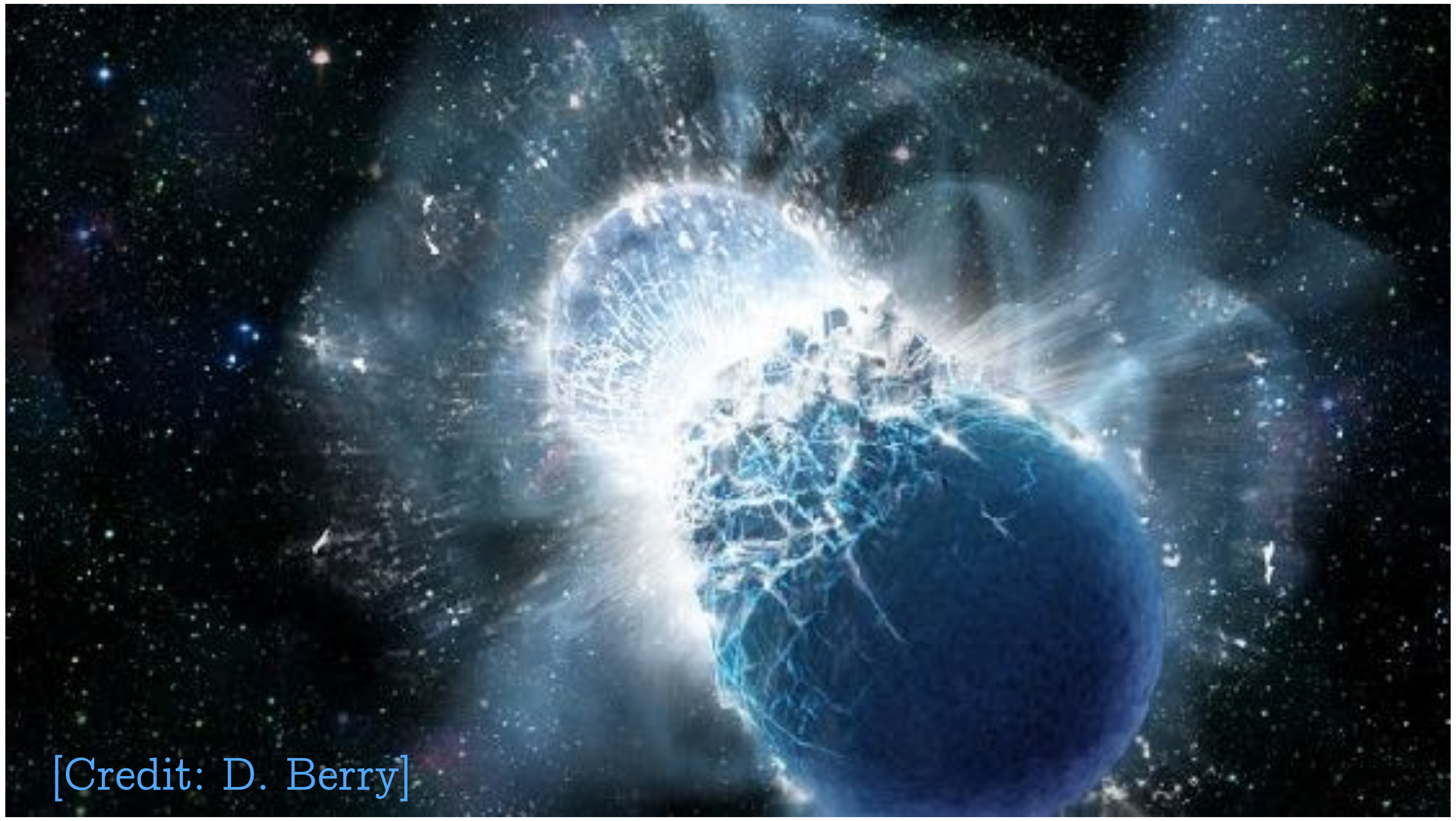
[Vujeva, **Ezquiaga**, Lo, Chan; *to appear*]



[Lo, Vujeva, **Ezquiaga**, Chan; 2024]

Multi-messenger lensing

- Observe multi-messenger lensed events, e.g. *binary neutron stars* with ground-based or *super-massive black hole binaries* with LISA



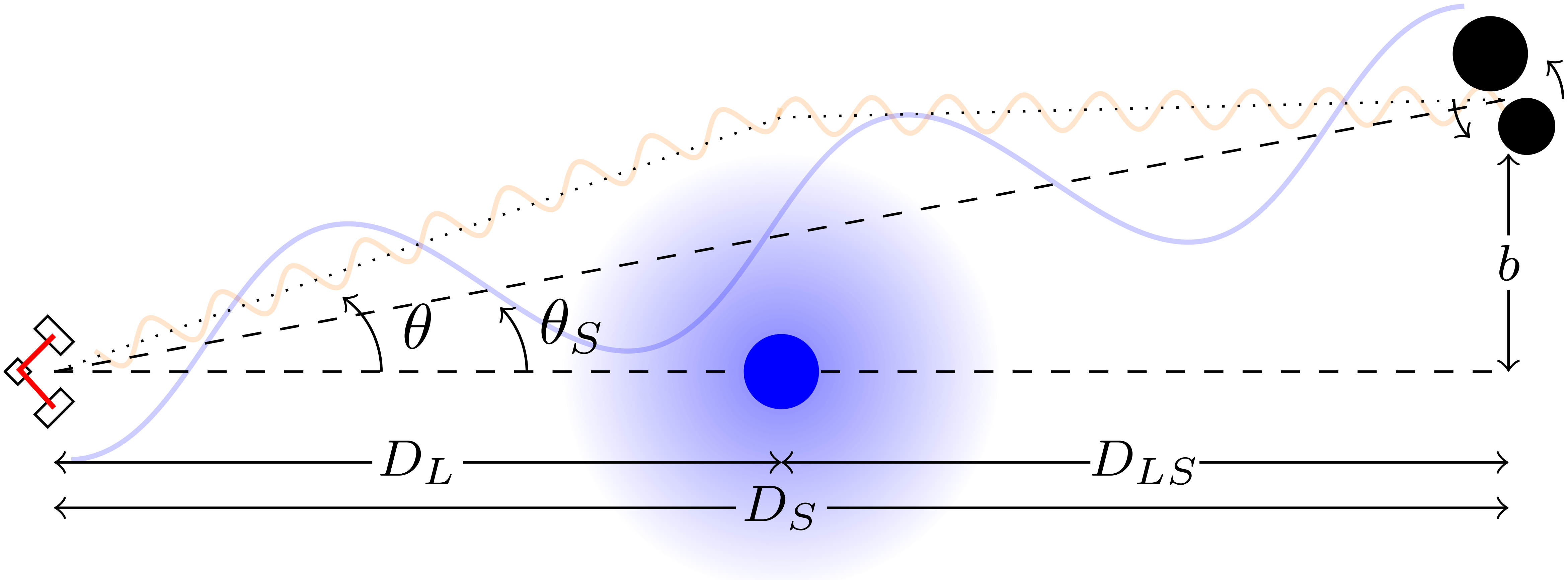
[see Smith's talk]

- Great target for future detectors!

- Will open many science cases

Multi-messenger lensing & wave optics

- Gravitational waves and photons could suffer lensing in different regimes



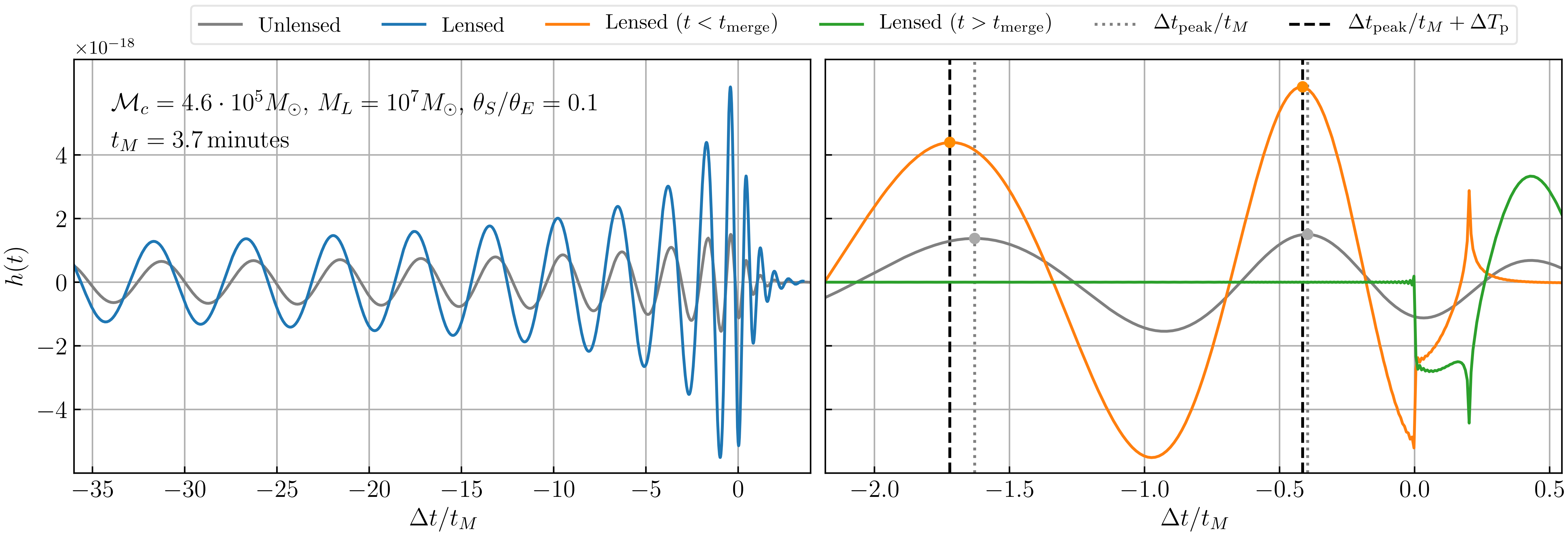
- *Phase* and *group velocity* may change in wave optics

$$t_p(\omega, \vec{\theta}_S) = -\frac{i}{\omega} \ln\left(\frac{F(\omega, \vec{\theta}_S)}{|F(\omega, \vec{\theta}_S)|}\right). \qquad t_g(\omega, \vec{\theta}_S) = t_p(\omega, \vec{\theta}_S) + \omega \frac{\partial t_p(\omega, \vec{\theta}_S)}{\partial \omega}$$

[**Ezquiaga**, Hu and Lagos; PRD'20]

Multi-messenger lensing & wave optics

- There is an *apparent superluminality* due to the waveform distortions

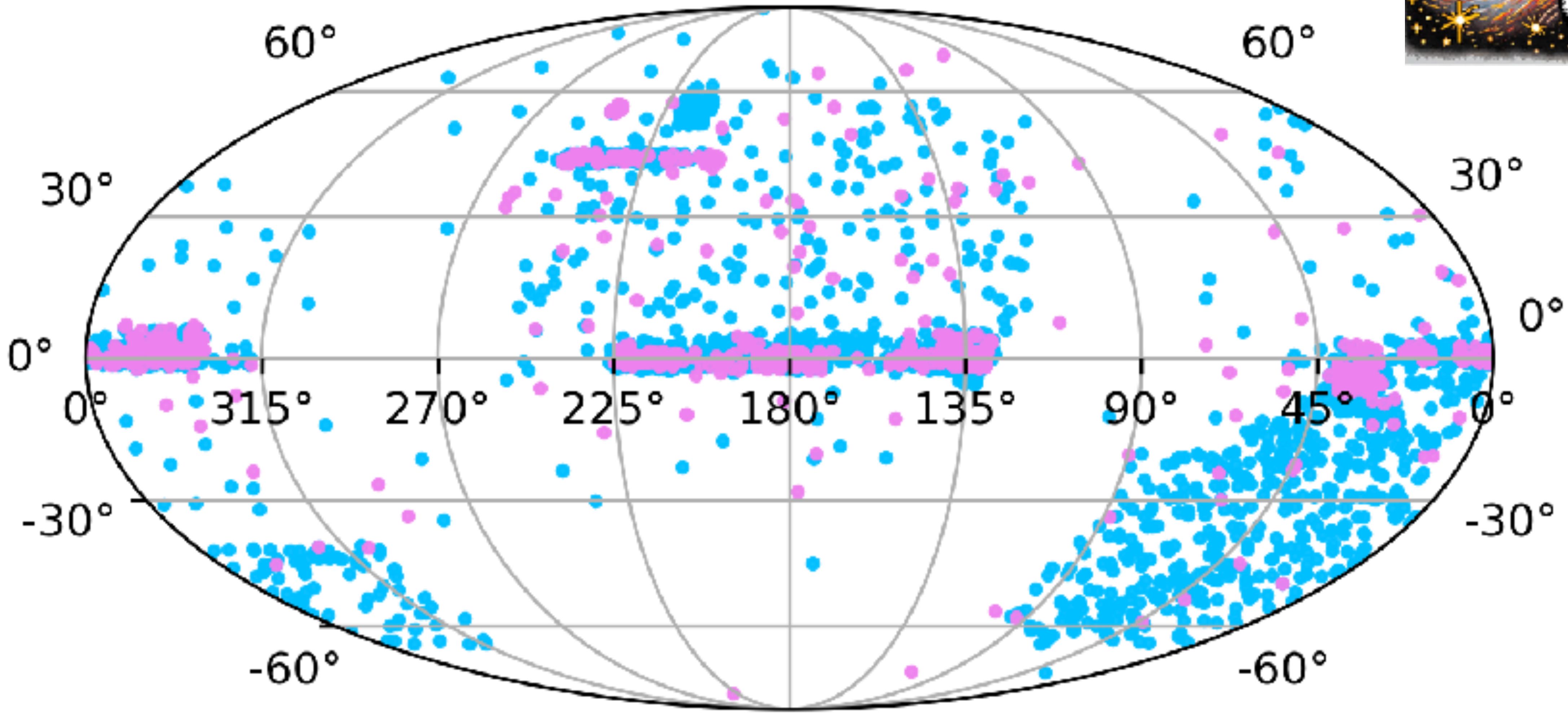


[example for super-massive black hole binary]

[**Ezquiaga**, Hu and Lagos; PRD'20]

Multi-messenger lensing

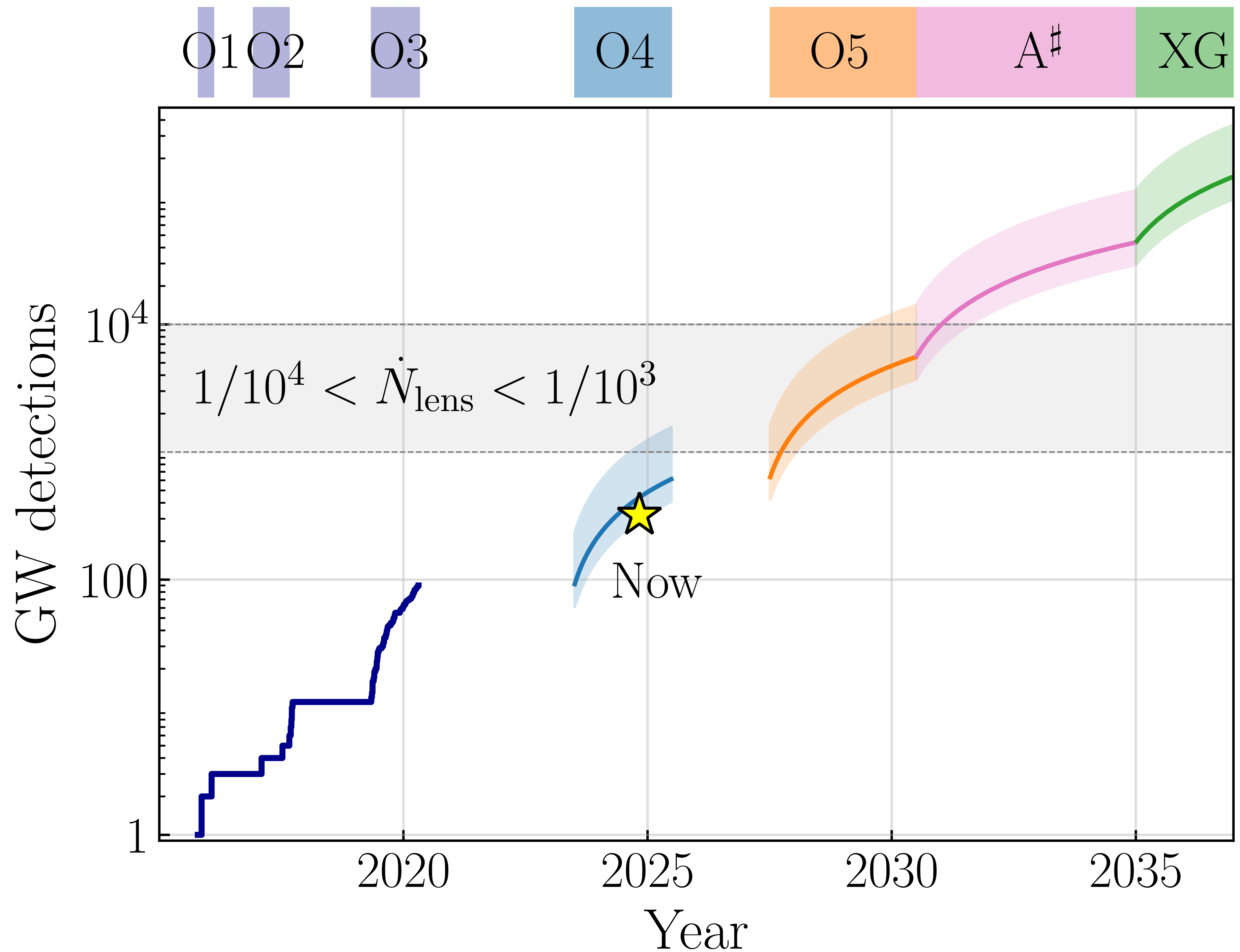
- Cross match GWs with *lens catalogs*
- Identify lensed host galaxy (*difficult!*)
- Watchlist for efficient lenses



https://github.com/lenscat/lenscat

GW lensing with next-generation detectors

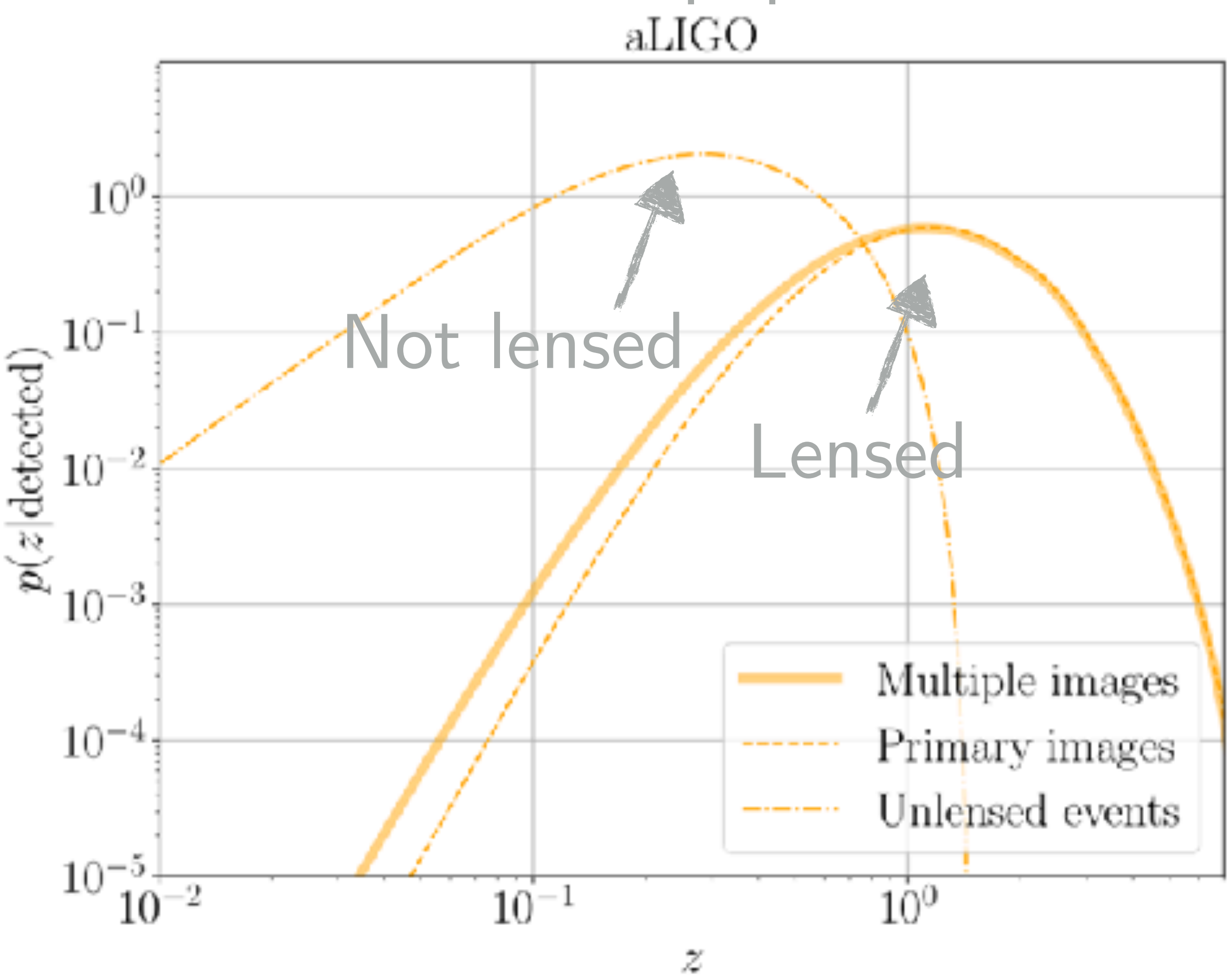
- Large number of detections enable statistical studies



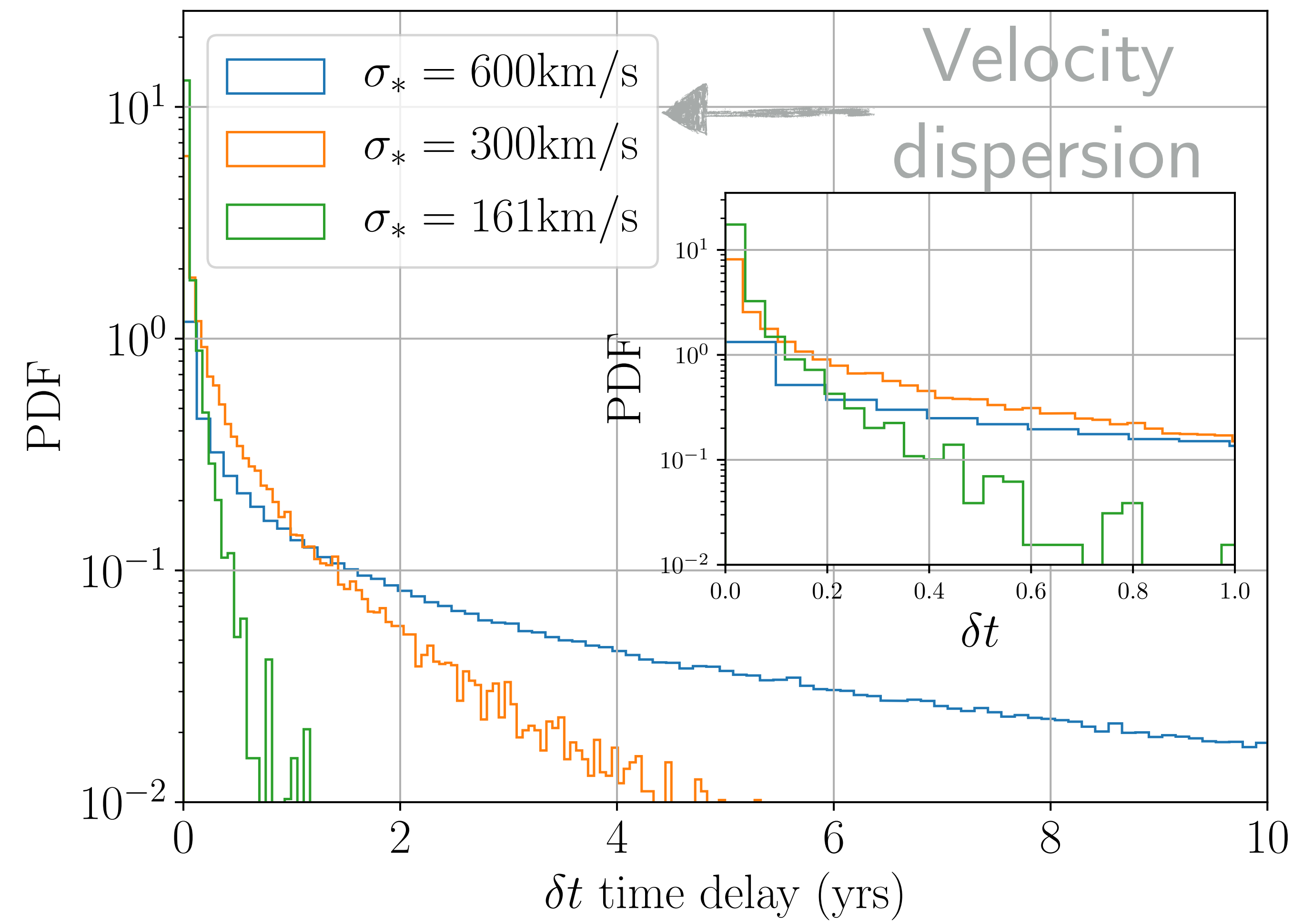
Populations & Cosmology

- Rates and time delay distributions inform about populations

Detected populations



Time delay distributions



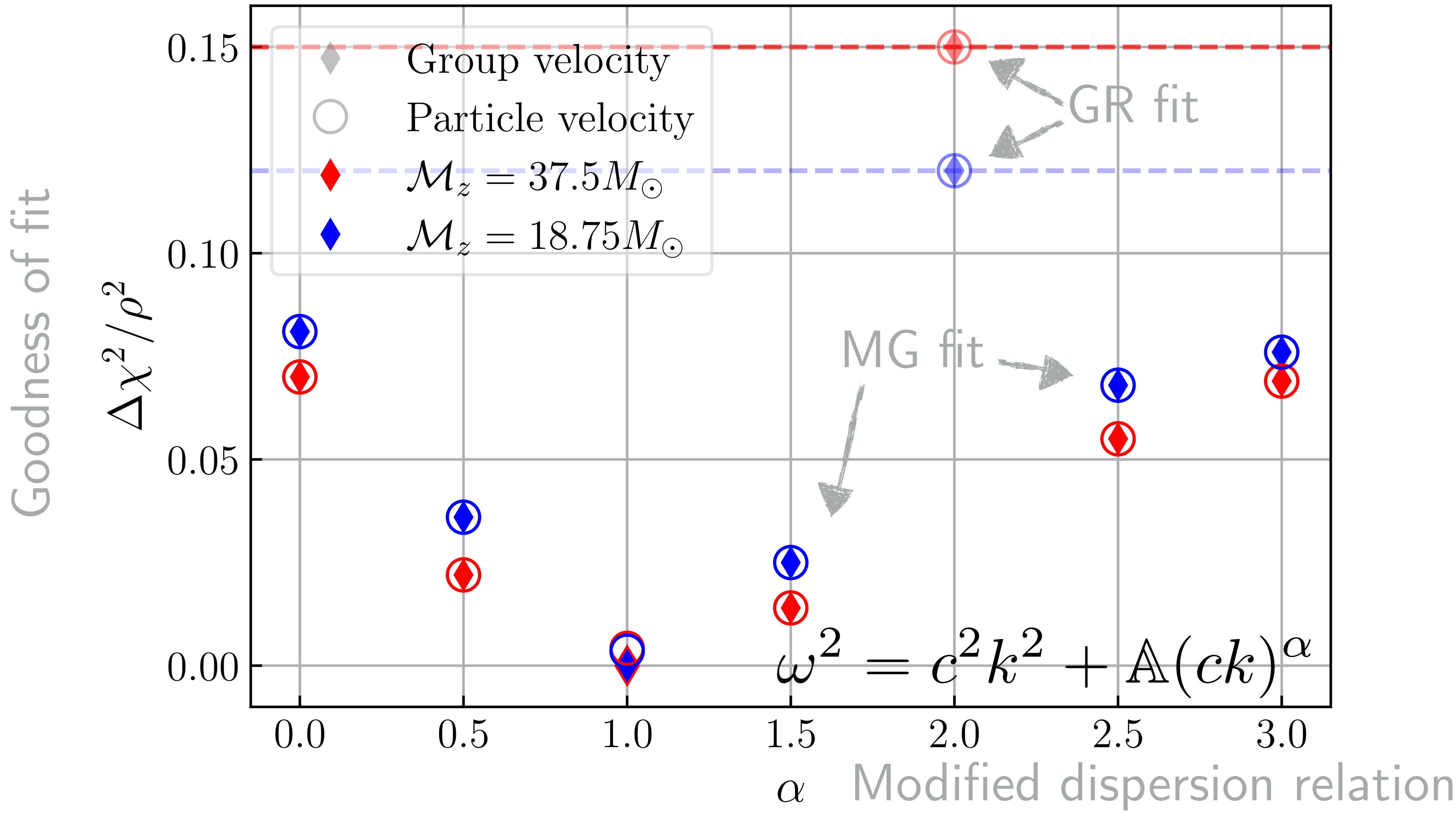
[Xu, **Ezquiaga**, Holz; ApJ'21]

- If you know the source and lens populations, rates and time delay distributions inform about cosmology [see Ajith's talk]

False violations of general relativity

- Lensed waveforms can be different from (unlensed) general relativity waveforms
- E.g. type II images

[**Ezquiaga**, *et al.*; JCAP'22]



Conclusions

Gravitational waves are precious cosmological probes:

- Well understood signals from general relativity
- Coherent detection of waveform
- Only distorted by gravitational lensing

- Current searches focus on repeated chirps and distorted waveforms
- No evidence so far, but first detections is *approaching*!

- Probing origin of the observed black holes and dark matter substructures with **gravitational wave lensing**

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VILLUM FONDEN

ezquiaga.github.io/joinus